Modeling of the Plane Film Flow in Alternating Electromagnetic Field

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Abstract: - The modeling of a plane film flow affected by alternating electromagnetic field (running EM wave) is considered in the paper. Basic parameters of film flow and specific peculiarities of parametrically excited oscillations in a film flow are studied and discussed for the theory, as well as for the diverse engineering and technological applications. The main attention is focused on the film flow spreading on a solid surface or in another liquid medium with comparably high velocities when inertia forces are playing together with capillary and electromagnetic ones (gravity forces are of lower impact due to high flow velocity). Scientific novelty of present study consists in the revealed new phenomena of the film flow oscillations and available decay or stabilization under appropriate practical statement.

Key-Words: - Film, Flow, Electromagnetic, Control, Excitation, Suppression, Instability, Decay

1 Plane film flow and the problem statement

Example of the plane film flow is formed by vertical plane jet spreading on a horizontal surface. It is supposed that jet is divided into two equal film flows to the right and to the left symmetrically as shown in Fig. 1, where b is a half-width of a jet, u_{00} is the speed of a jet in front of horizontal plate. For a flat jet it is easily established the connection of a film flow velocity u_0 with other parameters (b, u_{00}), considering an axis 0z of the Cartesian coordinate system as an axis of symmetry of the studied physical system.

1.1 Marvelous phenomenon of the film flows

It is not so simple question about radially spreading film formed by round vertical jet, which reveals decreasing of its thickness with a distance from origin starting from analysis of the conservation equation for mass flow rate. Actually the question about forming the film flow is not solved yet despite its seeming simplicity.

Everyone can experiment with a spoon put under water jet to see that jet transforms into a thin film flow abruptly forming from thick jet the very thin film. The process looks like a shock of the jet on a solid surface. Probably the problem has been not solved yet due to its mathematical complexity though phenomenon looks simple. Horizontal solid plate creates bifurcation in a jet flow, which transforms the jet into horizontal supersonic thin film flow. Despite both, jet and film flow are studied well, the transformation process from jet to film flow is unknown yet.

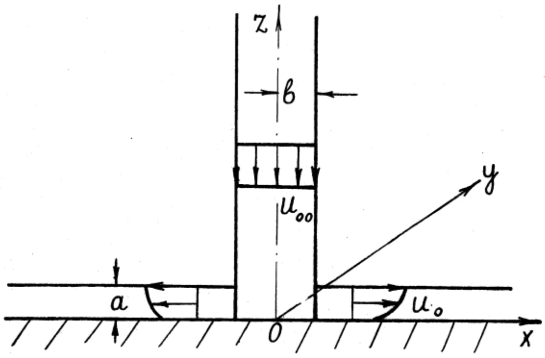

Fig. 1 Creation of the plane film flow:
b, u_{00} - half-width of vertical jet and flow velocity,
l- distance from the nozzle to the horizontal plate

1.2 Physical and mathematical models for the plane film flow

For example, in case of jet freely falling from height l over the horizontal plane it is possible to use approximate dependence for the jet velocity in the form of $u_{00}=\sqrt{2gl}$ and to determine the average film flow velocity from the integral correlations of the mass and impulse conservation:

$$au_0 = bu_{00}, \quad 0{,}5au_0^2 + 2\nu\left(\frac{\partial u}{\partial z}\right)_{z=0}(x-b) = 0{,}5bu_{00}^2, \quad (1)$$

where $u(z)$ is an instant flow velocity profile. Dissipation of the mechanical energy due to shock of a jet on a plate is neglected. The equations (1) give:

$$u_0 = (1/a)\int_0^a u(z)dz\,,\ \frac{b}{a} = 1 - \frac{4\nu}{u_{00}^2}\left(\frac{\partial u}{\partial z}\right)_{z=0}\left(\frac{x}{b} - 1\right), \quad (2)$$

where $u_0 / u_{00} = b/a$. For many real fluids including the liquid metals the value of kinematic viscosity coefficient ν is small, and the attainable for engineering practice approximation is: $u = 2z - z^2$, $(\partial u/\partial z)_{z=0} = 2$, so that the model of unperturbed system becomes simple.

In film flows the ratio of various forces significantly depends on the film thickness, physical properties of fluid and medium, type of intensity of external influences that defines big variety of the flow modes. Noticeable (sometimes - the main) role belongs to the capillary, viscous, electromagnetic (if the fluid is electro-conductive) forces [1]. And at a thickness of about the molecular sizes - also the Van-der-Waals force is substantial, which is not considered here since the film thickness considerably exceeds the molecular sizes that correspond to the applications considered by us.

Problems by parametric excitation and suppression of oscillations in film flows (stabilization of flows) due to periodic external influences were considered in connection with requirements of the MHD-granulation of metals [1-8], space technology and many other fields [9-11]. The main attention was paid by us to the phenomena of disintegration of film flows into the drops under influence of electromagnetic waves or vibrations, which have been taken as the basis for creation of the most perspective film MHD- [12] and vibrational [13] granulators having no analogs in the world. Thus, especially important are the regimes of a parametric resonance of the system giving the maximum technological effect at the minimum expenses of energy.

1.3 Description of the problem statement

According to the accepted physical model (Fig. 1) and the assumptions made about character of the studied physical system, the mathematical model for the plane film flow in a horizontal alternating electromagnetic field is built later on. And the statement for parametric oscillations on surface of the plainly film flow is the following. The continua (film flow and surrounding medium) are supposed viscous incompressible and film flow is electro conductive. Let consider that characteristics of the unperturbed system don't depend on *y* (flat system), the revolting force is caused by an electromagnetic wave of a form

$$H_z = H_m(z,t)\exp i(kx + my)\,. \quad (3)$$

The electromagnetic field is supposed to be solenoid, therefore $div\vec{H} = 0$ and a vertical electromagnetic wave can exist only under condition $\partial H_z / \partial z = 0$, when in expression (3) there is no dependence of a field from z. It is carried out if thickness of a film is significantly less than thickness of a skin-layer; otherwise the field has also the other components. At strong conductivity of the medium the alternating electromagnetic field in it spreads as a flat wave [14].

