Quantifying Uncertainty Costs in Renewable Energy Systems Considering Probability Function Behavior and CVaR at Low-Probability Generation Extremes using Deterministic Equations

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Abstract: - The increase and integration of renewable energy sources in electrical power systems implies an increase in uncertainty variables, both in costs and production, of economic dispatch (ED) and currently have a significant influence on wholesale electricity markets (MEM). Uncertainty costs refer to the quantification of additional expenses or economic losses associated with the variability inherent in the generation of renewable energy, such as wind, solar, or hydroelectric. Therefore, this article presents deterministic equations related to cost overestimation and underestimation, as well as CVaR, to model and evaluate the stochasticity of risks associated with the integration of renewable sources, allowing system operators and planners to make informed decisions. To mitigate or use said risks in energy systems with high penetration of elements, mainly smart networks. In this study, a mathematical analysis is carried out using the histogram spectrum formed by the power generated by the probability density function (PDF) for solar generation, although it is possible to consider other types of functions to determine energy generation. The objective of the proposed model is to provide another tool to the system operator for energy management and planning, which relieves a little of the weight of the computational load and at the same time presents more precision in the results by being able to work with a database. Historical data if these values are available. Commonly, for this type of analysis, values are estimated using probabilistic calculations by density functions when integrating these functions, or in other recent cases by estimating them by analytical methods of the same functions. A validation of the model is presented by comparing the result with the Monte Carlo simulation, developing the total cost of uncertainty only from "low probability generation extremes". Furthermore, the results are presented through analytical uncertainty cost functions (AUCF). This analysis includes the calculation of uncertainty costs for low and high-probability energy generation, determined by the Conditional Value at Risk (CVaR), using deterministic equations.

Key-Words: - analytical uncertainty, conditional value at risk, economic dispatch, histogram, low probability, mathematical modeling, Monte Carlo, probability density function, uncertainty cost, risk.

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1 Introduction

The integration of renewable energy sources into power systems is a crucial aspect of transitioning towards a more sustainable and resilient energy matrix. Renewable energy generation, such as solar and wind, possesses unique characteristics that can impact the stability and operation of electrical systems. These renewable energy sources (RESs) are becoming increasingly important to mitigate the environmentally harmful impacts of traditional energy sources. Despite the inherent advantages of pollution reduction and resource conservation, the inherent variability in renewable generation, influenced by factors like natural resource availability and weather conditions, poses challenges in the operation, and distribution of the power system and in terms of managing energy supply and demand, [1].

The main objective of using renewable sources in the main power system, in addition to reducing gas emissions and replacing conventional generation, is cost reduction. However, the use of a single renewable technology is not sufficient for the large daily demand, the same occurs if only a single main source of energy is used, due to its variability and imprecision in its generation.

The authors in [2] have suggested integrated scheduling of wind, thermal, and hydropower including spinning reserve. Several literature opt for sets of renewable sources, which is why the continuity of the electrical service is more efficient than opting only for solar and wind sources due to their uncertainty. In [3], the minimization of the power generation cost of the thermal power units is achieved by incorporating renewable sources, such as hydro, winds, and solar plants for 24 hours scheduled, and available transfer capability calculation is the prime objective. This mixture of non-conventional generations usually provides greater support to the network but is not exempt from improvements when replacing or adding other sources. Today there is no completely reliable method or technique for the integration of RESs into the network, however, several kinds of the literature raise the problem in the short, medium, and long term in real-time.

In [4], [5], they use the same methodology, however, they highlight the use of stochastic or deterministic optimization techniques. The article presents a short-term hydrothermal-wind complementary scheduling system (HTWCS) considering the uncertainty of wind energy, as well as several non-linear systems, formulated as a multi-objective optimization problem to optimize economic and environmental criteria. An improved multi-objective bee colony optimization (EMOBCO) algorithm is proposed to solve this problem, and the second paper a hydro-thermal-wind-solar hybrid energy system of the provincial power grid is taken as the background. A practical method for long-term coordination for this system is proposed.

In real-time economic dispatch (ED), due to accurate forecasting, the range of uncertainty is lower compared to the long-run scheduling, [6]. The economic dispatch of energy on power systems with high penetration of renewable generation is a mathematical problem of optimization. The paper [7], shows a mathematical analysis with probabilistic methods contrasted with an analytic development for controllable renewable systems to be included in the target functions of economic dispatch problems.

