Natural-frequency Analysis of Two-Roof Steel Frames via the Finite Element Method

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Abstract: - Steel frame structures are widely used in the fields of mechanics and construction, such as in factories and workshops with many different sizes and structures. Therefore, it is necessary to calculate the strength, stiffness, and stability of the frame structure, thereby determining the critical size and critical force that a structure can withstand. However, in reality, there are sometimes cyclical loads such as vibrations caused by compactors, vibrations caused by impacts, or by machines operating in the factory; in such cases, if the oscillation frequency of the load coincides with the free oscillation frequency of the frame, resonance may occur, which has a very harmful effect on the structure. Therefore, calculating and determining the oscillation frequency and the specific oscillation form of the frame structure will be the basis for avoiding resonance; or taking remedial measures when it occurs. In this study, the finite element method is used to determine the natural oscillation frequency and draw the corresponding natural oscillation forms of two-roof steel frames. In addition, the influence of geometric dimensions on the natural vibration mode of the beam is also calculated in this study. The calculation results can be used in the future to design structures with two-roof steel frame models.

Key-Words: - Frame, finite element, natural frequency, vibration mode, two-roof steel frames.

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1 Introduction

Steel frame structures are widely used in the fields of mechanics and construction, such as in factories and workshops with many different sizes and structures. With the characteristics e.g., simple, light, short construction time, easy to change, repair, and at the same time environmentally friendly while still ensuring safety; therefore, steel frame structures are quite popular today. Steel frame structures are also used to build houses in civil works with many outstanding advantages compared to other types of structures. To ensure safety in use, the structure needs to be durable enough, hard enough, and stable, and at the same time, it is necessary to determine the frequency and form of their oscillation to avoid resonance, [1], [2], [3].

Metal frame structures are usually made of high-strength steel or alloy steel, which are common materials and easy to buy on the market; therefore, they are convenient for repair, maintenance, or replacement. Material parameters including elastic modulus (E), Poisson's ratio (ν), and density (ρ) are usually determined, [4], [5]. Steel frames are usually made from shaped steel (I-beams, angle steel, or trough steel), manufactured in modules, and standardized so that the cross-sectional parameters of the structure have been determined, [4], [6].

These parameters are essential to calculate frame structures in order to determine strength, stiffness, and stability as well as natural oscillation frequencies, [7]. There are many methods to calculate frame structures, such as the analytical method, [8], experimental method, simulation by software [7], or use of finite element method, [9], [10], [11]. Nowadays, with the development of computational science, we can use the finite element method to calculate in a short time, easily change boundary conditions, large number of elements, complex structures, and can be used to simulate macroscopic structures or nanometer-sized structures to produce highly reliable calculation results, [9], [10], [11]. The stability of beam structures is studied by [12], [13], [14]; Structural vibrations are calculated by using the finite element method, [15]. The finite element method is used to calculate the free vibration of Z-shaped, L-shaped, and C-shaped structural beams, [16]; however, this method did not research two-roof steel frames. The two-roof steel frames are suitable for factories requiring wide beams. Therefore, in this study, we use the finite element method to determine the frequency and natural vibration mode of two-roof steel frames. In addition, the influence of geometric dimensions on the natural vibration mode of the beam is also calculated in this study.

2 Finite Element Method for Frame Problems

The load-bearing characteristics of a flat frame are tensile-compressive, and bending, [17]. The axis of the frame is a zigzag line, thus the coordinate system attached to the cross-section of the frame changes depending on each frame section (Figure 1). Due to the load-bearing characteristics of the frame at each cross-section, there are 3 internal force components including axial force, shear force, and bending moment that make 3 deformation components such as axial force (u), deflection (v) and rotation angle of the cross-section (θ). To calculate the frame by the finite element method, using a two-node link element, each node will have 3 displacement components characterizing the deformation along the link axis, deflection, and rotation angle of the cross-section, then the displacement of a two-node frame element $\{u\}_e = \{u_1, v_1, \theta_1, u_2, v_2, \theta_2\}^T$, is this displacement vector attached to the local coordinate system.