The characteristic thickness of the skin-layer was estimated for average values of magnetic viscosity coefficient ν_m, and corresponding δ were computed for 6 different liquid metals using the formula $\delta = \sqrt{2\nu_m / \omega}$. A number of such values ν_m near the melting temperatures of metals are given in the Table 1, where from it is seen that with a frequency of a field which isn't exceeding 1 kHz, the thickness a skin-layer is over $2\cdot 10^{-2}$ m that substantially prevails an average thickness of the films investigated by us.

Table 1 Thickness skin-layer against frequency

ω, Гц	*Al*	*Ga*	*Au*	*Sn*	*Hg*	*Fe*
1.0	0,570	0,65	0,71	0,89	1,26	1,48
50	0,080	0,09	0,10	0,13	0,18	0,21
10^3	0,018	0,02	0,022	0,028	0,040	0,047
10^6	$5{,}7\cdot 10^{-4}$	$6{,}5\cdot 10^{-4}$	$7{,}1\cdot 10^{-4}$	$8{,}9\cdot 10^{-4}$	$1{,}3\cdot 10^{-3}$	$1{,}5\cdot 10^{-3}$
10^9	$1{,}8\cdot 10^{-5}$	$2{,}0\cdot 10^{-5}$	$2{,}2\cdot 10^{-5}$	$2{,}8\cdot 10^{-5}$	$4{,}0\cdot 10^{-5}$	$4{,}7\cdot 10^{-5}$
ν_m	0,16	0,21	0,25	0,40	0,80	1,1

In the range of the frequencies 10^3-10^9 Hz the thickness of a film and a skin-layer can be of the same order, and then there is a need of the accounting of field's components by coordinates *x* and *y* that significantly complicates the mathematical model. At $\omega > 10^9$ Hz the skin-layer is much thinner than a film, as a first approximation it is possible to model it as a surface of zero thickness coinciding with interfacial boundary.

Now we are interested in the peculiarities of the film flow under alternating electromagnetic field and possibilities for the film surface control.

2 Mathematical models for surface oscillations in the plane film flow

Taking into account the stated above, the MHD-equations of the perturbed film flow including the known parameters of the unperturbed system, which can be set approximately by expressions (2) or more precise [10,15,16], in a linear approach are:

$$\rho_j\left[\frac{\partial u_j}{\partial t}+(2-j)(u_0\frac{\partial u_j}{\partial x}+u_j\frac{\partial u_0}{\partial x})\right]+$$

$$+\frac{\partial}{\partial x}\left(p_j+\frac{2-j}{2}\mu_m H_z^2\right)=\mu_j\left(\frac{\partial^2 u_j}{\partial x^2}+\frac{\partial^2 u_j}{\partial y^2}+\frac{\partial^2 u_j}{\partial z^2}\right),$$

$$\frac{\partial u_j}{\partial x}+\frac{\partial v_j}{\partial y}+\frac{\partial w_j}{\partial z}=0,\quad \rho_j\left[\frac{\partial v_j}{\partial t}+(2-j)u_0\frac{\partial v_j}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left(p_j+\frac{2-j}{2}\mu_m H_z^2\right)=\mu_j\left(\frac{\partial^2 v_j}{\partial x^2}+\frac{\partial^2 v_j}{\partial y^2}+\frac{\partial^2 v_j}{\partial z^2}\right),$$

$$\rho_j\left[\frac{\partial w_j}{\partial t}+(2-j)u_0\frac{\partial w_j}{\partial x}\right]+\frac{\partial p_j}{\partial z}=$$

$$=\mu_j\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{\partial^2 w_j}{\partial y^2}+\frac{\partial^2 w_j}{\partial z^2}\right), \qquad (4)$$

$$\frac{\partial H_z}{\partial t}+u_0\frac{\partial H_z}{\partial x}+H_z\frac{\partial u_0}{\partial x}=\nu_m\left(\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}\right).$$

Assuming the low-amplitude perturbations one can perform linearization of the equation array with respect to the above-mentioned unperturbed state of a system. The equations (4) are used for the description of the perturbed isothermal film flow and its parametric oscillations at the constant densities and viscosities of media (ρ_j, μ_j).

The revolting force is created by a vertical electromagnetic wave (3), which spreading regularities are influenced, in turn, by unperturbed film flow. Thus, perturbation of pressure *p* and velocity components u_1, v_1 are defined by energy pumping from an electromagnetic field to the conductive medium, and perturbation of vertical velocity component w_1 of a film flow is caused by changes of pressure. All this leads to oscillations of a film flow surface, which can be operated by means of the electromagnetic fields alternating by the regime stated. Therefore research of the main features of the electromagnetic field and media interaction is necessary.

The picture of the induced electric current in a film flow prone to an action of electromagnetic wave of the above-described form looks like presented in Fig. 2:

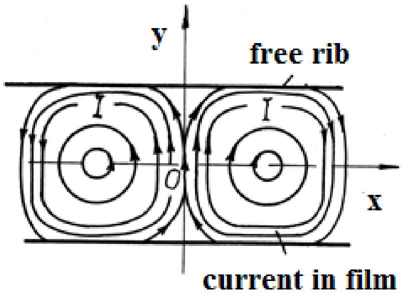

Fig. 2 Picture of induced current in a film flow

where the free ribs are cylinders of unloading the film from surface forces [16], which are the lines of minimal electric resistance. The last leads to a physical situation when the lines of the induced current are displaced to external borders of a film. Therefore the picture of spreading current is symmetric only in a middle part of a film flow where influence of regional effects is weakened. On sufficient distance from an initial section x=0 the lines of spreading current are almost parallel to an axis *0y* and thereof the electromagnetic forces affect a film flow in the direction of an axis *0x*.

Thus, fluid in a film flow is either accelerated or decelerated by coordinate *x* that causes the depression-compression wave by this coordinate leading to the corresponding deformation of a film flow surface. The analysis of practical cases shows [1,12,17] that for film flows the magnetic Reynolds numbers Re_m are small, therefore the induced electromagnetic field can be neglected and excluded from analysis. It allows solving independently the hydrodynamic equation array containing in their right hands the intensity of an electromagnetic field defined from the equations of field induction, which only contains the component of average film flow velocity for unperturbed state. This means that the equation for electromagnetic field is considered as the autonomous one.