In [8], analytical formulas of uncertainty penalty costs are calculated, for solar and wind energy and electric vehicles, through a mathematical expected value formulation.

The integration of renewable sources into the grid opens the way to various methods for their interconnection without affecting the stability of the main system, as well as new concepts such as costs due to the overestimation and underestimation presented by the uncertainties of renewable sources. The authors in [7], [8], [9] and [10], work on this concept analytically, improving the time and precision of the results that are commonly worked on by mathematical methods such as the Monte Carlo Method supported by the functions proposed for each generation. They present an analysis in the development of a new mathematical formulation with which it will be possible to determine, through probabilistic approaches, the cost that can be generated if a diversified electricity market exists, in which the demand can actively participate.

Uncertain behavior of renewable generation plants and PEV modeling can be done by probability distribution functions (PDFs), as shown in [11], where the wind speed for the plants was modeled by the Weibull PDF, and the solar irradiance was modeled by a lognormal distribution.

Probability Density Functions (PDFs) for determining solar and wind power are statistical tools that describe the distribution of energy generated by renewable sources based on the probability of occurrence. These functions allow modeling and predicting the variability in solar and wind energy production, which is crucial for the design and efficient management of renewable energy systems.

This article mentions normal and log-normal PDF for solar energy estimation and Weibull PDF for wind energy estimation. The main objective is the determination of the costs of uncertainties through a deterministic method, using as a calculation basis the histogram generated by the powers that are determined by the previous functions, which can be replaced by a historical database for greater precision, helping Thus, the system operator is responsible for the management and planning of costs and powers associated with a certain risk that is determined by the CVaR.

The commonly used probability density functions for wind and solar energy are presented in Section 1. Section 2 presents the formulation of the problem taking into account the uncertainties of costs and powers, and the conditional risk value. The Monte Carlo method and Analytical Low-Probability Generation Extremes are presented in Section 3, being the simulation and case study. Finally, Section 4 presents the conclusions due to the results shown in the analysis.

1.1 Normal and Log-normal PDF

In the context of analyzing solar power generation, probability density functions are essential for understanding the uncertainty associated with solar irradiance and power output. [12], utilized log-normal probability distribution functions for solar irradiance and normal distribution functions for the loading and unloading behavior of plug-in electric vehicles (PEVs) to develop uncertainty cost functions.

The article [9], develops a model with a normal distribution (eq. 1) power, presenting the validation for the uncertainty cost factor (UCF) by comparing the Monte Carlo simulation with the analytical proposal.

$$f(P) = \frac{1}{\sqrt{2\pi\phi^2}} * e^{\left(\frac{P-\mu}{\sqrt{2}\,\phi}\right)^2} \quad (1)$$

where f is the PDF of the demand, electric vehicle or solar power, P represents the power of the previous variables, μ and ϕ are the mean and standard deviation respectively of probabilistic behavior.

Several investigations have been done to find the probability distribution of irradiance, being the primary resource of solar energy photovoltaic. The function that best describes the behavior of this font is the function Log-normal probability distribution as [10] (eq. 2).

$$f_G(G) = \frac{1}{G\beta\sqrt{2\pi}} * e^{-\frac{(ln(G)-\lambda)^2}{2\beta^2}} \quad (2)$$

where f_G is the PDF log-normal of the irradiation, G represents the solar irradiance, λ and β are the mean and standard deviation respectively of probabilistic behavior.

1.2 Weibull PDF

The Weibull distribution is commonly used to model wind speed data for wind energy applications. The Weibull parameters, shape and scale, can be estimated using various numerical methods to characterize the wind resource at a given location. Shape and scale factors are commonly values already estimated by previous analyzes due to historical databases for the simplicity of wind generation studies. The application of Weibull distribution in wind data assessment can be extensively found, but the methods applied for estimating the parameters still need improvement.

According to the Weibull PDF with a shape factor (β) and scale factor (α), the wind speed distribution can be modeled as follows, as specified in [1], [13], [14]:

$$f_\upsilon(\upsilon) = \left(\frac{\beta}{\alpha}\right)\left(\frac{\upsilon}{\alpha}\right)^{(\beta-1)} e^{-(\upsilon/\alpha)^\beta}; \quad for\ 0 < \upsilon < \infty \quad (3)$$

where υ is the wind speed (m/s).