Because when subjected to force, the parts of the frame are subjected to tension-compression, and also bending; therefore, according to the principle of additive effects, the frame problem corresponds to the central tension-compression problem plus the horizontal plane bending problem of the two-node link elements (Figure 2); the element stiffness matrix of the frame element will be expanded from the stiffness matrix of the bending beam element, taking into account the influence of the axial force, [10].

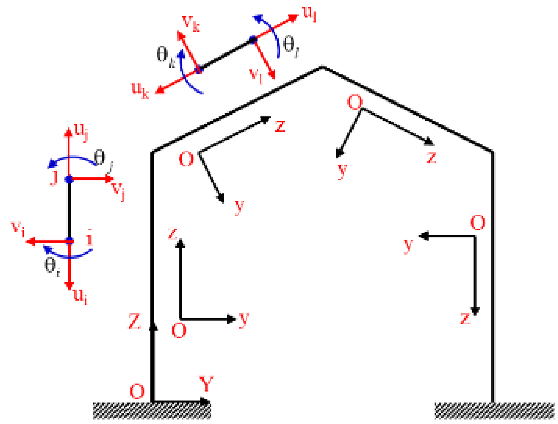

Fig. 1: Two-roof steel frames

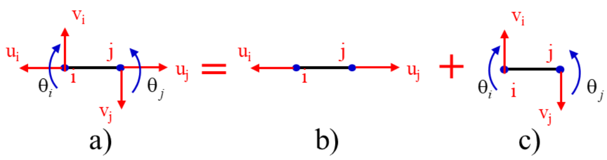

Fig. 2: 2-node frame element model. a) 2-node frame element, b) tension link element, c) plane transverse bending beam element

The stiffness matrix of the tensile link element is determined as follows, [10]:

$$[k_e]_{KN} = \frac{A_e E_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (1)$$

In which, A_e is the cross-sectional area, E_e is the elastic modulus, l_e is the length of the link element.

The stiffness matrix of the bending beam element is determined as follows, [10]:

$$[k_e]_D = \frac{EJ}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e & -6l_e & 2l_e \\ 12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e & -6l_e & 4l_e \end{bmatrix} \quad (2)$$

Because on a frame element, there are 2 nodes, each node has 3 displacements, the frame element stiffness matrix $[k_e]_{6x6}$ has a size of 6 rows, 6 columns. Expanding the matrix of tension bar elements and bending beam elements and then adding them together, we get the frame element stiffness matrix, [10]:

$$\left[\bar{k}_e\right]_{KH} = \begin{bmatrix} (AE/l_e) & 0 & 0 & (-AE/l_e) & 0 & 0 \\ 0 & 12EJ/l_e^3 & 6EJ/l_e^2 & 0 & -12EJ/l_e^3 & 6EJ/l_e^2 \\ 0 & 6EJ/l_e^2 & 4EJ/l_e & 0 & -6EJ/l_e^2 & 2EJ/l_e \\ (-AE/l_e) & 0 & 0 & (AE/l_e) & 0 & 0 \\ 0 & -12EJ/l_e^3 & -6EJ/l_e^2 & 0 & 12EJ/l_e^3 & -6EJ/l_e^2 \\ 0 & 6EJ/l_e^2 & 2EJ/l_e & 0 & -6EJ/l_e^2 & 4EJ/l_e \end{bmatrix} \quad (3)$$

The stiffness matrix of the frame element is calculated according to the formula in Eq. (3) for the local coordinate system attached to the element, the corresponding elements are shown in Figure 1. In the global coordinate system, the degrees of freedom are calculated corresponding to the local displacement through the rotation matrix [T], which is expressed as follows:

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

In which, θ is the angle; it is created by the axis of the frame element with the horizontal direction as shown in Figure 3.