For low-amplitude parametric oscillations of a surface of film flow in a linear approach, the task is reduced to a system of two differential equations describing distribution of an electromagnetic wave in the conductive medium and caused by it oscillations on a film surface (boundary separating the conductive and non-conductive media) [1,18-22]. For a further specification of physical and mathematical model by parametric fluctuations of the plainly spreading conductive film flow in the field of a progressive electromagnetic wave, it is required to stipulate a type of boundary and initial conditions satisfying the considered above physical situation.

2.1 Boundary conditions

Taking into account the above-stated in a linear approach by small perturbations one can seek for a system response to external influence in the form similar to (3) avoiding the statement of the initial conditions if only the behavior of a surface perturbations is of interest but not absolute value of perturbations' amplitude. Then for a full specification of mathematical model it is enough to set conditions on the boundaries of media using as an illustration of physical situation in Fig. 3, which on $\vec{n}, \vec{\tau}_x, \vec{\tau}_y$ are the instant normal and tangent unit vectors of the accompanying Cartesian coordinate system at a point $(x, y, a+\chi)$ on a film boundary with the surrounding medium.

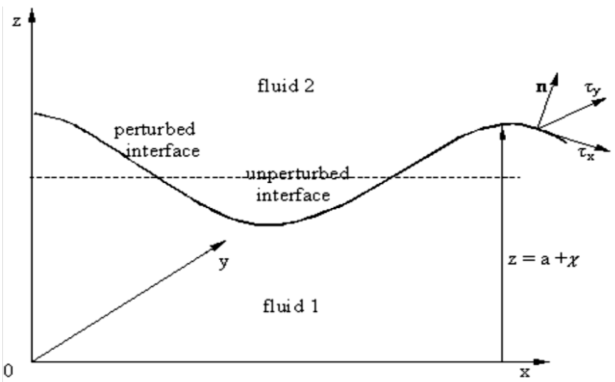

Fig. 3 Boundary perturbation between two fluids

On the lower boundary of a film flow the conditions by contact of viscous fluid with a solid surface or with the non-conductive surroundings is considered. The most widespread on a solid surface is the sticking condition though it not always adequately reflects a physical picture of the phenomena, especially in case of unmoisten surfaces (unwetting liquids). The correctness of this condition for tangent velocity components was being permanently raised since Stokes (1845), Lamb (1947), Zhukovsky (1948).

2.1.1 Slipping boundary condition

In the review [23] it is shown that on unmoisten solid surface the noticeable slipping of liquid is possible. J. Happel and G. Brenner [24] assumed that the most reasonable is the following hypothesis: the tangential velocity component of a liquid related to a solid body at a point on its surface is proportional to tangent tension at this point from the constant β' called by slipping friction coefficient. It was supposed that β' depends only on the nature of liquid and solid surface. Later on at different times J. Serrin [25], I.B. Bogoryad [26] and other researchers also disputed about applicability of slipping conditions on a wall in various problems of the viscous liquid. For example, Bogoryad has given mathematical substantiation of the condition of partial slipping.

Generally speaking, a question by calculation of liquid slipping on a solid surface is difficult, and recently the attempts by slipping definition in different physical situations were made:

• with use of a two-moment boundary condition in approach, linear on Knudsen's number, the speed of slipping for non-uniform by temperature and mass speed of the rarefied gas along a solid spherical surface has been calculated [27],

• the aerodynamic drag of a car was defined numerically solving the Navier-Stokes equations with a slipping boundary condition instead of a sticking condition [28],

• survey works on micro-hydrodynamics by a sticking condition and experimental studies of slipping on the boundary of solid surface.

2.1.2 Hysteresis of contact angle on solid surface

The flat film flow on a solid surface is unstable, even in a linear approach it is impossible to construct the correct theory due to hysteresis of a contact angle, possible flow separation on a surface, etc. [29].

2.1.3 Kinematic and dynamic conditions on a free film surface. Formula L.D. Landau

On the top film surface the following boundary conditions are considered. In absence of the media slipping we suppose on the unperturbed surface $z = a + \chi(x, y, t)$:

$$\tilde{u}_1 = \tilde{u}_2,\ \ \tilde{v}_1 = \tilde{v}_2,\ \ \tilde{w}_1 = \tilde{w}_2 = \frac{\partial \chi}{\partial t} + \tilde{u}_1 \frac{\partial \chi}{\partial x} + \tilde{v}_1 \frac{\partial \chi}{\partial y}. \qquad (5)$$

In this kinematic condition $\vec{\tilde{v}} = \vec{v} + \vec{v}_0$, where $\vec{v}_0$ is unperturbed velocity, $\vec{v}$ - its perturbation.

The dynamic condition on the interfacial boundary includes a balance of tangential and normal stresses, and at considerable curvature of the deformed interface it is necessary to consider the capillary pressure $p_\sigma = \sigma K_{cp}$ [30]:

$$K_{cp} = \frac{\frac{\partial^2 \chi}{\partial x^2}\left[1+\left(\frac{\partial \chi}{\partial y}\right)^2\right]+\frac{\partial^2 \chi}{\partial y^2}\left[1+\left(\frac{\partial \chi}{\partial x}\right)^2\right]-2\frac{\partial \chi}{\partial x}\frac{\partial \chi}{\partial y}\frac{\partial^2 \chi}{\partial x \partial y}}{\left[1+\left(\frac{\partial \chi}{\partial x}\right)^2+\left(\frac{\partial \chi}{\partial y}\right)^2\right]^{3/2}}. \qquad (6)$$

It is determined with use of the formulas of differential geometry for average curvature K_{cp} of a surface $z=a+\chi(x,y,t)$ at the point (x,y,t).

In linear approach expression (6) coincides with solution of the variation task on a minimum of total free energy done by L.D. Landau [31] for a case of small deviations of boundary from equilibrium situation. It is easy to be convinced that this expression doesn't contain terms of even orders by small-amplitude perturbation of a free surface. Therefore it is possible to interpret as a reason of successful application of linear approach at solution of the weakly non-linear tasks. At formulation of dynamic balance condition on a surface of boundary media in a linear approach it is possible to project all forces on the unperturbed surface *z=a*.

2.2 Non-linear boundary conditions

If a value of perturbation amplitude must be accounted when the normal and tangential vectors to an interface of media significantly deviate from their unperturbed position, it is necessary to project the acting stresses on the normal and tangent plane in a considered point.

