

Forecasting Electrical Energy Consumptions

K. STOILOVA¹, T. STOILOV¹, G. ANGELOVA¹, R. PETROV², I. KLISUROV²

¹Institute of Information and Communication Technologies – Bulgarian Academy of Sciences
1113 Sofia Acad. G. Bonchev str. bl.2,
BULGARIA

²Wildlife Rehabilitation and Breeding Centre-Green Balkans,
P.O. Box 27, 6006 Stara Zagora,
BULGARIA

Abstract: - Electrical energy management of an organization is related to the appropriate planning of required payments and allocation of resources accordingly. It is important for good management to account for past costs and to forecast future costs. An intelligent solution for the mathematical formalization of this process is presented in the study. The research object is the Wildlife Rehabilitation and Breeding Centre “Green Balkans” in Bulgaria, in which the correct allocation of resources is significant due to the minimal financial support. Mathematical models have been developed for forecasting electricity consumption based on data for the past three-year period. The applied methodology is based on linear regression analysis. Two mathematical models were synthesized, which were compared and analyzed depending on the length of the historical data interval. The models represent an intelligent solution for the Centre’s electrical energy management.

Key-Words: - Electrical energy management, Planning resources, Mathematical models, Statistical methods, Auto regression, Forecasting.

Received: April 9, 2024. Revised: September 3, 2024. Accepted: October 6, 2024. Published: November 7, 2024.

1 Introduction

In Bulgaria, a Wildlife Rehabilitation and Breeding Centre has been actively operating since 1992, which is a specialized unit of the activity of Green Balkans, related to the treatment, rehabilitation, reproduction, and return to nature of rare and endangered wild animals, as well as environmental education. During the cold months of the year, the center’s patients are about 100 per month, while during the warm months, they exceed 400. Patients, in addition to a lot of care and professional skills, require several resources such as food, medicine, and adequate heating, both in summer and winter, according to the different species of wild animals that are accepted at the Centre. The Centre’s management faces several problems in allocating its limited resources and funding. This requires proper use of available resources and the application of scientific methods such as planning, optimization, and forecasting for their redistribution. The present study aims to apply scientific approaches to the functioning of the Centre for its better management. Here, a management approach integrating forecasting and optimization is proposed to reallocate available resources.

Forecasting models are statistical tools designed to make predictions about future events based on historical data and trends. One of the most used forecasting methods is the Time Series Model. Time series represent a set of measurements of a certain variable made at regular intervals. Time is the independent evaluation variable. Time Series Model analyses historical data to predict future trends.

An overview of Time series analysis and forecasting is presented in [1]. It introduces the different Time series forecasting methods, starting with Time series decomposition, data-driven moving averages, and exponential smoothing, and discusses model-driven forecasts including regression, Autoregressive Integrated Moving Average methods, and machine learning-based methods using windowing techniques. Time Series forecasting can be classified into four broad categories of techniques [1]: Forecasting based on Time series decomposition, smoothing-based techniques, regression-based techniques, and machine learning-based techniques. In this study, according to this classification, models are based on regression analysis. The Time series model uses historical data and forecasting models like Auto regressive model (AR), the Moving Average (MA)

model, and their combination **Auto-Regressive Integrated Moving Average ARIMA** model [1].

Prediction models are the subject of research in many publications. In [2], the characteristics of five regression techniques for forecasting the sales of a retail chain are considered. The linear weighted method and an artificial neural network have been applied to predict the security of supply and demand in the forestry industry, [3]. An overview of new product forecasting techniques is presented in [4]. New research directions are proposed to improve the performance of new products, supporting managers in decision-making.

In [5], three models for forecasting future cash flows are compiled. The data used are from Tunisian trading companies. Learning-based short-term forecasting models for smart grids are reviewed in [6]. Different 41 models were used to predict wind speed based on a data set from the site of Jodhpur, India. In [7], solar forecasting approaches by applying machine-learning techniques are given. A predictive convolutional neural network model for source-load forecasting in smart grids is presented in [8]. Prediction of the Long-Term Electrical Energy Consumption in Greece Using Adaptive Algorithms is described in [9]. Three short term load forecasting models, which aim to predict system load over an interval of one day or one week based on Grey System theory are presented in [10]. Forecasting Electricity Price Using Seasonal Arima Model is given in [11]. A review and evaluation of current wind power prediction technologies is described in [12].