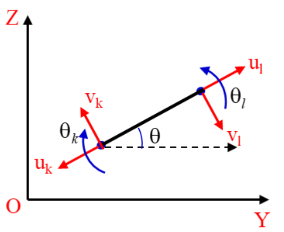

Fig. 3: Frame element in the global coordinate system

The frame element stiffness matrix in the global coordinate system is:

$$\left[k_e\right]_{KH} = [T]^T \left[\bar{k}_e\right]_{KH} [T] \quad (5)$$

From there, the overall stiffness matrix of the frame $[K]^{TT}_{3Nx3N}$ is the composite matrix of the element stiffness matrices, [9], [10], where N is the total number of divided nodes in the system. The overall stiffness matrix is used to solve frame problems using the finite element method.

The finite element equation system of the structural dynamics problem is established and written in matrix form as follows, [9], [10]:

$$[K]^{TT}\{q\} + [M]^{TT}\{\ddot{q}\} = \{f\} \quad (6)$$

In which, $[K]^{TT}, [M]^{TT}, \{q\}, \{f\}$ are the overall stiffness matrix, overall mass matrix, displacement vector, and corresponding nodal force vector of the entire frame, respectively. In this research, damping is completely ignored.

For the frame problem, the overall mass matrix is established from the frame element mass matrix. The method of establishing the element mass matrix of the frame element in the local coordinate system is similar to establishing the stiffness matrix through the tension bar element and the bending beam element, [9], [10]:

$$\left[m^*\right] = \frac{\rho A l_e}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l_e & 0 & 54 & -13l_e \\ 0 & 22l_e & 4l_e^2 & 0 & 13l_e & -3l_e \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l_e & 0 & 156 & -22l_e \\ 0 & -13l_e & -3l_e & 0 & -22l_e & 4l_e^2 \end{bmatrix} \quad (7)$$

In which, ρ is the density; A is the cross-sectional area of the frame; l_e is the element length.

Similarly, for the stiffness matrix, from the element mass matrix in local coordinates, calculate the element stiffness matrix on the global coordinate system through the rotation matrix [T] as follows:

$$\left[m_e\right]_{KH} = [T]^T \left[m^*\right]_{KH} [T] \quad (8)$$

If the right side $\{f\} = \{0\}$, equation (6) is called the undamped natural oscillation equation of the dynamic system, [9], [10], the solution in Eq. (8) is obtained as:

$$\{q\} = \{\bar{q}\}\sin(\omega t) \quad (9)$$

In which, $\{\bar{q}\}$ is the amplitude vector of oscillation at the corresponding nodes.

Calculating derivative (9) and substituting into (6) gets:

$$\left([K]^{TT} - \omega^2[M]^{TT}\right)\{\bar{q}\} = \{0\} \quad (10)$$

Because the existence condition of the solution is not trivial; therefore Eq. (10) can be rewritten as:

$$[K]^{TT} - \omega^2[M]^{TT} = \{0\} \quad (11)$$

If set $\lambda = \omega^2$, equation (11) has the form:

$$[K]^{TT} - \lambda[M]^{TT} = \{0\} \quad (12)$$

The solution of equation (12) is the problem of finding the eigenvalues λ; the oscillation angular frequency, calculated as $\omega = \sqrt{\lambda}$ (rad/s), is the oscillation frequency of the system. Corresponding to the eigenvalues are the eigenvectors λ, which are also the largest oscillation amplitudes at the nodes; the frames at the corresponding positions are the oscillation modes. The results of calculating the oscillation frequencies and oscillation modes of some frame structures are presented in the following section.

3 Results and Discussion

3.1 The Natural Vibration of the Structural Frame Γ

Problem parameters: A=10x10(mm); ρ=1000kg/m^3, E=100 GPa, number of frame elements N= 40, [9]. The results of determining the free oscillation frequency are shown in Table 1, the natural oscillation form is shown in Figure 4.