In order to get the proposed uncertainty cost functions, probability distribution functions (PDF) of the energy primary sources are considered: log-normal distribution for solar irradiance PDF, Rayleigh distribution for wind speed PDF and normal distribution for loading and unloading behaviour PDF of electric vehicles.

2 Problem Formulation

Renewable energies that are dependent on environmental factors, mainly such as solar irradiation and wind speed, present variability and uncertainty in energy production. Therefore, when this condition occurs in energy generation, operating costs are also related. Uncertainty costs quantify the variability that renewable sources introduce to the main system.

2.1 Uncertainty Powers

In the present study, it is assumed that the generation units can be operated by companies or by the end-users themselves, they are responsible for both energy production and buying/selling. This article presents an analysis of operational costs associated with the tails of probability distribution curves for each generation unit, specifically focusing on power levels with lower probability of occurrence. Depending on the probability density function (PDF) used, these power levels are influenced by parameters such as mean (μ) and standard deviation (σ), along with the conditional value-at-risk (CVaR).

2.1.1 Direct Cost and Uncertainty Solar Power

The direct cost function for the solar power plant is determined by the following expression.

$$C_{pv,i}(P_{pvs,i}) = C_o | C_u * P_{pvs,i} \quad (4)$$

where $P_{pvs,i}$ and $C_{w,i}$ are the scheduled power and direct cost coefficients of the ith solar power plant, respectively. The constants Co and Cu represent the costs associated with the uncertainty of solar energy.

The solar irradiance distribution can be modeled correctly using a lognormal PDF. Using the mean (μ) and standard deviation (σ) of the lognormal PDF, energy conversion for solar PV is defined in the following equation (2).

$$P_{pv}(G_s) = \begin{cases} P_{pvr}\left(\frac{G_s^2}{G_{std}R_c}\right), & for\ 0 < G_s < R_c \\ P_{pvr}\left(\frac{G_s}{G_{std}}\right), & for\ G_s \geq R_c \end{cases} \quad (5)$$

where R_c is the irradiance point, G_{std} is the solar irradiance in a standard environment, G_s is the available actual solar, and P_{pvr} is the rated power of the solar PV.

2.1.2 Direct Cost and Uncertainty Wind Power

The direct cost function for the wind power plant is determined by the following expression.

$$C_{w,i}(P_{ws,i}) = C_o|C_u * P_{ws,i} \quad (6)$$

where $P_{ws,i}$ and $C_{w,i}$ are the scheduled power and direct cost coefficients of the ith wind power plant, respectively. The constants C_o and C_u represent the costs associated with the uncertainty of wind energy, where their usage is determined by the wind power equation (2).

$$p_w(v) = \begin{cases} 0 & v < v_{in}\ and\ v > v_{out} \\ p_{wr}\left(\frac{v - v_{in}}{v_r - v_{in}}\right) & v_{in} <= v <= v_r \\ p_{wr} & v_r < v <= v_{out} \end{cases} \quad (7)$$

The wind speed distribution can be modeled correctly using a Weibull PDF. Therefore, for the estimation of wind power, it is determined using scale parameter (*c*) and shape parameter (*k*) or in real cases by historical data. For the case study, random values determined by *k* and *c* were used.

2.2 Uncertainty Cost Formulation

To define the behavior of the generation of some renewable source or demand in terms of probability distribution functions, the Log-normal function, Weibull distribution function, normal function [7], [8], and the beta function [9], commonly related to solar, wind generation and demand prediction, respectively.

The proposed deterministic method is presented to determine the penalty costs for each case, considering the risk conditioned at 10% and 90%, related to the cost of overestimation and underestimation, respectively.

2.2.1 Penalty Cost due to Underestimate

Penalty costs due to underestimation depend on the perspective of the operator. These costs arise when the actual generation power from a renewable source exceeds the programmed power value of the plant, leading to energy that cannot be delivered to the grid. Additionally, penalty costs occur when the electric power generator, typically the main grid, is unable to meet the energy demand of the users. Therefore, it is essential to apply penalties to costs associated with overestimating the available renewable energy or failing to meet demand requirements.