The unit tangential and normal vectors of the instant accompanying Cartesian coordinate system are presented in the form [30]:

$$\vec{n} = \frac{\left\{-\frac{\partial \chi}{\partial x},\, -\frac{\partial \chi}{\partial y},\, 1\right\}}{\sqrt{1+\left(\frac{\partial \chi}{\partial x}\right)^2+\left(\frac{\partial \chi}{\partial y}\right)^2}},\quad \vec{\tau}_x = \frac{\left\{1,\, 0,\, \frac{\partial \chi}{\partial x}\right\}}{\sqrt{1+\left(\frac{\partial \chi}{\partial x}\right)^2}},\quad \vec{\tau}_y = \frac{\left\{0,\, 1,\, \frac{\partial \chi}{\partial y}\right\}}{\sqrt{1+\left(\frac{\partial \chi}{\partial y}\right)^2}}, \qquad (7)$$

Using the (7), we determine the instant forces operating on an elementary plane with a normal $\vec{n}$. For this purpose, first the expressions for components of hydrodynamic stresses are written:

$$p_{nn}= n_x p_{nx}+ n_y p_{ny}+ n_z p_{nz}\,,\quad p_{\tau x}= \tau_{xx}p_{nx}+ \tau_{xy}p_{ny}+ \tau_{xz}p_{nz},$$

$$p_{\tau y}= \tau_{yx} p_{nx}+ \tau_{yy}p_{ny}+ \tau_{yz} p_{nz}\,, \qquad (8)$$

where are:

$$p_{nx} = n_x p_{xx} + n_y p_{yx} + n_z p_{zx}\,,\quad \vec{n} = \{n_x, n_y, n_z\}\,,$$
$$p_{ny} = n_x p_{xy} + n_y p_{yy} + n_z p_{zy}\,,\quad \vec{\tau}_x = \{\tau_{xx}, \tau_{xy}, \tau_{xz}\},$$
$$p_{nz} = n_x p_{xz} + n_y p_{yz} + n_z p_{zz}\,,\quad \vec{\tau}_y = \{\tau_{yx}, \tau_{yy}, \tau_{yz}\}.$$

Also the following expressions for the components of stress tensor are taken into account:

$$p_{xx}=2\mu\frac{\partial u}{\partial x}-p\,,\ p_{yy}=2\mu\frac{\partial v}{\partial y}-p\,,\ p_{xy}=p_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),$$

$$p_{zz}=2\mu\frac{\partial w}{\partial z}-p\,,\, p_{yz}=p_{zy}=\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right),\, p_{xz}=p_{zx}=\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right).$$

where indexes *j*=1,2 and tildes are omitted for simplicity. Here *p* is static pressure, μ- dynamic viscosity coefficient. With the equations thus obtained and accounting electromagnetic and gravity forces, the dynamic equilibrium conditions of the interfacial boundary are

$$\begin{aligned} &[p_{nn}]_2^1+[\rho]_2^1 g n_z\chi+[f]_2^1 n_z\chi=\sigma K_{cp}, \\ &[p_{\tau x}]_2^1+[\rho]_2^1 g\tau_x\chi+[f]_2^1 \tau_x\chi=0, \\ &[p_{\tau y}]_2^1+[\rho]_2^1 g\tau_y\chi+[f]_2^1 \tau_y\chi=0, \end{aligned} \qquad (9)$$

where $[\ ]_2^1$ mean a jump of corresponding parameter on the boundary, e.g. $[p]_2^1=p_1-p_2$.

Taking into account expressions (6)-(9) we can present a condition of local dynamic balance of the perturbed interfacial boundary in rather general view. In a projection on the normal to an interface of media, on the tangent plane in the direction *x* and in a projection to the tangent plane in the direction of y, respectively:

$$\begin{aligned}
&(p_2-p_1)\left[1+\left(\frac{\partial\chi}{\partial x}\right)^2+\left(\frac{\partial\chi}{\partial y}\right)^2\right]^{\frac{3}{2}}+2\left[1+\left(\frac{\partial\chi}{\partial x}\right)^2+\left(\frac{\partial\chi}{\partial y}\right)^2\right]\cdot \\
&\cdot\left\{\frac{1}{2}\left[(\rho_1-\rho_2)g\chi+\frac{1}{2}\mu_m H_z^2\right]+\left[\mu_2\left(\frac{\partial v_2}{\partial z}+\frac{\partial w_2}{\partial y}\right)-\mu_1\left(\frac{\partial v_1}{\partial z}+\frac{\partial w_1}{\partial y}\right)\right]\frac{\partial\chi}{\partial y}+\right. \\
&+\left[\mu_1\left(\frac{\partial u_1}{\partial x}\frac{\partial\chi}{\partial x}-\frac{\partial u_1}{\partial z}-\frac{\partial w_1}{\partial x}\right)+\mu_2\left(\frac{\partial u_2}{\partial z}+\frac{\partial w_2}{\partial x}-\frac{\partial u_2}{\partial x}\frac{\partial\chi}{\partial x}\right)\right]\frac{\partial\chi}{\partial x}+ \\
&+\left[\mu_1\left(\frac{\partial v_1}{\partial x}+\frac{\partial u_1}{\partial y}\right)-\mu_1\left(\frac{\partial v_2}{\partial x}+\frac{\partial u_2}{\partial y}\right)\right]\frac{\partial\chi}{\partial x}\frac{\partial\chi}{\partial y}+\mu_1\frac{\partial w_1}{\partial z}-\mu_2\frac{\partial w_2}{\partial z}+ \\
&\left.+\left(\mu_1\frac{\partial v_1}{\partial y}-\mu_2\frac{\partial v_2}{\partial y}\right)\left(\frac{\partial\chi}{\partial y}\right)^2\right\}=\sigma\left[\frac{\partial^2\chi}{\partial x^2}+\frac{\partial^2\chi}{\partial y^2}+\frac{\partial^2\chi}{\partial x^2}\left(\frac{\partial\chi}{\partial y}\right)^2+\right. \\
&\left.-2\frac{\partial\chi}{\partial x}\frac{\partial\chi}{\partial y}\frac{\partial^2\chi}{\partial x\partial y}+\frac{\partial^2\chi}{\partial y^2}\left(\frac{\partial\chi}{\partial x}\right)^2\right],
\end{aligned} \qquad (10)$$