The purpose of this research is to integrate intelligent solutions into the management policy of the Centre. Achieving this goal is related to solving the following tasks: choosing an appropriate method to formalize a better management policy; determining an appropriate mathematical formalization; numerical simulations; and model validation. The work consists of the following sections: Mathematical models; Integrating linear regression models with a shifting predictive approach; Numerical simulations and results; and Conclusions.

2 Mathematical Models

The quantitative formalization of the forecasting process is based on statistical analysis and more specifically on linear autoregression (AR) analysis. Our goal is to construct first- and second-order linear autoregression models (so-called $AR(1)$ and $AR(2)$) and to analyze their behavior.

2.1 $AR(1)$ Linear Regression Model

The $AR(1)$ model has the following formalizations

$$y(t+1) = a + by(t) \quad (1)$$

In (1), the known value is $y(t)$, the future unknown value is $y(t+1)$ and parameters a and b are to be determined.

When there is a set of m data values, dependence (1) can be represented as a system of m linear equations:

$$y_1(t+1) = a + by_1(t) \quad (2)$$

$$y_2(t+1) = a + by_2(t)$$

⋮

$$y_m(t+1) = a + by_m(t).$$

In the system of equations (2), the coefficients a and b are unknown. They should be determined in such a way as to provide the best approximation for the linear system. In this case, we will apply the method of least squares for the approximation. With known coefficients a and b , we denote the predicted value by y_i^p

$$y_i^p(t+1) = a + by_i(t). \quad (3)$$

The difference between the actual and predicted value is denoted by δ

$$\delta_i = y_i - y_i^p. \quad (4)$$

We aim to minimize the approximation error δ by applying the least squares method, whose formalization is by (5):

$$\min_{a,b} \delta = \sum_{i=1}^m (y_i - y_i^p)^2 \quad (5)$$

The $AR(1)$ model of type (3) can be presented in matrix form:

$$Y = \gamma \bar{Y}, \quad (6)$$

where the matrices are in the form:

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}; \quad \gamma = \begin{bmatrix} b \\ \vdots \\ b \end{bmatrix}; \quad \bar{Y} = \begin{bmatrix} 1 & y_0 \\ \vdots & \vdots \\ 1 & y_{m-1} \end{bmatrix}. \quad (7)$$

Problem (5) for minimizing the error when applying the matrix form (6) of the $AR(1)$ model leads to:

$$\min_Y \delta = (Y - \bar{Y}\gamma)^T (Y - \bar{Y}\gamma) = Y^T Y - Y^T \bar{Y}\gamma - \gamma^T \bar{Y}^T Y + \gamma^T \bar{Y}^T \bar{Y}\gamma. \quad (8)$$

To solve (8), we differentiate both sides of (8):

$$\frac{d\delta}{dy}=0= -2\bar{Y}^T Y + 2\bar{Y}^T \bar{Y} \gamma . \quad (9)$$

The unknown regression coefficients γ or (a, b) are determined from (9):

$$\gamma = (\bar{Y}^T \bar{Y})^{-1} \bar{Y}^T Y . \quad (10)$$

In this way, the model $AR(1)$ in matrix form (6) has certain linear coefficients a and b according to (10), so that the future values can be determined based on the available data from the dependence (6) or (2).

Further, the predicted values are determined using the formalization of the $AR(1)$ model (3) or (6) and the unknown coefficients γ (a and b) for the approximation is determined with optimal accuracy based on the method of least squares (5).