In summary, if the generating unit provides a larger amount of actual energy than the scheduled power (8), the surplus power may be unused, and the grid operator is liable for the penalty cost. The penalty cost associated with the surplus can be referred to as follows (9):

$$P_{a,i} > P_{s,i} \quad (8)$$

$$E[C_{u,i}(P_{s,i}, P_{a,i})] = C_{u,i} * (P_{a,i} - P_{s,i}) \quad (9)$$

Presenting the corresponding function for each unit depending on the natural resource, the cost is (10):

$$E[C_{u,i}(P_{s,i}, P_{a,i})] = C_{u,i} * \int_{P_{s,i}}^{P_{ar,i}} (P_{a,i} - P_{s,i})\, f_P(P_{a,i})\, d\,P_{a,i} \quad (10)$$

2.2.2 Penalty Cost due to Overestimate

Due to the intermittent and uncertain nature of primary sources (irradiation or wind) for energy production, there is a possibility that the generating unit will not be able to generate scheduled power. If the actual power supplied by the renewable generation unit is less than the scheduled power by the operator (11), the system will require reserve energy sources to maintain supply continuity to consumers. Therefore, the penalty cost due to energy scarcity should be assumed by the reserve units and can be defined as follows (12):

$$P_{a,i} < P_{s,i} \quad (11)$$

$$E[C_{o,i}(P_{s,i}, P_{a,i})] = C_{o,i} * (P_{s,i} - P_{a,i}) \quad (12)$$

Presenting the corresponding function for each unit depending on the natural resource, the cost is (13):

$$E[C_{o,i}(P_{s,i}, P_{a,i})] = C_{o,i} * \int_{P_{s,i}}^{P_{ar,i}} (P_{s,i} - P_{a,i})\, f_P(P_{a,i})\, d\,P_{a,i} \quad (13)$$

2.3 Conditional Value at Risk (CVaR)

The development of an electricity system with high penetration of energy from renewable resources requires considerable flexibility to cover the risk of energy curtailment and shortages, [15]. This paper explores how the system generation portfolio of a pool of diverse renewable sources can be appropriately designed to balance overall planning costs and operational flexibility constraints. The proposed study is basically based on the production of renewable energy with little probability presenting some risk when interacting with the main grid. An index based on the conditional value at risk (CVaR) method is introduced to quantify the risk of any renewable source, being a parameter or parameters necessary to determine or plan the needs of the system, for example, the capacity or need of a energy storage system.

The CVaR is a tool of risk measurement, compared with the VaR (risk value), it only considers the risk information under confidence level, while the risk information behind the confidence level is ignored. The CVaR measures the average loss behind the confidence level, and the inclusion of tail risks can better reflect the portfolio risks. The framework of the CVaR is demonstrated in the Figure 1.

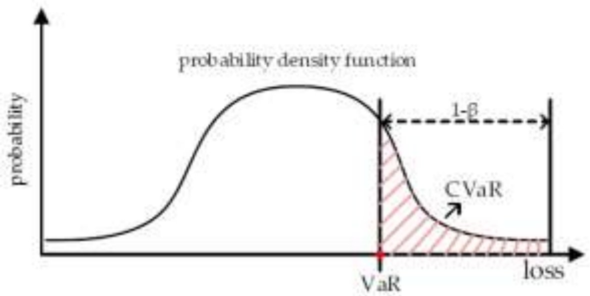

Fig. 1: Framework of the conditional risk value (CVaR), [16]

VaR can only determine a risk situation under the given confidence level and doesn't consider the risk tail, so there are certain limitations in its practical applications, [17].

$$\alpha_\beta(x) = min\left\{\alpha \in R; \int_{f(x,y)\leq\alpha} \rho(y)dy \geq \beta\right\} \tag{14}$$

Denote the loss function as f (x, y), where x and y denote the probability density function of the decision variable and the random variable, respectively. The probability density function of y is defined as ρ(y), then the VaR value at confidence level is given as α_β.

$$\phi_\beta(x) = \frac{1}{1-\beta} \int_{f(x,y)\geq\alpha_\beta(x)} f(x,y)\rho(y)dy \tag{15}$$

The Conditional Value at Risk (CVaR) can describe the distribution of risk outside the confidence level. The equation (15) describes the VaR ($\alpha_\beta(x)$) value and CVaR ($\phi_\beta(x)$) value of the portfolio problem, where $\phi_\beta(x)$ is the CVaR value when the loss is greater than $\alpha_\beta(x)$.