Table 1. Frame oscillation frequency Γ

No.	One end clamped, one end free, ω (rad/s)			One end clamped, one end supported, ω (rad/s)	Two clamped ends, ω (rad/s)
	Present results	Reference [9]	Error with [9], %	Present results	Present results
1	34.41	34	1.2	71.55	460.3
2	93.49	92	1.6	451.89	667.79
3	460.33	455	2.3	652.5	1491.37

In this study, the natural oscillation frequency of the frame was compared with the calculation result, [9], when the frame has one clamped end; in Table 1, the largest error is 2.3%, which shows that the calculation program is reliable. The data in Table 1 shows that the natural oscillation frequency of the frame with two clamped ends is the largest when comparing the corresponding natural oscillation frequencies. In Figure 4, the first three oscillation modes of the frame were drawn corresponding to the boundary conditions. From there, we determined the regions with the largest free oscillation amplitude, specifically for the case of the frame with one free end or one end as a support. The free end and the support side have the largest oscillation amplitude; in contrast to the case of double-end clamps, the largest oscillation amplitude occurs on the inside depending on the specific oscillation form. If the oscillation form has one antinode, the midpoint of the frame segments has the largest amplitude. If there are two antinodes, the points with the largest amplitude are 1/4 of the frame length away from the two ends.

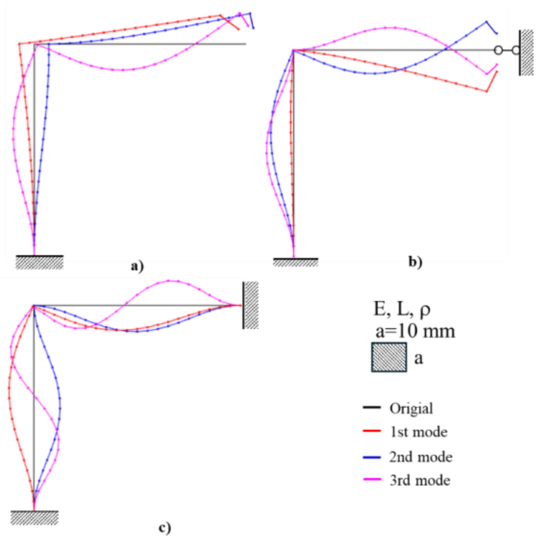

Fig. 4: Oscillation mode of the frame with different boundary conditions. a) One end clamped, one end free; b) One end clamped one end supported; c) two clamped ends

3.2 Natural Oscillations of Two-Roof Steel Frame

The two-roof steel frame model is made from I-shaped cross-section steel (Figure 5), with the following material parameters i.e., steel code I_{40}, width b=400mm; h_1=155mm, cross-sectional area A=71.4cm^2, moment of inertia J_x= 18930 cm^4, density ρ=7857kg/m^3, elastic modulus E=2.1x10^{4}kN/cm^2, [4]; frame dimensions i.e., height H=4m; width B, tilt angle α (degree); frame elements are divided into equal lengths l_e= 0.04m; the frame is clamped at both ends. The results of calculating the frequency and dynamic form of this model are summarized in Table 2 and Figure 5.

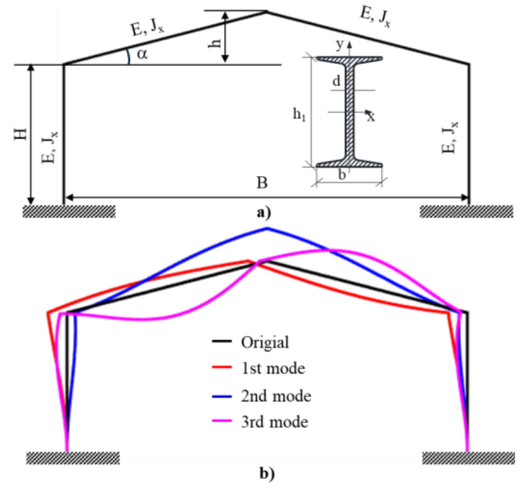

Fig. 5: Oscillation of steel frame model, I-section. a) Frame model; b) mode of oscillation

