On Covering Properties in Intuitionistic Fuzzy Topological Spaces: A survey

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Abstract: - We show here some results on covering properties in intuitionistic fuzzy topological spaces. In 1983, K.T. Atanassov proposed a generalization of the notion of fuzzy set: the concept of intuitionistic fuzzy set. D. Çoker constructed the fundamental theory on intuitionistic fuzzy topological spaces, and D. Çoker and other mathematicians studied compactness, connectedness, continuity, separation, convergence and paracompactness in intuitionistic fuzzy topological spaces. Finally, G.-J Wang and Y.Y. He showed that every intuitionistic fuzzy set may be regarded as an L-fuzzy set for some appropriate lattice L. Nevertheless, the results obtained by above authors are not redundant with other for ordinary fuzzy sense.

Key-Words: - Mathematics; Topology; Fuzzy topology; Fuzzy sets; Atanassov's intuitionistic fuzzy sets; Covering properties

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1 Introduction

In 1983, K.T. Atanassov proposed a generalization of the notion of fuzzy set: the concept of "intuitionistic fuzzy set" (IFS) [1]. Some basic results on intuitionistic fuzzy sets were published in [2, 3], and the book [4] provides a comprehensive coverage of virtually all results until 1999 in the area of the theory and applications of intuitionistic fuzzy set. D. Çoker and M. Demirci [5] defined and studied the basic concept of intuitionistic fuzzy point, later D. Çoker [6, 7] constructed the fundamental theory on "intuitionistic fuzzy topological spaces" (IFTSs), and D. Çoker and other mathematicians [8-38] studied compactness, connectedness, continuity, separation, convergence and paracompactness in intuitionistic fuzzy topological spaces. Finally, G.- J. Wang and Y.Y. He [39] showed that every intuitionistic fuzzy set may be regarded as an *L*-fuzzy set for some appropriate lattice *L*. Nevertheless, the results obtained by above authors are not redundant with other for ordinary fuzzy sense.

On the other hand, currently, various authors continue working about intuitionistic fuzzy topological spaces. Indeed, K.T. Atanassov and co-workers studied relation with various logic properties [40-45], and many authors studied some properties in intuitionistic fuzzy topological spaces [46-56].

In this paper, we do a survey about covering properties in intuitionistic fuzzy topological spaces.

Papers authored by us about this subject, were exposed previously in [57].

2 Definitions and Results

Firstly, we list previous definitions:

Definition 1. Let X a nonempty set. An intuitionistic fuzzy set (IFS) in X is an object from the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define of degree of membership and the degree of non- membership of an element $x \in X$, respectively, and for each $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$

Definition 2. [3] Let X be a nonempty set, and A be an IFS in X. We determine for it the four numbers

$$\text{K}=\max_{x\in X}\{\mu_A(x)\} \qquad \text{L}=\min_{x\in X}\{\nu_A(x)\}$$

$$\text{k}=\min_{x\in X}\{\mu_A(x)\} \qquad \text{l}=\max_{x\in X}\{\nu_A(x)\}$$

And the IFS $C(A)=\{\langle x,K,L\rangle \mid x\in X\}$ and $I(A)=\{\langle x,k,l\rangle \mid x\in X\}$ called closure and interior of A

Theorem 1. [23] Let X be a non-empty set, $\mathscr{S}$ the family of all IFS in X, and the mapping $\varphi : \mathscr{S} \to \mathscr{S}$

$A \to C(A)$

If we denote $\mathscr{F} = \{\text{F IFS} \mid C(F)=F\}$ then $\tau = \{\overline{\text{F}} \mid \text{F} \in \mathscr{F}\}$ is an IFT on X (in Çoker's sense).

Remark 1. [23] The closure operator of Atanassov defines a Çoker's intuitionistic fuzzy topology, but if we take an IFTS the closures of IFS of it in Atanassov's sense are not necessarily IFCSs in the IFTS.

Theorem 2. [23] Let X be a non-empty set, $\mathscr{S}$ the family of all IFSs in X, and $\psi : \mathscr{S} \to \mathscr{S}$, $A \to \psi(A)$ be a mapping which verify:

(1) $A \subset \psi(A)$

(2) $\psi(\psi(A)) = \psi(A)$

(3) $\psi(0_{\sim}) = 0_{\sim}$

(4) $\psi(A \cup B) = \psi(A) \cup \psi(B)$

Then, if $\mathscr{F} = \{F\ IFS \mid \psi(F) = F\}$ and $\tau = \{\overline{\text{F}} \mid \text{F} \in \mathscr{F}\}$ we have that τ is an IFT on X in Çoker's sense, and, for every IFS A in X, $\psi(A)$ is the closure of A in (X,τ).

Proposition 1. [26] Let (X,τ), (Y,ς) be IFTSs, $f : X \to Y$ be a map and p either an IFP or VIFP in X. Then f is fuzzy continuous at p if and only if every intuitionistic fuzzy net s in X that converges to p in (X,τ) has the property that $f \circ s$ converges to $f(p)$ in (Y,ς).

Note. It is clear that, the result by Wang and He [40] does not make this proposition redundant. The reason for this is fundamentally, that Çoker's definition of ε-neighborhoods for IFPs and VIFPs are not equivalent to the corresponding concept of neighborhood of a fuzzy point, based on the containment of fuzzy points in fuzzy sets.

