TRIAD-Aided EKF for Satellite Attitude Estimation and Sensor Calibration

MEHMET ASIM GOKCAY
Hezârfen Aeronautics and Astronautics Technologies Institute for Space Sciences
Turkish National Defense University
Yeşilyurt, 34149, Bakırköy, Istanbul
TURKEY

CHINGIZ HAJIYEV
Faculty of Aeronautics and Astronautics
Istanbul Technical University
Ayazağa, 34469, Maslak, Istanbul
TURKEY

Abstract: - Increasing demand for the space operations, space industry turns its face to cost effective solutions. Small satellites, due to their size and cost, are receiving interest from many organizations. Two of most common sensors that are being used in nanosatellites are magnetometers and sun sensors. In this work, magnetometer and sun sensor measurements are fused together with the TRIAD algorithm to produce body angle estimation. Combining with gyroscopes measurements, an Extended Kalman Filter is used to estimate body angles, angular velocities, gyroscope and magnetometer biases.

Key-Words: Extended Kalman filter, satellite attitude estimation, magnetometer, sun sensor, gyroscope

Received: November 29, 2021. Revised: November 2, 2022. Accepted: November 29, 2022. Published: December 31, 2022.

1 Introduction

Increasing usage of small satellites led to an increase in research about their hardware and software technologies. Attitude determination and control (ADC) is a key part for any type of mission [1]. It directly affects the mission success. Due to their size, sensor variety is considerably lower than regular size satellites. This fact is a challenge for ADC researchers. Commonly used sensors are used for body angle and angular velocity estimations and Euler angles are used to represent satellite attitude. Many different methods have been proposed for body angle estimation [9]. In this work, sun sensor and magnetometer are fed to the TRIAD algorithm to produce body angle estimations [2],[3],[4],[5],[8]. Combining with gyroscope measurements, these TRIAD outputs are then fed to an extended Kalman filter as measurements in order to estimate body angles, angular velocities, gyroscope and magnetometer biases. These estimated biases are fed back to sensor models for calibration.

2. Equations of Motion

To define satellite motion, kinematic and dynamic equations are derived. In this paper, the DCM is constructed using 2-1-3 rotation.

$$A_{213}(\theta,\phi,\psi) = A_3(\psi)A_1(\phi)A_3(\theta)$$
$$= \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad (2.1)$$

After matrix multiplication, dcm has been derived.

$$\begin{bmatrix} c\psi c\theta + s\phi s\psi s\theta & c\phi s\psi & c\theta s\phi s\psi - c\psi s\theta \\ c\psi s\phi s\theta - c\theta s\psi & c\phi c\psi & s\psi s\theta + c\psi c\theta s\phi \\ c\phi s\theta & -s\phi & c\phi c\theta \end{bmatrix} \quad (2.2)$$

where c(.) and s(.) are cosine and sine functions respectively. From 2-1-3 rotation matrix, Euler angles can be extracted from matrix elements with equations below.

$$\psi = \operatorname{atan2}\left(\frac{A(1,2)}{A(2,2)}\right) \quad (2.3)$$

$$\theta = \operatorname{atan}\left(\frac{A(3,1)}{A(3,3)}\right) \tag{2.4}$$

$$\phi = \operatorname{atan}\left(\frac{-A(3,2)\cos(\psi)}{A(2,2)}\right) \tag{2.5}$$

where atan and atan2 are both arctangent functions. atan2 is the arc tangent function with two arguments for complete coverage of four quadrants.

2.1 Kinematic equations

Obtaining a frame from another frame is possible with rotation matrices. Using the fact that angular velocities are additive, angle rates can be determined in terms of these velocities. Considering 2-1-3 rotation, the initial frame rotates about the y axis by θ. Second rotation is about x′ by ϕ and final rotation is about w by ψ. Summing angular velocity vectors for each rotation [1]

$$\omega = \dot{\theta}\vec{y} + \dot{\phi}\vec{x}' + \dot{\psi}\vec{w} \tag{2.6}$$

Taking the components of ω in $\vec{u}, \vec{v}, \vec{w}$

$$\begin{aligned} \omega_u &= \dot{\theta}\vec{y}\vec{u} + \dot{\phi}\vec{x}'\vec{u} \\ \omega_v &= \dot{\theta}\vec{y}\vec{v} + \dot{\phi}\vec{x}'\vec{v} \\ \omega_w &= \dot{\theta}\vec{y}\vec{w} + \dot{\phi}\vec{x}'\vec{w} + \dot{\psi} \end{aligned} \tag{2.7}$$