2.2 AR(2) Linear Regression Model

In the second-order linear regression model $AR(2)$, the predicted value at time $t+1$ depends on the available data both at the previous time t and on the data at the earlier time $t-1$. The $AR(2)$ model has the following mathematical notation:

$$y(t + 1) = a + b_1 y(t) + b_2 y(t - 1) \quad (11)$$

where $y(t+1)$ is the forecasting value. The unknown parameters are a, b_1 and b_2 that must be determined. Since the unknown parameters are three, at least four historical data are needed to make a prediction. For the $AR(1)$ model, the unknown parameters are two, so at least three historical data must be available.

An $AR(2)$ model can be represented as a system of m linear equations when m data values are available:

$$\begin{aligned} y_1(t + 1) &= a + b_1 y_1(t) + b_2 y_1(t - 1) \\ y_2(t + 1) &= a + b_1 y_2(t) + b_2 y_2(t - 1) \\ &\vdots \\ y_m(t + 1) &= a + b_1 y_m(t) + b_2 y_m(t - 1). \end{aligned} \quad (12)$$

The unknown parameters a, b_1, b_2 can also be determined here by the method of least squares in order to better approximate the forecast. We denote the predicted value from (12) by $y_i^p(t + 1)$:

$$y_i^p(t + 1) = a + b_1 y_i(t) + b_2 y_i(t - 1). \quad (13)$$

The error of approximation δ represents the difference between the actual value y and the predicted value y^p

$$\delta_i = y_i - y_i^p .$$

We want to minimize the error of approximation δ . For this purpose, we apply the method of least squares with the following formalization:

$$\min_{a,b_1,b_2} \delta = \sum_{i=1}^m (y_i - y_i^p)^2 . \quad (14)$$

We can present the model $AR(2)$ of (13) in matrix form:

$$Y = \gamma \bar{Y} ,$$

$$\text{where } = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}; \gamma = \begin{bmatrix} b_1 & b_2 \\ \vdots & \vdots \\ b_1 & b_2 \end{bmatrix} \quad \bar{Y} = \begin{bmatrix} 1 & y_0 \\ \vdots & \vdots \\ 1 & y_{m-1} \end{bmatrix} . \quad (15)$$

The least squares problem (14) in matrix form is given by:

$$\min_{\gamma} \delta = (Y - \bar{Y}\gamma)^T (Y - \bar{Y}\gamma) = Y^T Y - Y^T \bar{Y}\gamma - \gamma^T \bar{Y}^T Y + \gamma^T \bar{Y}^T \bar{Y} \gamma . \quad (16)$$

The solution is obtained by differentiating both sides of (16):

$$\frac{d\delta}{d\gamma}=0= -2\bar{Y}^T Y + 2\bar{Y}^T \bar{Y} \gamma .$$

After transformations, the unknown parameters γ are determined by the dependence:

$$\gamma = (\bar{Y}^T \bar{Y})^{-1} \bar{Y}^T Y . \quad (17)$$

Relation (17) is further applied to determine the unknown linear regression coefficients a, b_1, b_2 (or γ) of the $AR(2)$ model.

3 Integrating Linear Regression Models with a Shifting Predictive Approach

The Wildlife Rehabilitation and Breeding Centre Green Balkans provided us with data on the electricity consumption of the center for 3 years or 36 months. We use this data as consumption history to predict the next costs of the Centre. It is important for the manager and the operative at the Centre to know what resources to budget for the next month when planning expenses. To formalize the prediction process, we will apply the two linear regression models $AR(1)$ and $AR(2)$ proposed above.

The available electricity consumption data (in kW) are for three years (36 months), given in Table 1. The prediction approach consists of the following. When we use the $AR(2)$ model, the unknown regression coefficients are three: a, b_1, b_2

and to determine them, at least 4 historical data are needed. For the $AR(1)$ model, the unknowns are 2: a , b , so the history data could be at least two to determine the unknown coefficients a and b . Since the data of the first two months are included in the determination of the unknown coefficients of (2), the prediction can start as early as the third month. Let the historical data interval include four records for better approximation. We use the first 4 months' data from Table 1 as history to predict the energy consumption for the fifth month y_5^p . The predicted value is then compared with the actual value for the fifth month, which is given in Table 1, and the error δ of the prediction is determined. According to the applied least squares method, we minimize this error δ (see (8) and (16)) for both models and as a result determine new values of the linear regression coefficients for $AR(1)$ and $AR(2)$.