Definition 3. [24]
Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$ be two IFSs. We say that A quasi-coincides with B, denoted by AqB, if μ_A quasi-coincides with μ_B and $\nu_A{}'$ quasi-coincides with $\nu_B{}'$. (See also [20] and [46-47])

Theorem 3. [24] Let (X,τ) be an IFTS, let p be an IFP of X and let $\mathscr{U}_Q(p)$ be the family of all the Q-nighborhoods of p in (X,τ), then:

(1) $N \in \mathscr{U}_Q(p)$ implies that pqN.

(2) $N_1, N_2 \in \mathscr{U}_Q(p)$ imply that $N_1 \cap N_2 \in \mathscr{U}_Q(p)$.

(3) if $N \in \mathscr{U}_Q(p)$ and $N \subset M$, then $M \in \mathscr{U}_Q(p)$.

(4) if $N \in \mathcal{U}_Q(p)$, then exists $M \in \mathcal{U}_Q(p)$, $M \subseteq N$, such that, for every IFP e which quasi-coincides with M, we have that $M \in \mathcal{U}_Q(e)$.

Definition 4. [22] An IFTS (X,τ) will be called regular if for each IFP p and each IFCS C such that, $p \cap C = 0_{\sim}$, there exist IFOSs M and N such that $p \subseteq M$, $C \subseteq N$, and $M \cap N = 0_{\sim}$.

Definition 5. [22] An IFTS (X,τ) will be called normal if for each IFCSs C_1 and C_2 such that $C_1 \cap C_2 = 0_{\sim}$ there exist IFOSs M_1 and M_2 such that $C_i \subseteq M_i$ (i=1,2) and $M_1 \cap M_2 = 0_{\sim}$.

Proposition 2. [22] Let (X,τ) be a T_2 IFTS. If (X,τ) is normal, then also, it is a regular IFTS.

Compactness in intuitionistic fuzzy topological spaces is defined and studied in [6, 11-13, 19, 29, 31-33, 35].

Definition 6. [6] Let (X,τ) be an IFTS. It is called fuzzy compact if every fuzzy open cover of (X,τ) has a finite subcover.

Çoker [6] obtained some properties on compactness in intuitionistic fuzzy topological spaces analogous to them for compactness in topological spaces. Çoker and Eş [12-13], Hanafy [19], Ramadan [29], Thakur [38] and other authors [28], [31], [33-35], [37] defined and studied some variations of this concept.

Definition 7. [27] Let (X,τ) be an IFTS and $\mathcal{U} = \{\langle x, \mu_{G_j}, \nu_{G_j} \rangle \mid j \in J\}$ and $\mathcal{V} = \{\langle x, \mu_{A_i}, \nu_{A_i} \rangle \mid i \in I\}$ be two families of IFOSs in X. We will say that $\mathcal{V}$ refines $\mathcal{U}$ (or $\mathcal{V}$ is a refinement of $\mathcal{U}$), if for each $i \in I$ there exists some $j \in J$ such that $\langle x, \mu_{A_i}, \nu_{A_i} \rangle \subseteq \langle x, \mu_{G_j}, \nu_{G_j} \rangle$.

Definition 8. [27] Let (X,τ) be an IFTS and $\mathcal{U} = \{\langle x, \mu_{G_j}, \nu_{G_j} \rangle \mid j \in J\}$ be a family of IFSs in X. We will say that $\mathcal{U}$ is locally finite in an IFS A of X if, for each intuitionistic fuzzy point $p \in A$, there exists an ε-neighbourhood N of p such that $N \cap \langle x, \mu_{G_j}, \nu_{G_j} \rangle = 0_{\sim}$ for all $j \in J$ in the complement of a finite subset of J.

Definition 9. [27] If (X,τ) is an IFTS and A is an IFS of X, we will say that A is paracompact if for each fuzzy open cover $\mathcal{U} = \{\langle x, \mu_{G_j}, \nu_{G_j} \rangle \mid j \in J\}$ of A and for each $r \in (0,1]$, there exists a refinement of $\mathcal{U}$ which is locally finite in A and a fuzzy open cover of $A - r$. We will say that X is paracompact if $1_{\sim}$ is a paracompact IFS.

Note. There is no problem with these concepts and the result of Wang and He [39], because here we used ε-neighbourhoods.

Proposition 3. [27] If (X,τ_0) is a fuzzy topological space in Lowen's sense and $\tau = \{\langle x, \mu_A, 1-\mu_A \rangle \mid A \in \tau_0\}$ the associate IFT on X. If c_1 is a $*$-paracompact fuzzy set of (X,τ_0), then (X,τ) is a paracompact IFTS.

3 Conclusion

Atanassov´s intuitionistic fuzzy set is a still emerging concept with very interesting applications to various areas as decision making, information theory, intelligent systems, pattern recognition, medical diagnosis,… Anyone can quickly check this looking for the subject "intuitionistic fuzzy set" in some database. And also occurs for intuitionistic fuzzy topological spaces. Future research in this field could show yet other surprising new applications in various scientific or technological areas (for example, some papers on intuitionistic fuzzy topological spaces authored by us, are been cited for other authors in papers on differential equations, logic, algebra, pattern recognition, decision making,...)

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