In order to derive kinematic equations, (2.7) needs to be determined. $\vec{y}\vec{u}, \vec{y}\vec{v}$ and $\vec{y}\vec{w}$ can be calculated from DCM. $\vec{y}$ indicates that second column of the DCM.

$$\vec{y}\vec{u} = \cos\phi\sin\psi \tag{2.8}$$

$$\vec{y}\vec{v} = \cos\phi\cos\psi \tag{2.9}$$

$$\vec{y}\vec{w} = -\sin\phi \tag{2.10}$$

For second rotation, $R_3(\psi)R_1(\phi)$ matrix needs to be calculated. Taking first column of the second rotation matrix

$$\vec{x}'\vec{u} = \cos\psi \tag{2.11}$$

$$\vec{x}'\vec{v} = -\sin\psi \tag{2.12}$$

$$\vec{x}'\vec{w} = 0 \tag{2.13}$$

Hence, angular velocity equations become

$$\omega_u = \dot{\theta}\cos\phi\sin\psi + \dot{\phi}\cos\psi \tag{2.14}$$

$$\omega_v = \dot{\theta}\cos\phi\cos\psi - \dot{\phi}\sin\psi \tag{2.15}$$

$$\omega_w = -\dot{\theta}\sin\phi + \dot{\psi} \tag{2.16}$$

In matrix form,

$$\omega_{bo} = \begin{bmatrix} \cos\phi\sin\psi & \cos\psi & 0 \\ \cos\phi\cos\psi & -\sin\psi & 0 \\ -\sin\phi & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} \tag{2.17}$$

Taking inverse of the matrix, Euler angle rates can be determined in terms of angular velocities in body frame with respect to reference frame [1]. Rate equations are given below

$$\dot{\phi} = \omega_u\cos\psi - \omega_v\sin\psi \tag{2.18}$$

$$\dot{\theta} = (\omega_u\sin\psi + \omega_v\cos\psi)\sec\phi \tag{2.19}$$

$$\dot{\psi} = \tan\phi(\omega_u\sin\psi + \omega_v\cos\psi) + \omega_w \tag{2.20}$$

Rotations need to be defined with respect to inertial frame. For this reason, a transformation matrix is needed. Using the relation below, angular velocities with respect to the inertial frame can be calculated.

$$\begin{bmatrix} \omega_u \\ \omega_v \\ \omega_w \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} - A\begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix} \tag{2.21}$$

where $\omega_0 = \sqrt{\frac{\mu}{r^3}}$ is angular velocity at the altitude r. μ is gravitational constant of the Earth and r is the distance between center of mass of the satellite and the Earth, $r = |\vec{r}|$.

2.2 Satellite dynamics

Attitude dynamics is related with time derivative of the angular momentum vector.If the origin of the body frame is selected as the center of mass, c, then [1],

$$h_c = J\vec{w} \tag{2.22}$$

where h is angular momentum vector, w is angular velocity vector in body frame wrt inertial frame and J is moment of inertia matrix. J is defined as,

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{2.23}$$

The relation between time derivative of the angular momentum and the angular velocity is,

$$\dot{\vec{h}}_c = \dot{\vec{h}}_b + \vec{w} \times (\vec{h}_c) \tag{2.24}$$

where $\dot{\vec{h}}_b$ is the time derivative of angular momentum as seen in a body-fixed frame. Knowing the time rate of change of the angular momentum is equal to external torque, T_c and using (2.22)

$$T_c = \dot{\vec{h}}_b + \vec{w} \times (J\vec{w}) \tag{2.25}$$

Rewriting this equation considering time derivative of the w is same in both body-fixed and reference frames,

$$\dot{\vec{h}}_b = J\dot{\vec{w}} \tag{2.26}$$

$$\dot{\vec{w}} = J^{-1}\left[T_c - \vec{w} \times \left(J\vec{w}\right)\right] \tag{2.27}$$

The external torque can be defined as the sum of the gravity gradient torque, aerodynamic torque, magnetic torque and solar pressure disturbance. In this thesis, only magnetic torque is considered as external torque.

$$T_c = T_m \tag{2.28}$$

Finally, rewriting (2.25) for discrete time,

$$w_{x_{k+1}} = w_{x_k} + \frac{\Delta t}{J_x}\left[w_z w_y \left(J_y - J_z\right) + T_m\right] \tag{2.29}$$

$$w_{y_{k+1}} = w_{y_k} + \frac{\Delta t}{J_y}\left[w_x w_z \left(J_z - J_x\right) + T_m\right] \tag{2.30}$$

$$w_{z_{k+1}} = w_{z_k} + \frac{\Delta t}{J_z}\left[w_x w_y \left(J_x - J_y\right) + T_m\right] \tag{2.31}$$