3 Study Case and Simulation

3.1 Monte Carlo Simulation

Monte Carlo (MC) simulation uses random sampling and statistical modeling to estimate mathematical functions and mimic the operations of complex systems. The MC method has gained widespread acceptance for validating physical models involving variables with associated probability density distributions (e.g., solar radiation) [18], [19]. Through Monte Carlo simulation, the behavior of overestimation and underestimation instances was studied for a predetermined power value (Ps), considering randomly generated values of solar irradiance and wind speed, generated by the log-normal, normal, and Weibull distribution functions, respectively. The random generation of these values will be used to obtain the generation powers of the unit in question.

The outcome will establish values within the generated scenarios (N=100000) for the Cost Overestimation (Co) and Cost Underestimation (Cu), depending on the average power value (Ps).

$$E[C_{o,i}(P_{s,i}, P_{r,i})] = C_{o,i} * (P_{s,i} - P_{r,i}) \tag{16}$$

$$E[C_{u,i}(P_{s,i}, P_{r,i})] = C_{u,i} * (P_{r,i} - P_{s,i}) \tag{17}$$

Equations (1), (2) and (3) represent the probability functions that will be used for the determination of the generated powers for each renewable source. The obtained values were considered for practical purposes using the MatLab software commands "lognrnd" and "wblrnd". For this framework, Table 1 and Table 2 shows the initial values for the simulation were:

Table 1. PV Solar

Rated power output	(Psr)	65 MW
Scheduled power	(Ps)	20 MW
Solar irradiation std	(Gstd)	1000 W/m^2
Certain irradiation	(Rc)	150 W/m^2
Maximum power	(Wmax)	100 MW
Mean	(μ)	6
Standard deviation	(σ)	0.25

After an elapsed simulation time of around 2.1644 second for solar power and 3.4711 second for wind power, multiple statistical parameters were obtained. It includes expected values and variances associated with the different cost functions that were modeled for the photovoltaic (PV) and wind turbine (WT) generation.

Table 2. WT Wind

Rated power output (Wsr)	150 MW
Scheduled power (Ws)	20 MW
Cut-in speed (Vi)	5 m/s
Rated output speed (Vr)	15 m/s
Cut-out speed (Vo)	45 m/s
Linear coefficient of the WT (a, b)	(15, -75)
Rayleigh distribution scale parameter (σ)	15.9577
Weibull Scale Parameter (√2* σ)	22.5676
Weibull Shape Parameter (k)	2

Figure 2 and Figure 3 show the results obtained by MC for solar and wind power, respectively. The costs of overestimation and underestimation present a certain relationship regarding their technology or power curves.

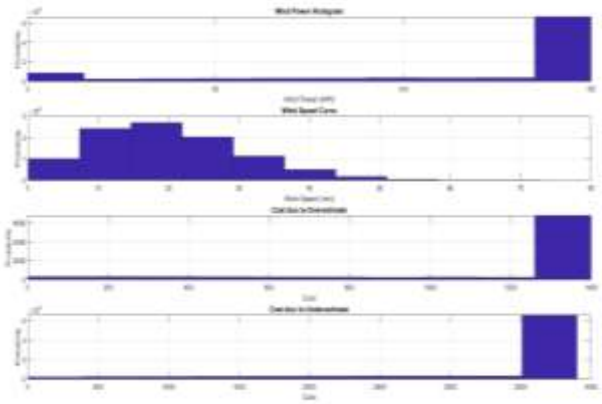

Fig. 2: MC simulation for Solar power

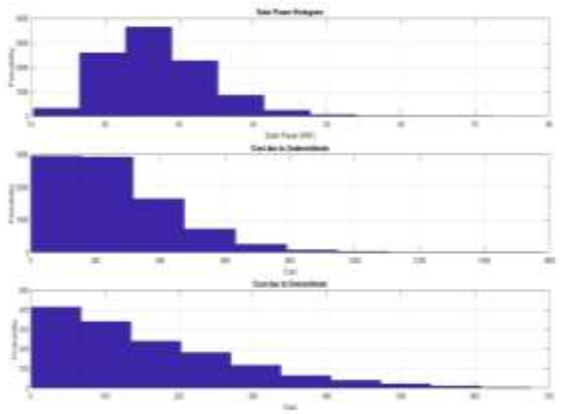

Fig. 3: MC simulation for Wind power

Figure 4 and Figure 5 show the solar and wind power values, respectively.