Table 1. Electricity consumption in the Centre

Month	2021	2022	2023
1 January	2855	4029	3441
2 February	2498	1351	2710
3 March	2860	2873	2430
4 April	2713	2424	3050
5 May	2103	2498	3236
6 June	2412	2453	3417
7 July	2391	2545	3887
8 August	2938	3367	3294
9 September	2287	1691	3297
10 October	2570	2600	3052
11 November	2511	3075	3383
12 December	3072	1815	3329

These values are entered into the new regression models $AR(1)$ and $AR(2)$, an offset of one month is made and for months 2-5 (4-month history), the data from Table 1 is used to predict the expenditure for the 6th month. In other words, at each step of the sliding procedure, new linear regression coefficients are determined for the two models $AR(1)$ and $AR(2)$ in an optimal way (according to the optimization problem) and, accordingly, the predicted values are different from the previous ones. The approach to sequentially shift the origin of the data history and apply the prediction is schematically presented in Figure 1.

The historical data period varies from 4 to 8 months to assess the impact of this interval on forecast accuracy. A comparison is made for the accuracy of the approximation depending on the different lengths of the historical data.

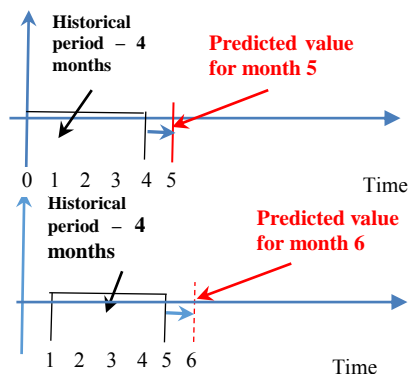


Fig. 1: Forecasting with the sliding procedure at history 4 months

The next study, again changing the period of history, is related to the accuracy of the two $AR(1)$ and $AR(2)$ models. A comparison is made between the two models and the results are presented graphically.

4 Numerical Simulations and Results

The proposed prediction policy is based on the integration of several methods:

- Linear regression analysis, through which the $AR(1)$ and $AR(2)$ models are compiled,
- Least squares method, which minimizes the prediction error and objectively determines the values of the regression coefficients as a solution to an optimization problem, and
- A sliding prediction procedure to sequentially move the data used as history.

This prediction approach is applied to both $AR(1)$ and $AR(2)$ models. A comparison was made of the obtained results in terms of prediction accuracy. The influence of the size of the data interval n used as history for the prediction was also investigated.

4.1 Simulations and Graphical Results for the $AR(1)$ Model with a History of 4 Months ($n=4$)

To simplify the simulations without reducing the accuracy, relative data were used. The data were normalized, as for each year from Table 1 the data were summed and divided by 12.

The actual values of electricity consumption are presented in the black solid line in Figure 2. The black dashed line is the average of the actual costs. The predicted costs according to the $AR(1)$ model are in the blue solid line, and their average value is in the blue dashed line.



Fig. 2: Actual and predicted values for model $AR(1)$, $n=4$

From Figure 2 it follows that the predicted values are almost like the real ones. Figure 3 shows the difference between real and predicted values for model $AR(1)$, $n=4$. A dashed line indicates the average value of the difference.

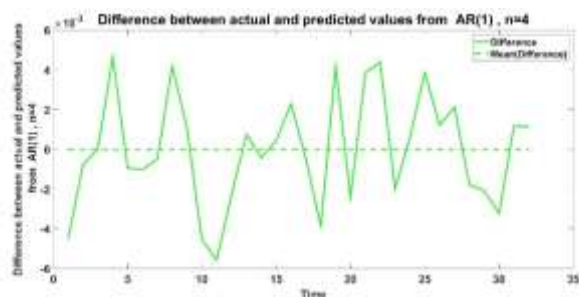


Fig. 3: Dynamics of prediction error with a model $AR(1)$, $n=4$

The magnitude of the vertical axis is multiplied by 10 to the minus 3rd power, indicating a very small prediction error, with a mean value of zero.