Equations (2.14)-(2.16) and (2.29)-(2.31) describe the satellite attitude motion

3 Models of Sensor Measurements

3.1 Magnetometer Measurement Model

Magnetometer is the most commonly used sensor particularly in nanosatellite applications. For magnetic field vector, dipole model is used. Sensor model is given below,

$$\begin{bmatrix} B_x(\phi,\theta,\psi,t) \\ B_y(\phi,\theta,\psi,t) \\ B_z(\phi,\theta,\psi,t) \end{bmatrix} = A \begin{bmatrix} B_1(t) \\ B_2(t) \\ B_3(t) \end{bmatrix} + b_m + \eta_m \tag{3.1}$$

where $B_1(t)$, $B_2(t)$ and $B_3(t)$ indicates Earth magnetic field vector components in orbit frame and given by [5],

$$B_1(t) = \frac{M_e}{r^3}\left\{\cos\left(\omega_0 t\right)\left[\cos\left(\varepsilon\right)\sin(\iota) - \sin(\varepsilon)\cos(\iota)\cos(\omega_e t)\right] - \sin\left(\omega_0 t\right)\sin(\varepsilon)\sin\left(\omega_e t\right)\right\} \tag{3.2}$$

$$B_2(t) = -\frac{M_e}{r^3}\left[\cos\left(\varepsilon\right)\cos\left(\iota\right) + \sin\left(\varepsilon\right)\sin\left(\iota\right)\cos\left(\omega_e t\right)\right] \tag{3.3}$$

$$B_3\left(t\right) = \frac{2M_e}{r^3}\left\{\sin\left(\omega_0 t\right)\left[\cos\left(\varepsilon\right)\sin\left(\iota\right) - \sin\left(\varepsilon\right)\cos\left(\iota\right)\cos\left(\omega_e t\right)\right] + 2\cos\left(\omega_0 t\right)\sin\left(\varepsilon\right)\sin\left(\omega_e t\right)\right\} \tag{3.4}$$

where $M_e = 7.943 \times 10^{15}$ *Wb.m* is magnetic dipole moment of the Earth, $\varepsilon = 11.7^{\circ}$ is the magnetic dipole tilt, $\omega_e = 7.29 \times 10^{-5}$ *rad/s* is the spin rate of the Earth, ι is the orbit inclination and $\mu = 3.98601 \times 10^{14}\ \mathrm{m^3/s^2}$ is the Earth Gravitational constant.

$B_x(\phi,\theta,\psi,t)$, $B_y(\phi,\theta,\psi,t)$ and $B_z(\phi,\theta,\psi,t)$ show the Earth magnetic field vector components in body frame as a function of body angles and time. The magnetometer bias vector is given as $b_m = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T$. Bias vector is model as rate of change of the bias vector is constant in time.

$$\dot{b}_m = 0 \tag{3.5}$$

The noise term, η_m, is added linearly to the model and modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_m^2\delta_{kj} \tag{3.6}$$

where I_{3x3} is the identity matrix, σ_m is the standard deviation of magnetometer errors and δ_{kj} is the Kronecker symbol.

3.2 Sun Sensor Measurement Model

In order to construct a sun sensor model, a sun direction vector is used. Using VSOP87 theory, a direction cosine matrix is calculated which shows the sun's position relative to Earth in ECI frame [6]

$$\begin{bmatrix} s_{B_x} \\ s_{B_y} \\ s_{B_z} \end{bmatrix} = A \begin{bmatrix} s_{E_1} \\ s_{E_2} \\ s_{E_3} \end{bmatrix} + \eta_s \tag{3.7}$$

Construction of the sun direction vector, s_E requires two assumptions.Comparing the distance between Sun-Earth, 1 AU, and Earth-satellite, satellite altitude is negligible. Therefore the satellite's sun direction vector is always parallel to Earth's sun direction vector. The other assumption is taking the right ascension node of the Sun's orbit as zero.