$$\begin{aligned}
&\left[1-\left(\frac{\partial\chi}{\partial x}\right)^2\right]\left[\mu_1\left(\frac{\partial w_1}{\partial x}+\frac{\partial u_1}{\partial z}\right)-\mu_2\left(\frac{\partial w_2}{\partial x}+\frac{\partial u_2}{\partial z}\right)\right]+ \\
&\left\{2\left[\mu_1\left(\frac{\partial w_1}{\partial z}-\frac{\partial u_1}{\partial x}\right)+\ +\mu_2\left(\frac{\partial u_2}{\partial x}-\frac{\partial w_2}{\partial z}\right)+\right.\right. \\
&\left.\left.\left[\mu_2\left(\frac{\partial w_2}{\partial y}+\frac{\partial v_2}{\partial z}\right)-\mu_1\left(\frac{\partial w_1}{\partial y}+\frac{\partial v_1}{\partial z}\right)\right]\frac{\partial\chi}{\partial y}\right\}\frac{\partial\chi}{\partial x}+\right. \\
&+\left[\mu_2\left(\frac{\partial v_2}{\partial x}+\frac{\partial u_2}{\partial y}\right)-\mu_1\left(\frac{\partial v_1}{\partial x}+\frac{\partial u_1}{\partial y}\right)\right]\frac{\partial\chi}{\partial y}+
\end{aligned}$$

$$\left[(\rho_1-\rho_2)g\chi+\frac{1}{2}\mu_m H_z^2\right]\cdot$$

$$\cdot\left\{\left[1+\left(\frac{\partial\chi}{\partial x}\right)^2+\left(\frac{\partial\chi}{\partial y}\right)^2\right]\left[1+\left(\frac{\partial\chi}{\partial x}\right)^2\right]\right\}^{\frac{1}{2}}\frac{\partial\chi}{\partial x}=0,$$

$$\left[1-\left(\frac{\partial\chi}{\partial y}\right)^2\right]\left[\mu_1\left(\frac{\partial v_1}{\partial z}+\frac{\partial w_1}{\partial y}\right)-\mu_2\left(\frac{\partial v_2}{\partial z}+\frac{\partial w_2}{\partial y}\right)\right]+$$

$$+\left[\mu_2\left(\frac{\partial w_2}{\partial x}+\frac{\partial u_2}{\partial z}\right)-\mu_1\left(\frac{\partial w_1}{\partial x}+\frac{\partial u_1}{\partial z}\right)\right]\frac{\partial\chi}{\partial x}\bigg\}\frac{\partial\chi}{\partial y}+$$

$$+\left[\mu_2\left(\frac{\partial u_2}{\partial y}+\frac{\partial v_2}{\partial x}\right)-\mu_1\left(\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x}\right)\right]\frac{\partial\chi}{\partial x}+\left[(\rho_1-\rho_2)g\chi+\right.$$

$$\left.+\frac{1}{2}\mu_m H_z^2\right]\left\{\left[1+\left(\frac{\partial\chi}{\partial x}\right)^2+\left(\frac{\partial\chi}{\partial y}\right)^2\right]\left[1+\left(\frac{\partial\chi}{\partial y}\right)^2\right]\right\}^{\frac{1}{2}}\frac{\partial\chi}{\partial y}=0.$$

The equations (10) become significantly simpler for two-dimensional perturbations, when $\partial/\partial y=0$ or $\partial/\partial x=0$. In the equations obtained for local dynamic balance of the perturbed interface of media, the terms containing $\vec{v}_{j0}$ and not containing $\vec{v}_j$ can be excluded if consider the following boundary conditions imposed on unperturbed system:

$$z=a,\quad \mu_1\frac{\partial u_{10}}{\partial z}=\mu_2\frac{\partial u_{20}}{\partial z},\quad \mu_1\frac{\partial v_{10}}{\partial z}=\mu_2\frac{\partial v_{20}}{\partial z},$$

$$\mu_1\left(\frac{\partial u_{10}}{\partial y}+\frac{\partial v_{10}}{\partial x}\right)=\mu_2\left(\frac{\partial u_{20}}{\partial y}+\frac{\partial v_{20}}{\partial x}\right),\quad w_{10}=w_{20}=0,$$

$$p_{10}=p_{20}+2\left(\mu_1\frac{\partial w_{10}}{\partial z}-\mu_2\frac{\partial w_{20}}{\partial z}\right).$$

Apparently from the aforesaid, the mathematical model of physical process in a general view is cumbersome and thereof is not so much suitable for practical calculations. Therefore various simplified linear and rather simple non-linear mathematical models are used in most cases [32-35], and the systems of type (10) serve as initial ones by derivation of simple equation arrays based on use of the additional data or hypotheses of physical character of the phenomena.

2.3 The linearized boundary conditions

In case of low-amplitude perturbations of system the linearization procedure for the boundary conditions and equations gives considerable simplification of its mathematical model. So, for linearization of boundary conditions (5), (10) it is necessary to expand the presented functions in the vicinity of $z=a$ by small parameter (as such χ can be taken), omit in the received expressions the terms of the second and higher orders, and then it yields:

$$z=a,\quad \vec{v}_1=\vec{v}_2,\quad w_1=w_2=\frac{\partial\chi}{\partial t}+u_{j0}\frac{\partial\chi}{\partial x}+v_{j0}\frac{\partial\chi}{\partial y};$$

$$p_1=p_2+(\rho_1-\rho_2)g\chi+\frac{\mu_m}{2}H_z^2-\sigma\left(\frac{\partial^2\chi}{\partial x^2}+\frac{\partial^2\chi}{\partial y^2}\right)+$$

$$+2\left(\mu_1\frac{\partial w_1}{\partial z}-\mu_2\frac{\partial w_2}{\partial z}\right); \qquad (11)$$

$$2\left[2\left(\mu_2\frac{\partial u_{20}}{\partial x}-\mu_1\frac{\partial u_{10}}{\partial x}\right)+\left(\mu_2\frac{\partial v_{20}}{\partial y}-\mu_1\frac{\partial v_{10}}{\partial y}\right)\right]-\mu_2\left(\frac{\partial w_2}{\partial x}+\frac{\partial u_2}{\partial z}\right)+$$

$$+\mu_1\left(\frac{\partial w_1}{\partial x}+\frac{\partial u_1}{\partial z}\right)+\frac{\mu_m}{2}H_z^2\frac{\partial\chi}{\partial x}=0,$$

$$\mu_1\left(\frac{\partial w_1}{\partial y}+\frac{\partial v_1}{\partial z}\right)-\mu_2\left(\frac{\partial w_2}{\partial y}+\frac{\partial v_2}{\partial z}\right)+$$

$$2\left[2\left(\mu_2\frac{\partial v_{20}}{\partial y}-\mu_1\frac{\partial v_{10}}{\partial y}\right)+\left(\mu_2\frac{\partial u_{20}}{\partial x}-\mu_1\frac{\partial u_{10}}{\partial x}\right)\right]+\frac{\mu_m}{2}H_z^2\frac{\partial\chi}{\partial y}=0,$$