With the same interval of 4 months for forecasting history, model $AR(2)$ was applied.

4.2 Simulations and Graphical Results for the $AR(2)$ Model with a History of 4 Months ($n=4$)

The actual values of electricity consumption are presented in the black solid line in Figure 4. The black dashed line is the average of the actual costs. The predicted costs according to the $AR(2)$ model are in the red solid line, and their average value is in the red dashed line.

The predicted values are close to the actual values according to Figure 4. The prediction with this model is quite precise because the mean values of the actual and predicted values are almost the same (dashed lines).

Figure 5 shows the difference between real and predicted values (the prediction error) for model $AR(2)$, $n=4$. A dashed line indicates the average value of the difference.

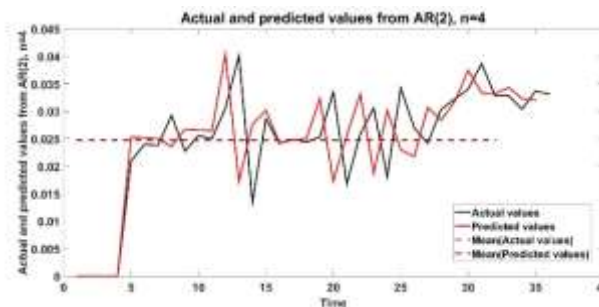


Fig. 4: Actual and predicted values for model $AR(2)$, $n=4$

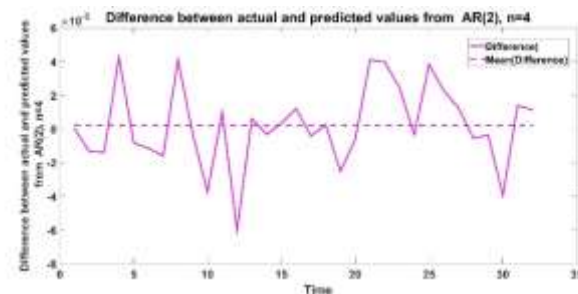


Fig. 5: Dynamics of prediction error for model $AR(2)$, $n=4$

Analogous simulations were done for both models on historical data for 6 months, $n=6$.

4.3 Simulations and Graphical Results for the $AR(1)$ Model with a History of 6 Months ($n=6$)

The actual values of electricity consumption are presented in the black solid line in Figure 6. The black dashed line is the average of the actual costs. The predicted costs according to the $AR(1)$ model are in a blue solid line, and their average value is in a blue dashed line.



Fig. 6: Actual and predicted values for model $AR(1)$, $n=6$

Figure 7 shows the difference between real and predicted values for model $AR(1)$, $n=6$. A dashed line indicates the average value of the difference.

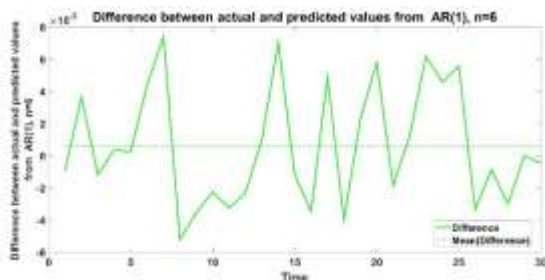


Fig. 7: Dynamics of prediction error for a model AR(2), $n=6$

This graph shows that the average error is no longer zero as in the prediction when $n=4$, but slightly larger. If at $n=4$ the average value of the error is zero, then at $n=6$ it is $0.5 \cdot 10^{-3}$. This means that a larger interval as a history leads to a more inaccurate prediction with the same model.

4.4 Simulations and Graphical Results for the AR(2) Model with a History of 6 Months ($n=6$)

Figure 8 presents the actual values of electricity costs denoted by the black solid line and the predicted values red solid line for the AR(2) model. The mean values are given in black dashed and red dashed lines, respectively.