Model is using Julian Date (JD). The reference epoch of the first order model is the January 1st 2000, 12:00:00 pm. Converting this date to JD would be 2451545. Satellite's epoch is selected as March 16th 2017, 22:46:22. The first step of calculating the direction vector is to find the mean anomaly of the Sun. All of the angles that are given below are in degrees.

$$T_{TDB} = \frac{JD_{UTC} - 2451545}{36525} \tag{3.8}$$

$$M_{Sun} = 357.5277233° + 35999.05034 T_{TDB} \tag{3.9}$$

Secondly, the ecliptic longitude of the Sun, $\lambda_{ecliptic}$ is calculated.

$$\lambda_{ecliptic} = \lambda_{M_{Sun}} + 1.914666471 \sin\left(M_{Sun}\right) + 0.019994643 \sin\left(2M_{Sun}\right) \tag{3.10}$$

where $\lambda_{M_{Sun}}$ is the mean longitude of the sun. It can be calculated with the equation below.

$$\lambda_{M_{Sun}} = 280.460 + 36000.770 T_{sat} \tag{3.11}$$

Initially, T_{sat} is satellite's epoch. It will increase by one second until satellite reach at the end of its first period. Lastly, the angle between Earth's orbit and equator planes, obliquity of the ecliptic needs to be calculated.

$$\varepsilon = 23.439291 - 0.0130042 T_{TDB} \tag{3.12}$$

The sun direction vector,

$$s_E = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix} \tag{3.13}$$

The noise term, η_s, is added linearly to the model and modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_s^2\delta_{kj} \tag{3.14}$$

where I_{3x3} is the identity matrix, σ_s is the standard deviation of sun sensor errors and δ_{kj} is the Kronecker symbol.

3.3 Gyroscope Measurement Model

The gyro model is constructed with satellite dynamic equations. A commonly used model for gyro measurements given by

$$\hat{\omega}_{BI} = \omega_{BI} + b_g + \eta_g \tag{3.15}$$

where b_g is the gyro bias vector and the η_g is modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_g^2\delta_{kj} \tag{3.16}$$

where I_{3x3} is the identity matrix, σ_g is the standard deviation of gyro errors and δ_{kj} is the Kronecker symbol. Bias vector is given as

$$\dot{b}_g = \eta_b \tag{3.17}$$

where η_b is also the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{bk}\eta_{bj}^T\right] = I_{3x3}\sigma_{gb}^2\delta_{kj}. \tag{3.18}$$

Here, σ_{gb} is the standard deviation of gyro biases.

4 Filter Design

In this section, the design methods that are used for the attitude determination process are given. Initially, a deterministic attitude determination process, the algebraic method, also known as TRIAD method, is given. The outputs of the TRIAD method are then used in extended Kalman filters [7].

After each subsection, results are presented. For simulation, an imaginary satellite is selected. Its orbital attributes are given in Table 1.

Table 1 : Orbital Parameters.

RAAN (deg)	Argument of Perigee (deg)	Inclination (deg)	Altitude (km)
310.94	207.4	97.65	400

Using (2.29)-(2.31) and (2.21), satellite's mathematical model is constructed. The satellite's inertia matrix is given below.

$$J = \begin{bmatrix} 2.1\times10^{-3} & 0 & 0 \\ 0 & 2\times10^{-3} & 0 \\ 0 & 0 & 1.9\times10^{-3} \end{bmatrix} \quad (4.1)$$

4.1 TRIAD Method

A rotation matrix describes the attitude of a spacecraft with respect to a known reference frame. It takes at least two measured vectors to determine the orientation of the vehicle. This directed cosine matrix has nine elements but only three quantities are sufficient to build the matrix. Therefore, with two measurements, four different quantities are obtained. This leads the problem to be overdetermined.The TRIAD algorithm discards the part of the measurements so that a solution can be found [2].

The algebraic method constructs two triads of orthonormal vectors. In this thesis, two triads are expressed by sun sensor and magnetometer unit vectors in body and reference frames. The magnetometer measurement vector is denoted by B and the sun sensor vector is denoted by S. For initial base vector, a more accurate sensor is selected to be exact.

$$\vec{u} = \vec{S} \quad (4.2)$$

$$\vec{u}_b = \vec{S}_b \quad (4.3)$$

$$\vec{u}_r = \vec{S}_r \quad (4.4)$$

b and r subscripts denote the body and reference frames. For the second base vector, a unit vector that is perpendicular to the first base vector is constructed.

$$\vec{v} = \vec{S} \times \vec{B} \quad (4.5)$$

$$\vec{v}_b = \frac{\vec{S}_b \times \vec{B}_b}{\left|\vec{S}_b \times \vec{B}_b\right|} \quad (4.6)$$

$$\vec{v}_r = \frac{\vec{S}_r \times \vec{B}_r}{\left|\vec{S}_r \times \vec{B}_r\right|} \quad (4.7)$$