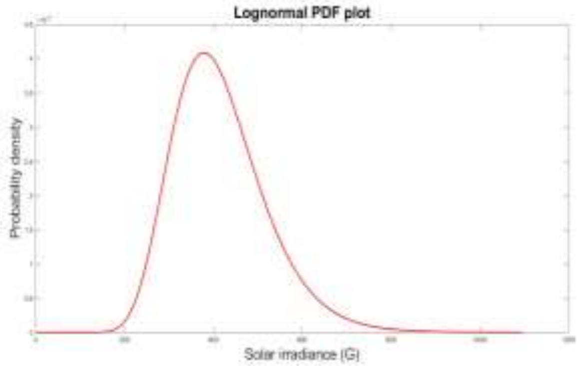

Fig. 4: Solar irradiation

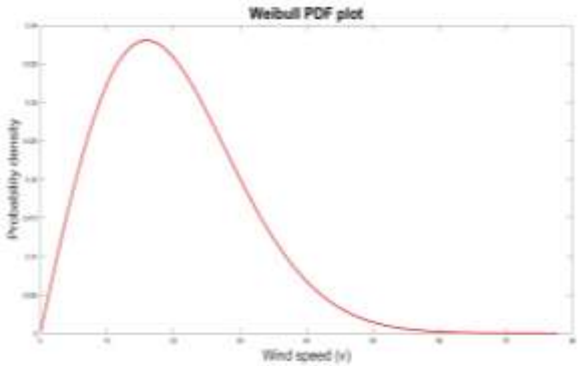

Fig. 5: Wind speed

3.2 Analytical Low-Probability Generation Extremes in Solar power and Wind Power

The equations (5) determine the power generated about the irradiance of the place. However, CVaR values are previously calculated (15) to determine the risk points of interest.

The confidence level plays a very important role in the calculation of CVaR, which determines the minimum and maximum values of the data to be analyzed. In this work, only the data related to CVaR will be considered, obtaining the cost of overestimation and underestimation determined by the scheduled power.

For this analysis, the points of interest are related to the management and need for energy storage, establishing the parameters for said actions as mentioned in section 2.3.

Figure 6 shows the CVaR at 10%, determining the probability of energy generation on a smaller scale, for example, it could be demonstrated that the solar device is located in a place with cloud cover, leading to little production and having to take into account the need of the installation of an energy storage system. The same occurs with the value recorded for a CVaR at 90%, whose value would indicate the maximum solar production of the solar panel, indicating that it is in a place with good solar irradiation. The analysis of these values is determined concerning the observer, in this case, the network operator. The same happens with Figure 8 showing the CVaR at 10% and 90% for wind power.

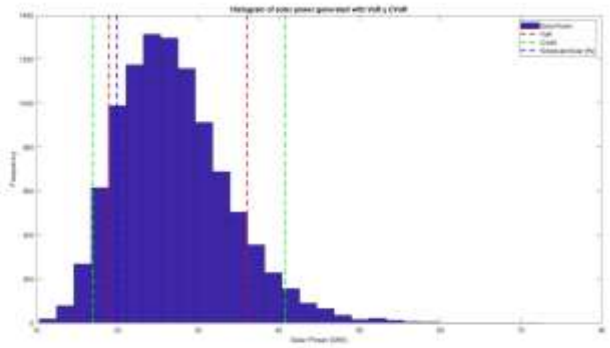

Fig. 6: Solar power with CvaR 10% and 90%

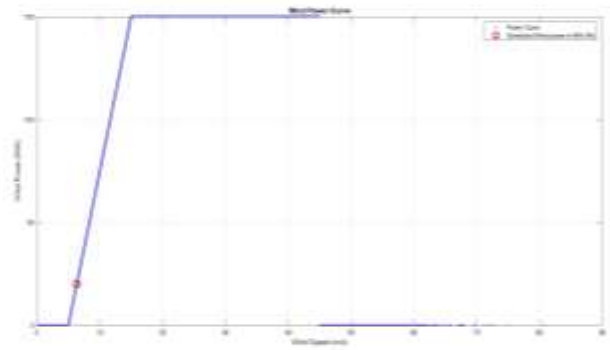

Fig. 7: Power Curve of Wind Energy

The power curve (Figure 7) is related to the wind speed, which is determined by the equations (7). This curve presents a certain increasing behavior due to the characteristics of the wind turbine. Unlike the behavior for solar production, where its generation depends on the intensity of solar irradiation, the wind turbine will not always generate energy even if there is a presence of wind in the area, due to the physical characteristics of the wind turbine. However, considering non-production values even with the presence of the primary source, can provide data on improvements in the system.