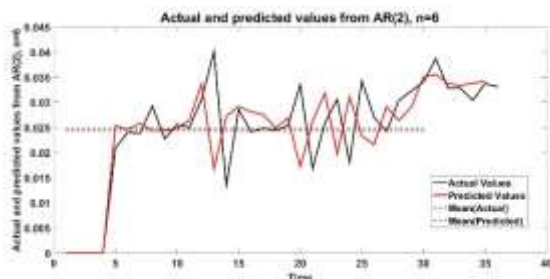


Fig. 8: Actual and predicted values for model AR(2), $n=6$

Figure 9 shows the difference between real and predicted values (the prediction error) for model AR(2), $n=6$. A dashed line indicates the average value of the difference.



Fig. 9: Dynamics of prediction error for model AR(2), $n=6$

Comparing the average error values when $n=4$ (Figure 5) and $n=6$ (Figure 9) follows the same conclusion for the model AR(2) as for AR(1). As the size of the historical interval increases, the accuracy of the prediction decreases.

Simulations were made at $n=7$ and $n=8$ months. Here we will present the results when $n=8$.

4.5 Simulations and Graphical Results for the AR(1) Model with a History of 8 Months ($n=8$)

The actual values of electricity consumption are presented in the black solid line in Figure 10. The black dashed line is the average of the actual costs. The predicted costs according to the AR(1) model are in the blue solid line, and their average value is in the blue dashed line.

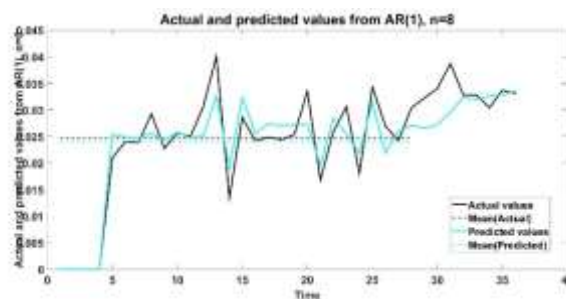


Fig. 10: Actual and predicted values for model AR(1), $n=8$

Figure 11 shows the difference between real and predicted values for model AR(1), $n=8$. A dashed line indicates the average value of the error.

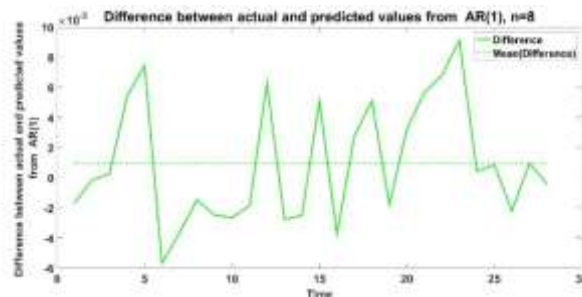


Fig. 11: Dynamics of prediction error for model AR(1), $n=8$

This graph shows that the average error is no longer zero as in the prediction when $n=4$, but slightly larger. If at $n=4$ the average value of the error is zero, then at $n=6$ it is $0.5 \cdot 10^{-3}$, and at $n=8$ it is about $1 \cdot 10^{-3}$. Therefore, the larger history interval leads to a more inaccurate forecast.

4.6 Simulations and Graphical Results for the AR(2) Model with a History of 8 Months ($n=8$)

The same notations are used as in the previous simulations. Figure 12 compares the actual (in black) and predicted (in red) values and their respective average values (with dashed line).

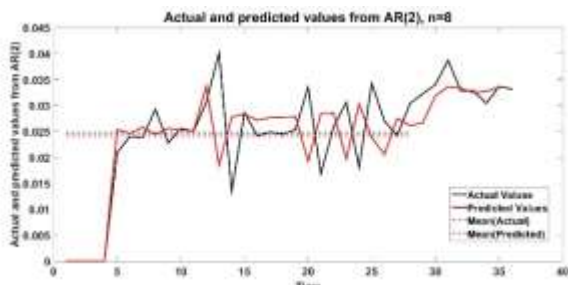


Fig. 12: Actual and predicted values for model AR(2), $n=8$

The variation of the error for the AR(2) model at $n=8$ is given in Figure 13.