The model of a two-roof steel frame made of I_{40} steel, the corresponding material parameters, and the results of calculating the natural oscillation frequency in Table 2 and Table 3 show that the oscillation frequency of the frame is quite large. The natural oscillation forms of the steel frame are shown in Figure 5. From the 3 natural oscillation forms of this steel frame, it predicts that the positions or areas will have the largest oscillation amplitude when the frame oscillates. Corresponding to the first oscillation form, the corner point of the frame has the largest horizontal oscillation amplitude; with the second oscillation form, the top of the frame oscillates with the largest amplitude; in the third case, the largest oscillation amplitude is in the middle of the sloping roof. From that result, in the process of manufacturing and assembling the structure, it is necessary to have measures to strengthen the connection to create solidity for the structure.

3.3 Effect of Width on the Natural Frequency of the Beam

Table 2. Effect of width *B* on the natural frequency of two-roof steel frames

Mode	H=4m, α=15^0, width B changes					
	TH21 B=9m	TH22 B=12m	TH23 *B*=15m	TH24 *B*=20m	TH25 *B*=25m	TH26 *B*=30m
1	108.58	89.44	75.31	57.78	44.73	34.91
2	196.09	127.86	92.58	62.32	46.8	37.58
3	472.51	293.65	202.81	129.17	94.72	75.89
4	803.23	479.24	314.51	180.3	116.33	81.10
5	936.93	848.25	588.93	347.03	228.1	161.89

Table 2 shows the results of calculating the natural frequency of two-roof steel frames when *H* is fixed (H = 4m) and the tilt angle α is 15°, with the width *B* varying from 9m to 30m.

Figure 6 shows that when the width *B* increases, the corresponding natural oscillation frequency of the frame decreases. This is completely consistent with reality. Due to this, when the distance between the beams increases, the stiffness of the structure decreases. Corresponding to the calculation cases, the graph has a large slope when the length of structure B changes from 9m to 30m; at this time, the width of the frame changes, and the corresponding natural frequency changes significantly. Meanwhile, when the size changes from 15m to 30m, the graph has a small slope; that means the influence of the width of the structure on the oscillation frequency is not obvious. For each structure, when the first natural oscillation frequency is the smallest, the value will increase correspondingly with the next natural frequency.

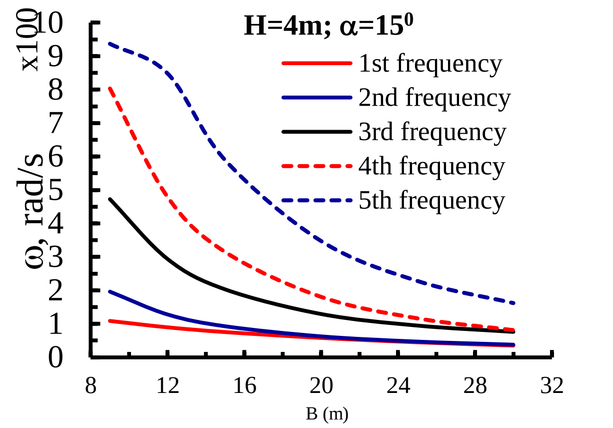

Fig. 6: Variation of the first five natural frequencies with width B (*H*=4m, α=15°)

3.4 Effect of Tilt Angle α on Natural Frequency

Table 3 shows the results of calculating the natural frequency of two-roof steel frames when the width *B* and height *H* are fixed, i.e., B = 9m, H = 4m, with the tilt angle α changing from 10° to 30°.