The histograms in Figure 6 and Figure 8 for the solar and wind generation powers are determined by the equations:

$$E1\left[C_{o,i}\left(P_{s,i}, CVaR_{10}\right)\right] = \left(counts\ *\ C_{o,i}\right) * \left(P_{s,i} - \left(\frac{(Bin_b + Bin_a)}{2}\right)\right) \quad (18)$$

$$E2\left[C_{o,i}\left(P_{s,i}, CVaR_{10}\right)\right] = \left(\frac{counts\ *\ C_{o,i}}{Bin_b\ +\ Bin_a}\right) * \left(\left(P_{s,i} * (CVaR_{10} + Bin_a)\right) - \left(\frac{CVaR_{10}^2}{2}\right) + \left(\frac{Bin_a^2}{2}\right)\right) \quad (19)$$

$$E3\left[C_{u,i}\left(P_{s,i}, CVaR_{90}\right)\right] = \left(counts\ *\ C_{u,i}\right) * \left(\left(\frac{(Bin_b + Bin_a)}{2}\right) - P_{s,i}\right) \quad (20)$$

$$E4\left[C_{u,i}\left(P_{s,i}, CVaR_{90}\right)\right] = \left(\frac{counts * C_{u,i}}{Bin_b + Bin_a}\right) * \left(\left(\frac{CVaR_{90}^2}{2}\right) - \left(\frac{Bin_b^2}{2}\right) - \left(P_{s,i}\ *\ (CVaR_{90}\ + Bin_b)\right)\right) \quad (21)$$

Each equation determines part of the structure of the histogram determined by its probability (counts) related by its minimum and maximum limits (Bin). The sum of the equations (18, 19) shows the uncertainty cost of overestimation, on the other hand, the sum of the equations (20, 21) shows the uncertainty cost of underestimation.

This method calculates the cost of overestimation and underestimation in a time of 0.0551 and 0.3097 seconds for solar and wind generation, respectively, regardless of the function or renewable system to be used. The points of interest are determined by the system operator with a confidence level for the CVaR as a reference, or by the operator himself. Table 3 and Table 4 present the results in comparison to MC simulation.

The CVaR calculation is a risk index that determines the expected losses that slightly exceed the VaR. That is, expected losses greater than or equal to VaR. In other words, risk cost for high or minimum production, which may or may not use the system. In the case of wind energy, given that said production depends on wind speed, presenting uncertainty in the primary source, the CVaR value can coincide with the maximum production value due to its behavior in the power curve.

The CVaR should be adjusted according to the operator's needs or can highlight the maximum or minimum generations for the power management or storage capacity seen above. For practical purposes, it was decided to have a confidence level of 0.7 for wind energy. In summary, the value of the confidence level represents a piece of information that indicates the percentage of risk to be analyzed. In the case of wind generation, the power curve is determined by the wind speed, which regardless of its value at that moment. If said speed is equal to or greater than the cutting speed established by the manufacturer or the equipment itself, the power will be unique, presenting a certain error when calculating the risk, since the percentile or quartile of the operation establishes the maximum or minimum generated. , having to adjust the percentage value so that it presents a degree of error in the risk by displaying a probability value.

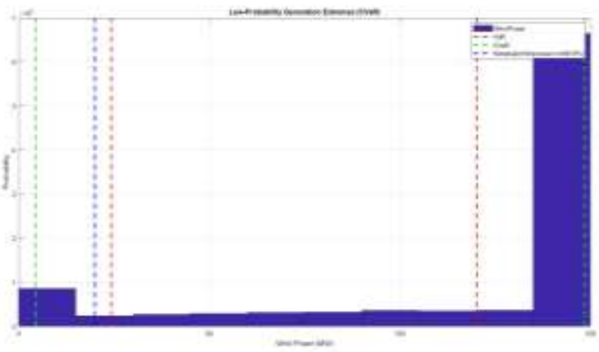

Fig. 8: Wind power with CvaR 10% and 30%

Table 3. Results of Solar power

	MC simulation (sec)	Analytical Method CVaR Solar Power (MW)	Error %
$CVaR_{10}$	17.0025		N/A
$CVaR_{90}$	40.8575		N/A
Co	13.6078	14.2643	0.0482
Cu	28.1748	28.4930	0.0113
time	1.5248	0.0551	N/A