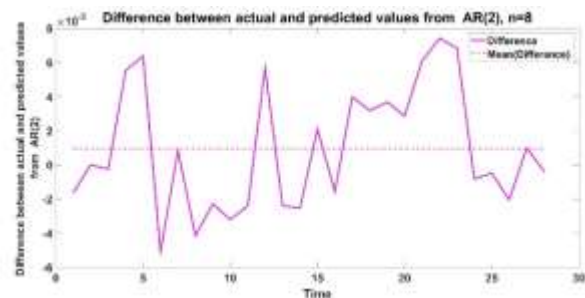


Fig. 13: Dynamics of prediction error for model AR(2), $n=8$

Comparing the average error values when $n=4$ (Figure 5), $n=6$ (Figure 9), and $n=8$ (Figure 13) follows the same conclusion: prediction accuracy decreases as the historical interval increases. As a reason for the decrease in forecast accuracy when the historical interval increases can be said the following. For a smaller time interval, the total error between actual and forecast value has a smaller value because a shorter period is considered. Slow processes are taken into account and faster processes are averaged.

As a confirmation of this conclusion, Figure 14 presents an illustration of the variation of the prediction error as a function of the size of the story. For the AR(1) model, the error variation is in the blue solid line, with the mean value in the blue dashed line. For the AR(2) model, these changes are in red.

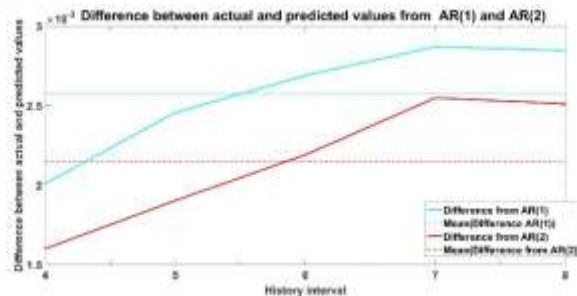


Fig. 14: Comparison of prediction error for AR(1) and AR(2) depending on the size of the historical interval

The following conclusions can be drawn from the comparison of the prediction error of the two models. The smallest prediction error for both models is at the smallest historical interval: $n=4$. As the size of the historical interval increases, the error increases for both models. The explanation of this dependence is that with a shorter history, the next value is predicted more accurately because the dynamics of the process for the next time value is preserved.

From the comparison between the model AR(1) and AR(2) it follows that the prediction error is greater in the model AR(1) compared to the model AR(2). Here the explanation is that with AR(2) two previous values are taken from the data while with AR(1) only one value at the previous time is used, therefore the model AR(2) is more accurate compared to AR(1).

5 Conclusion

The study develops an intelligent solution for the Organization Manager to better plan future resources. Based on the 36-month electricity consumption data, two types of models have been developed to forecast future costs. Statistical linear regression was applied to formalize the process. A sliding-sequential forecasting procedure was applied at different historical data intervals from 4 to 8 months. The predicted values are compared with the existing values. The minimized prediction error is determined. As a result of the optimized error, new values of the linear regression coefficients are determined for each step of the sliding procedure. This results in higher model accuracy/less prediction error. This error results from the application of the least square method and represents the solution of the optimization problem (5). According to this solution, the approximation coefficients used for forecasting for the next month are determined. For each simulation, the prediction error is plotted. This error represents a maximum of ± 0.006 which we

rate as a very good result. The accuracy of the forecast was analyzed depending on the size of the historical interval. A comparison is made between the two models AR(1) and AR(2). The second model gives better predictive accuracy). Further research is related to the inclusion of more predictive indicators.