Similarly the equation array (4) is linearized too:

$$\rho_j\frac{\partial v_j}{\partial t}+(2-j)(\vec{v}_0\nabla\vec{v}_j+\vec{v}_j\nabla\vec{v}_0)+\nabla\left(p_j+\frac{2-j}{2}\mu_m H^2\right)=\mu_j\Delta\vec{v}_j,$$

$$div\vec{v}_j=0,\quad \frac{\partial H}{\partial t}+\vec{v}_0\nabla H=\nu_m\Delta H\cdot \qquad (2.1.12)$$

The equation array (12) and conditions (11) give the linearized boundary-value task for definition of parametric oscillations of a conductive film flow under the vertical electromagnetic wave. Thus, as far as follows from (4), in the last two equations of system (11) the components of electromagnetic force have the second order by perturbation, they can be omitted. Then boundary conditions for the conductive and non-conductive media are identical.

Conditions on the other interfacial boundaries have been already partly discussed. Further they are concretized more in detail in relation to the considered physical situations. For the solid boundary $z=0$ conditions of the perturbation attenuation are set. If $z=0$ is a free surface, where the conditions of type (11) ought to state. Similar boundary conditions are on the top surface of the non-conductive medium: by considerable thickness of layer the perturbations in it are extinguished completely. By small thickness of a layer this condition is at contact of the non-conductive medium with a solid surface.

In tasks about spreading of perturbations (waves) the adequate treatment of initial conditions isn't always possible and, besides, it is often important to know not an absolute value of perturbation but the rate of its increase in time and (or) in space. As a rule the uniform initial conditions are used:

$$t=0, \qquad \vec{v}_j=0, \qquad p_j=0, \qquad \chi=0. \qquad (13)$$

Non-uniform initial conditions usually lead to more cumbersome results, which, however, have no essential qualitative distinction in comparison with a case of uniform conditions; therefore their use by many researchers admits inexpedient.

The equilibrium state of a number of the real physical systems is described with a sufficient accuracy by rather simple approximate functions, therefore the mathematical model (1)-(13) type is done significantly simpler and non-linear problems are successfully solved on computer. The examples of such tasks are considered further.

3 Excitation of surface oscillations by vertical electromagnetic wave

The problem by excitation of low-amplitude perturbations on a film flow surface by means of vertical progressive electromagnetic wave (3) using the above-stated linearized equations and boundary conditions is considered. For convenience of the task solution and increase of its generality the boundary task is transformed to dimensionless form.

Choose the characteristic scales of length, velocity, time, pressure and intensity of an electromagnetic field, respectively $b, u_0, b/u_0, \rho_1 u_0^2, H_0$ and put for simplicity $u_0 = const$. Then (11)-(13) are presented as

$$div\vec{v}_j=0, \quad \frac{\partial w_j}{\partial t}+(2-j)\frac{\partial w_j}{\partial x}+\rho_{1j}\frac{\partial p_j}{\partial x}=\frac{1}{\mathrm{Re}_j}\Delta w_j,$$

$$\frac{\partial u_j}{\partial t}+(2-j)\frac{\partial u_j}{\partial x}+\rho_{1j}\frac{\partial}{\partial x}(p_j+\frac{2-j}{2}Al\cdot H^2)=\frac{1}{\mathrm{Re}_j}\Delta u_j,$$

$$\frac{\partial v_j}{\partial t}+(2-j)\frac{\partial v_j}{\partial x}+\rho_{1j}\frac{\partial}{\partial y}(p_j+\frac{2-j}{2}Al\cdot H^2)=\frac{1}{\mathrm{Re}_j}\Delta v_j,$$

$$\frac{\partial H}{\partial t}+\frac{\partial H}{\partial x}=\frac{1}{\mathrm{Re}_m}\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right); \qquad (14)$$

Boundary conditions are:

$$z=\varepsilon, \quad p_1=p_2+\frac{1-\rho_{21}}{Fr^2}\chi+\frac{Al}{2}H_m^2-\frac{1}{We}\left(\frac{\partial^2\chi}{\partial x^2}+\frac{\partial^2\chi}{\partial y^2}\right)+$$

$$+2\left(\frac{1}{\mathrm{Re}_1}\frac{\partial w_1}{\partial z}-\frac{\rho_{21}}{\mathrm{Re}_2}\frac{\partial w_2}{\partial z}\right); \quad w=\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial x}; \qquad (15)$$

$$\mu_{j1}\left(\frac{\partial w_j}{\partial x}+\frac{\partial u_j}{\partial z}\right)=idem, \quad \mu_{j1}\left(\frac{\partial w_j}{\partial y}+\frac{\partial v_j}{\partial z}\right)=idem, \quad \vec{v}_1=\vec{v}_2;$$

$$z=0, \quad \vec{v}_1=0; \qquad z=\gamma, \quad \vec{v}_2=0; \qquad (16)$$

$$t=0, \qquad \vec{v}_j=0, \qquad p_j=0, \qquad \chi=0, \qquad H=H_0, \qquad (17)$$

where $Al=\mu_m H_0^2/(\rho_1 u_0^2)$, $Fr=u_0/\sqrt{gb}$, $We=\rho_1 b u_0^2/\sigma$, $\mathrm{Re}_j=u_0 b/\nu_j$, $\mathrm{Re}_m=u_0 b/\nu_m$ are the Alfven, Froude, Weber, Reynolds and magnetic Reynolds numbers, respectively; $H_0=H(0)$, and γ - dimensionless total thickness of the layers of conductive and non-conductive media, $\varepsilon=a/b$, $\rho_{ij}=\rho_i/\rho_j$, $\mu_{j1}=\mu_j/\mu_1$, *j*=1,2. For simplification, the dimensionless values keep the same view as the dimension ones.