Table 3. Effect of tilt angle α on the natural frequency of two-roof steel frames

Mode	***B*=9m; *H*=4m, α changes**				
	TH11 α=10^0	**TH12 α=15^0**	**TH13 α=20^0**	**TH14 α=25^0**	**TH15 α=30^0**
1	110.94	108.58	105.64	102.10	97.92
2	190.82	196.09	198.99	199.41	197.28
3	481.5	472.51	459.99	444.14	425.12
4	861.87	803.23	719.73	636.51	558.77
5	957.82	936.93	937.59	937.99	933.88

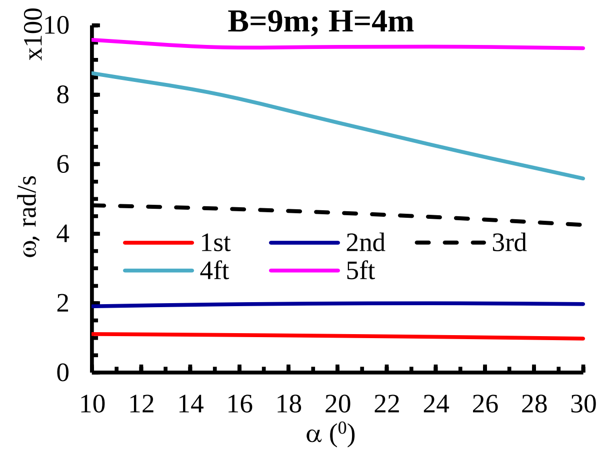

Fig. 7: Effect of tilt angle α on natural frequency of two-roof steel frames

Figure 7 shows the influence of the roof slope angle of the structure while the width and height of the structure have constant dimensions of *B*=9m; *H*=4m; during the calculation, the slope angle changes from $\alpha=10^0$ to $\alpha=30^0$. The calculation results show that the slope angle of the structure does not affect much the natural oscillation frequency of the frame structure, which is clearly shown when the graph shows the corresponding horizontal frequency lines. However, with the 4th frequency, when the slope angle increases, this frequency decreases; the relationship between the frequency and the slope angle of the frame through the linear equation has the form ω=-15.5α +1025.2. In this case, the graph also clearly shows that the first oscillation frequency of the structures is the smallest, that value increases corresponding to the following oscillation frequencies.

4 Conclusion

By the Γ frame oscillation calculation program, the results have determined the natural oscillation frequency without damping, and the comparison results with [9], show that the calculation program has high reliability. On the other hand, with the change of different boundary conditions, when the frame is clamped at both ends, the natural oscillation frequency is the largest. The oscillation form has been drawn corresponding to the boundary conditions, thereby showing the difference in amplitude and Γ shape when the frame oscillates. In this study, the natural oscillation frequency of the two-sided inclined roof steel frame model with an I-shaped cross-section (code I_{40}) has been determined. The corresponding natural oscillation forms have also been determined. By the natural oscillation form, the positions with the largest free oscillation amplitude have been predicted. We have studied the influence of the roof slope and the width of the structure on the natural oscillation frequency. The research results show that when the width of the structure changes from 9m to 15m, the natural oscillation frequency of the structures has a large difference (the slope of the graph is large); when the size changes from 9m to 30m, the natural oscillation frequency of the structure does not change much (the graph is almost horizontal). When changing the slope angle of the roof, the corresponding natural oscillation frequency does not change much. With the 4th oscillation frequency, when the slope angle changes, the oscillation frequency decreases; when linearizing the relationship between the slope angle and the fourth oscillation frequency, we have the equation ω=-15.5α+1025.2. The results also demonstrate that the first natural oscillation frequency is the smallest, this value increases with the following oscillation frequencies. These results can be used in practice to design structures with two-roof steel frame models.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

This paper originated from an idea by Nguyen Van Trang and Nguyen Huu Tu. The simulation and calculation phases were carried out by Nguyen Van Thanh and Dang Van Manh. The manuscript was written by Nguyen Huu Tu, with support from Nguyen Van Trang. Nguyen Van Trang is the contact author, responsible for submitting articles and responding to comments. All authors have reviewed and agreed to the content.

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Conflict of Interest

The authors have no conflicts of interest to declare.