References:

- [1] V. Kotu ,B. Deshpande, Chapter 12 - Time Series Forecasting, In *Data Science* (Second Edition), 2019, pp.395-445, [Online]. <https://www.sciencedirect.com/science/article/pii/B9780128147610000125> (Accessed Date: October 29, 2024).
- [2] A. Mitra, A. Jain, A. Kishore, P. Kumar, A Comparative Study of Demand Forecasting Models for a Multi-Channel Retail Company: A Novel Hybrid Machine Learning Approach, *Oper. Res. Forum* 3, 58, 2022, <https://doi.org/10.1007/s43069-022-00166-4>
- [3] X. Zhao, S. Yue, Analysing and forecasting the security in supply-demand management of Chinese forestry enterprises by linear weighted method and artificial neural network, *Enterprise Information Systems*, 15(9), 2020, pp. 1280–1297. <https://doi.org/10.1080/17517575.2020.1739343>
- [4] M. Mas-Machuca, M. Sainz, C. Martínez-Costa, A review of forecasting models for new products, *Intangible Capital* 10(1), 2014, DOI: 10.3926/ic.482
- [5] A. Telmoudi, H. Noubbigh, J. Ziadi, Forecasting of Operating Cash Flow: Case of the Tunisian Commercial Companies, *International Journal of Business and Management*, vol.5(10), 2010, pp. 198-210, [Online]. <https://pdfs.semanticscholar.org/9859/06e45fc11ef4ed0493e6d606124c1aa1d491.pdf> (Accessed Date: October 29, 2024).
- [6] V. K. Saini, R.Kumar, A. S. Al-Sumaiti, Sujil A., E.Heydarian-Forushani, Learning based short term wind speed forecasting models for smart grid applications: An extensive review and case study, *Electric Power Systems Research J.*, vol. 222, 2023, Art#222, 109502, [Online]. <https://www.sciencedirect.com/science/article/pii/S0378779623003917> (Accessed Date: October 29, 2024).
- [7] S. K. Singh, A. K. Tiwari, H.K. Paliwal, A state-of-the-art review on the utilization of machine learning in nanofluids, solar energy generation, and the prognosis of solar power, *J. Engineering Analysis with Boundary Elements*, vol. 155, 2023, pp. 62-86, [Online]. <https://www.sciencedirect.com/science/article/pii/S0955799723003119?via%3Dihub> (Accessed Date: October 29, 2024).
- [8] D. Khoury, F. Keyrouza, A predictive convolutional neural network model for source-load forecasting in smart grids, *WSEAS Transactions on Power Systems*, vol.14, 2019, Art #22, pp. 181-189.
- [9] S. Sp. Pappas, S. Adam, Prediction of the Long-Term Electrical Energy Consumption in Greece Using Adaptive Algorithms, *WSEAS Transactions on Power Systems*, vol.13, 2018, pp. 291-299.
- [10] S. Sreekumar, J. Verma, A. Sujil, R. Kumar, An Approach Towards Real Time Short Term Load Forecasting Using Grey Index Models For Smart Grid Framework, *WSEAS Transactions on Power Systems*, vol.11, 2016, Art #19, pp. 147-155.
- [11] H. Joshi, V. Pandya, C. Bhavsar, M. Shah, Forecasting Electricity Price Using Seasonal Arima Model and Implementing RTP Based Tariff in Smart Grid, *WSEAS Transactions on Power Systems*, vol. 11, 2016, Art. #6, pp. 43-51.
- [12] S. Saroha, S. K. Aggarwal, A Review and Evaluation of Current Wind Power Prediction Technologies, *WSEAS Transactions on Power Systems*, vol. 10, 2015, Art. #1, pp. 1-12.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Krasimira Stoilova, compiled the first model, the optimization and simulation, and writing the original draft.
- Todor Stoilov compiled the research methodology, formal analysis, and creation of the second model.
Galia Angelova was responsible for supervision.
- Rusko Petrov collected the data included in the simulations
- Ivailo Klisurov was responsible for the resources.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This research has received funding from the Bulgarian Ministry of Education and Science under the National Science Program *Intelligent Animal Husbandry*, grant agreement № D01-62/18.03.2021, and from the EU's Horizon Europe Widening program via COALition project "*Promoting Innovation Excellence in Transformation of Coal Regions to Climate-Neutral, Thriving Economies*", grant agreement № 101087022.)

Conflict of Interest

The authors have no conflicts of interest to declare.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0

https://creativecommons.org/licenses/by/4.0/deed.en_US