The first boundary condition (16) corresponds to a film spreading on a solid surface. For free film flow instead of this condition by *z=0* should be considered also boundary condition of (15) type. Because $u_a=H_0\sqrt{\mu_m/\rho_1}$ is a velocity of spreading the Alfven waves, the Alfven number can be presented as a ratio of characteristic velocities: $Al=u_a^2/u_0^2$.

4 Parameters of electromagnetic wave

With account of the above and equations (3), from the last equation of the system (14) results

$$H_m=H_{m0}\exp\left\{-\left[ik+\left(m^2+k^2\right)/\mathrm{Re}_m\right]t\right\}, \qquad (18)$$

where $H_{m0}=H_m(0)$, $k=k_r+ik_i$, $m=m_r+im_i$. With account the expression $H_z=H_{m0}\exp i(kx+my-\omega t)$ and (18), yields the dispersive equation of the form $\omega=k-i\left(m^2+k^2\right)/\mathrm{Re}_m$, where from:

$$\omega_r=k_r+2\frac{m_r m_i+k_r k_i}{\mathrm{Re}_m}, \quad \omega_i=k_i+\frac{m_i^2+k_i^2-m_r^2-k_r^2}{\mathrm{Re}_m}, \qquad (19)$$

and then expression for the real part of the strength of magnetic field follows in the form

$$H_r=H_{m0}\exp(\omega_i t-m_i y-k_i x)\cos(k_r x+m_r y-\omega_r t). \qquad (20)$$

In general case the formula (20) describes progressive electromagnetic waves spreading in two-dimensional space (by each coordinate with its own speed) and exponentially growing (or decreasing) by amplitude in a space and (or) time. If $\omega=m_i^2+k_i^2+k_i\mathrm{Re}_m<m_r^2+k_r^2$, then $\omega_i<0$ by

$$t<\mathrm{Re}_m\left(m_i y+k_i x\right)/\left(k_i\mathrm{Re}_m+m_i^2+k_i^2-m_r^2-k_r^2\right).$$

If $\omega=m_i^2+k_i^2+k_i\mathrm{Re}_m>m_r^2+k_r^2$, then $\omega_i>0$ by

$$t>\mathrm{Re}_m\left(m_i y+k_i x\right)/\left(k_i\mathrm{Re}_m+m_i^2+k_i^2-m_r^2-k_r^2\right).$$

The phase and group spreading speeds of the progressing by *x* and *y* waves, with account of (19), (20) are, respectively:

$c_x^H = 1 + 2\frac{m_r m_i + k_r k_i}{k_r \mathrm{Re}_m}$, $c_y^H = \frac{k_r}{m_r} c_x^H$, $C_x^H = 1 + \frac{2k_i}{\mathrm{Re}_m}$, $C_y^H = \frac{2m_i}{\mathrm{Re}_m}$.

Important characteristics of the progressive magnetic field is slipping $s^H = \left(\left|\vec{c}^H\right| - \left|\vec{v}\right|\right) / \left|\vec{c}^H\right|$, where $\vec{c}^H = \left\{c_x^H, c_y^H\right\}$, as far as mutual influence of the electromagnetic and hydrodynamic fields is determined by magnetic Reynolds number $\mathrm{Re}_{ms} = \mathrm{Re}_m \cdot s^H$. Analysis shows that by $k_r m_r > 0$ the waves by x and y are spreading in positive direction if $\mathrm{Re}_m + 2m_i m_r / k_r > -k_i$. If $m_i = k_i = 0$, then for $\forall t > 0$ the wave amplitude decreases, and $c_x^H = 1$, $c_y^H = k_r / m_r$, $\omega_i = -\left(m_r^2 + k_r^2\right) / \mathrm{Re}_m$, where from follows that with decrease of wave length (increase m_r, k_r) the rate of amplitude decrease is strongly growing by time, and wave energy is transferred only by x.

Similar influence is done by magnetic Reynolds number: the lower is Re_m, the higher is phase speed of the electromagnetic wave spreading, and in case of infinitely high conductivity of it becomes infinite too. Then the wave group speed (the speed of energy transfer) is infinitely growing as well. The components of running speed of the electromagnetic wave in a space are interconnected as

$$v_y^H + k_i / m_i \cdot v_x^H = \omega_i / m_i. \tag{21}$$

From the equations (19)-(21) can be shown that in case of spreading of the short waves by x and y their decrease in time is identical while in a first case the speed of wave spreading differ on the value of velocity of unperturbed film flow (here it is 1).

The phase speeds of the long waves are substantially higher than the ones of the short waves: $c_x^H \sim m_r$, $c_y^H \sim k_r$. For example, for short waves by $k_r = m_r$ it is got $c_{x,y}^H = 1 + 2\frac{m_i + k_i}{\mathrm{Re}_m}$ and decrement of such wave decrease in time is twice bigger than corresponding value in a previous case.

By $m_i \gg 1, k_i \gg 1$ yields, correspondingly:

$$c_x^H \sim c_y^H \sim 2\frac{m_i}{\mathrm{Re}_m}, \quad C_y^H \gg C_x^H, \quad C_y^H = \frac{2m_i}{\mathrm{Re}_m};$$

$$c_x^H \sim c_y^H \sim 2\frac{k_i}{\mathrm{Re}_m}, C_x^H \gg C_y^H, C_x^H \sim \frac{2k_i}{\mathrm{Re}_m}, v_x^H \sim \frac{k_i}{\mathrm{Re}_m}, v_y^H \sim \frac{m_i}{\mathrm{Re}_m},$$

where from follows that spreading velocity of the constant crest amplitude is twice less then phase speed of the waves, and a group speed by one coordinate coincides with the phase speed being substantially higher the one by other coordinate. Thus, the wave energy has preferable spreading direction despite the waves' phase speeds by x and y are of the same order.