Table 4. Results of Wind power

	MC simulation (sec)	Analytical Method CVaR Wind Power (MW)	Error %
$CVaR_{10}$	4.1161		N/A
$CVaR_{30}$	148.4147		N/A
Co	99.3260	95.4860	0.0387
Cu	2453.7	2346.0	0.0439
time	2.5813	0.3097	N/A

Table 5 shows the results obtained from MC simulation, analytical Method, and Analytical Method CVaR using a normal function. Times decrease significantly with the proposed methods.

Table 5. Results of Solar power with normal function

	MC simulation (sec)	Analytical Method Solar Power (MW)	Analytical Method CVaR Solar Power (MW)
$CVaR_{10}$	8.1297		
$CVaR_{90}$	9.8788		
Co	1.6025	1.6064	1.6487
Cu	1.9002	1.8774	1.9526
time	2.9160	0.0137	0.0166

4 Conclusion

Due to the uncertainty generated by renewable sources for their production of electrical energy, the operator of the electrical system on the part of the main network or the part of microgrids, virtual power plants, distributed systems, etc., must-have tools, techniques, or methodologies. That can minimize the risk related to the integration of these renewable sources into any system or variable loads in the economic dispatch of electricity, pointing to the interaction of energy exchange. This article shows analytical advances, which can be an important part of the decision-making that the operator makes every day. This proposed mathematical formulation can be incorporated into optimization techniques to obtain dynamic economic dispatch models, due to the variability of the system. The proposed analytical method, in addition to improving the computational response time with an error of 0.05 seconds, has the practicality of obtaining acceptable cost values due to overestimation and underestimation, regardless of the probability density function that describes the behavior of the energy production from renewable energy sources, for example, solar and wind, described by the Log-normal and Weibull function, respectively. The method of this article works in a more tactile way, due to the calculations that are handled in the histogram graph. The proposed method can handle the probability of a historical database having as a reference a programmed power about its probability. Unlike previous articles related to the topic, this method uses the power histogram generated from the Monte Carlo Method using probability density functions to perform the estimated calculation for the associated costs and risks, instead of integral functions which have great computational weight, however, the use of a historical database can replace this step. The use of the historical database provides us with greater precision in the results obtained by using real data, for the analysis and for practicality the Monte Carlo Method was chosen as mentioned above.

The main objective of this work is the analysis for the management, planning, and determination of the use of these sources in the main network or in any other system to be incorporated. The risk present in the energy generated by these sources usually indicates certain behavior due to its low probability of generation on a smaller or larger scale. For example, the values associated with the powers generated through an optimization process in a distribution system that presents a high CVaR concerning its VaR can give us indications of the need to have greater generation at the risk of not being able to cover the user demand, or on the contrary, by presenting a relatively low CVaR compared to its VaR, it indicates stability in the system by being able to meet the user's needs, and could also lend itself to the installation of a battery bank for reserve use. Due to the high probability of generation.

In the context of a photovoltaic system, a high CVaR could be interpreted as the average of the generated power that is expected to be lost in cases where the generated power falls below the level established by the VaR. It is a useful measure for understanding additional risk beyond VaR and can be used to make informed decisions about risk management strategies.

The operator's point of view is an important piece of information because the cost of the overestimated/underestimated risk would depend on the element or system to be analyzed as it is associated with the risk that it presents in its operation. Both the analytical method and the proposed method about risk present very acceptable values compared to the MC simulation, however, the determination of risks helps the operator to improve the system. The decision-making that the operator, in charge of the electrical system, makes is vital for the stability of the system itself and even more so having a criterion of the estimated risk, presenting the possibility of helping to better manage the energy generated, stored, or required, however , the use of this analysis could present certain limitations due to the different systems being analyzed and even more so if they are analyzed together, as presented in the wind analysis (section 3.2), which is why the point of view of the operator in charge of the system is of vital importance for the formulation of the problem.

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