The established parameters of electromagnetic waves are important for analysis of their action on conductive medium and selection of the control fields, which supply an achievement of the requested behavior of the controlled continua, e.g. liquid metal film flow. The analysis performed has shown that short electromagnetic wave with constant by x, y amplitude is available if and only if magnetic field is rapidly decreasing in time, so that it is similar to the impulse packet having abrupt back front of type $\exp\left(-k_r^2 t / \mathrm{Re}_m\right)$.

5 Solution of the boundary problem

Assuming the small-amplitude perturbations, we seek solution of the boundary problem (14)-(17) as a linear response of the system to an external action. Following the superposition principle, the excited by electromagnetic wave (3) surface waves in a film flow are considered as proportional to the function $\exp 2i\left(kx + my\right)$ with corresponding amplitudes – functions of time. Statement of the boundary conditions by x,y is not needed, parameters of medium are prone to similar perturbations differing with additional amplitude dependence of coordinate z. This approach is attainable when the wave amplitude reaches substantial value so that the non-linear effects become playing.

Solution of the boundary problem (14)-(17) was convenient to search with use of the integral transformation methods [36]. The averaging of highly-oscillating terms under integrals was performed by the second scheme [37]. The approximate solution obtained is close enough to the exact solution on the infinite interval of time and its fidelity grows with increase of k_r (according to the assumptions made $k_r \ll 1/\varepsilon$). It has the form:

$$\zeta = \frac{Ha\sqrt{Al}\left\{\exp\left[-\frac{8k}{\varepsilon^2}\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2} + \frac{8k^2}{We}\right)t\right] - \exp\left(-\frac{4k^2}{\mathrm{Re}_m}t\right)\right\}}{\left[\pm 2\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2} + \frac{8k^2}{We}\right)(1-i) - k\frac{\varepsilon^2}{\mathrm{Re}_m}\right]\exp(2ikt)}. \tag{22}$$

Analysis of the solution (22) shows that vertical electromagnetic wave (18) causes perturbation of the film flow surface, which has a part similar to the exciting wave force and the other one, different. By real k (speed of the wave spreading is equal to the velocity of film flow, $\mathrm{Re}_{ms} = 0$) the surface wave is alone (similar by form to the exciting force). By $\rho_{21} > 1 + \frac{8k^2}{Oh^2 \cdot Ga^2}$ the exponential growing of the amplitude of film surface oscillations in time is available, by $\rho_{21} < 1 + \frac{8k^2}{Oh^2 \cdot Ga^2}$ the influence of the second medium is only the quantitative one.

The amplitude of film surface oscillations excited by electromagnetic field is proportional to $Ha\sqrt{Al}$, and parametric resonance of the system is achieved by (for simplicity a case of real value k is taken):

$$\mathrm{Re}_1 = \frac{8Be\left[(1-\rho_{21})Ga^2 + 8k^2/Oh^2\right]^2}{k\varepsilon^2\left\{\varepsilon/Be \mp 8\left[(1-\rho_{21})Ga^2 + 8k^2/Oh^2\right]\right\}}. \quad (23)$$

where Be, Ga, Oh are the Batchelor, Galileo and Ohnesorge numbers, respectively. Ought to account that (23) is correct by $\mathrm{Re}_1 \gg 1$. In condition obtained the single dynamic criterion Re_1 is expressed through the parameters of system and wave number k, $Be = \nu/\nu_m$, $Ga = \sqrt{gb}b/\nu$, $Oh = b\sqrt{\rho g/\sigma}$.

Examples of computations by (23) for aluminum melt: $\rho_1 = 2{,}4\cdot10^3$ kg/m^3, $\nu = 1{,}21\cdot10^{-6}$ m^2/s, $\nu_m = 0{,}164$ m^2/s, $\sigma = 0{,}5$ N/m, $\rho_1 = 2{,}4\cdot10^3$ kg/m^3, $\rho_2 = 10^3$ kg/m^3, g =9,8m/s^2, ε =1, Be=7,4$\cdot10^{-6}$, ρ_{21}=0,42 are given in Figs 4, 5:

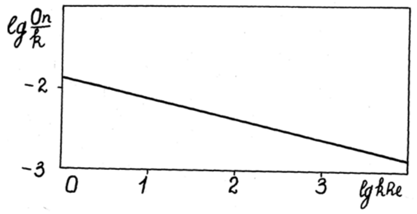

Fig. 4 Region of the film flow instability: Ohnesorge number depending on the Reynolds number

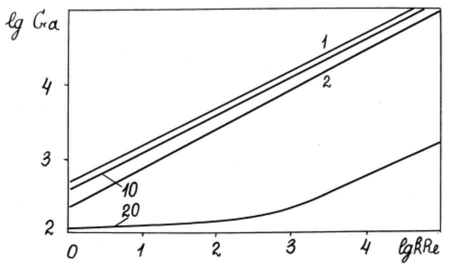

Fig. 5 The characteristics of parametric resonance for conductive film flow by $Ga \gg 1/Oh$: curves 1, 2 bifurcate from 10 (non-conductive liquid)

where from follows that for conductive liquid the both sets of curves are bifurcating from the one corresponding to the non-conductive liquid.

The Eigen oscillations from (22) have the form $\zeta_* = \zeta_0 \exp\left\{\left[(\omega_* - 1)i \pm \omega_*\right]t\right\}$, $\omega_* = \frac{8k}{\varepsilon^2}\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2} + \frac{8k^2}{We}\right)$, they are available growing and falling with time. The increase (decrease) of oscillations is strengthen with increase of k and Re_1. Here is $\zeta_0 = \zeta(0)$.

6 Conclusions by the results obtained

- by electromagnetic excitation the short electromagnetic wave has to be like a pack of impulses with the steep back front $\sim \exp(-k_r^2 t/\mathrm{Re}_m)$;
- vertical progressive electromagnetic wave causes two waves on a film surface, one of which is similar to a wave of the exciting force;
- amplitude of surface oscillations is $\sim Ha\sqrt{Al}$, and a parametric resonance (film decay) is reached at $\mathrm{Re} = \mathrm{Re}^*$ determined by a formula obtained depending of k, ρ_{21}, Be, Ga, Oh;
- for the conductive liquid two families of curves for the film decay are bifurcating from the one corresponding to non-conductive liquid.

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