And the final base vector to complete the triad,

$$\vec{w} = \vec{u} \times \vec{v} \quad (4.8)$$

$$\vec{w}_b = \vec{u}_b \times \vec{v}_b \quad (4.9)$$

$$\vec{w}_r = \vec{u}_r \times \vec{v}_r \quad (4.10)$$

Three base vectors form a complete orthogonal coordinate system. It is important to note that two vectors, S and B can not be parallel, $\left|\vec{S}.\vec{B}\right| < 1$ .Constructing the direction cosine matrix,

$$A^{br} = \left[\vec{u}_b \quad \vec{v}_b \quad \vec{w}_b\right]\left[\vec{u}_r \quad \vec{v}_r \quad \vec{w}_r\right]^T \quad (4.11)$$

Equation (4.11) results in a 3x3 matrix. Using equations (2.3)-(2.5), body angles can be obtained. Final step of the algebraic method is to find covariance of the algorithm. Calculating with the formula given below, covariance matrix is established [4],

$$P = \frac{\sigma_m^2 \vec{S}_b \vec{S}_b^T + \sigma_s^2 \vec{B}_b \vec{B}_b^T}{\left\|\vec{S} \times \vec{B}\right\|} + \sigma_s^2 \vec{v}_b \vec{v}_b^T \quad (4.12)$$

Covariance and body angles that are obtained from the TRIAD algorithm are used as measurement inputs to the kalman filter. These results are combined with gyro measurements in order to achieve better accuracy.

4.2 Extended Kalman Filter

In this section, the traditional EKF, which is based on the nonlinear measurements, is introduced. The satellite's rotational motion about its mass center is formulated using the discrete-time nonlinear state space model

$$x(k+1) = f\left[x(k)\right] + w(k) \quad (4.13)$$

$$z(k) = h\left[x(k)\right] + v(k) \quad (4.14)$$

where $f[\cdot]$ and $h[\cdot]$ are the nonlinear dynamic and measurement functions respectively, $x(k)$ is the n dimensional state vector at time k, $w(k)$ is the zero-mean Gaussian noise with covariance $Q(k)$, $z(k)$ is the d dimensional measurement vector, $v(k)$ is the zero-mean Gaussian noise with covariance $R(k)$. It is assumed that both noise vectors $w(k)$ and $v(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean,

$$E[w(k)] = E[v(k)] = 0, \ \forall k \tag{4.15}$$

Filter algorithms based on the described system and measurements in (4.13)- (4.14) can be given. The approximations in the prediction and update stages of the filter can be found based on the EKF. The estimation value can be found as,

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \{z(k+1) - h[\hat{x}(k+1/k)]\} \tag{4.16}$$

The extrapolation value of the dynamic function can be found as

$$\hat{x}(k+1/k) = f[\hat{x}(k)] \tag{4.17}$$

Filter-gain of the EKF is,

$$K(k+1) = P(k+1/k)H^T(k+1) \times [H(k+1)P(k+1/k)H^T(k+1) + R(k)]^{-1} \tag{4.18}$$

where $H(k+1) = \dfrac{\partial h[\hat{x}(k+1/k)]}{\partial \hat{x}(k+1/k)}$ is the partial derivatives of the measurement function with respect to the states.

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k)]}{\partial \hat{x}(k)} P(k/k) \times \frac{\partial f^T[\hat{x}(k)]}{\partial \hat{x}(k)} + Q(k) \tag{4.19}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = [I - K(k+1)H(k+1)]P(k+1/k) \tag{4.20}$$

The filter expressed by the formulas (4.17) - (4.20) is called the extended Kalman filter.

The state vector considered in this study is,

$$x = \begin{bmatrix} \phi & \theta & \psi & \omega_x & \omega_y & \omega_z & b_{m_x} & b_{m_y} & b_{m_z} & b_{g_x} & b_{g_y} & b_{g_z} \end{bmatrix}^T \tag{4.21}$$

Body angles from the TRIAD algorithm, angular velocities from gyros and magnetic field measurements from magnetometers are taken as measurement vector. 9-state measurement vector is given below

$$z = \begin{bmatrix} \phi & \theta & \psi & \omega_x & \omega_y & \omega_z & B_x & B_y & B_z \end{bmatrix}^T$$

5 Simulations

A 12-state EKF is used for attitude estimation and magnetometer and gyro bias calibration. Orbit of the satellite is chosen as circular with an altitude of 550 km. Other orbital parameters are given in the magnetometer measurement model section.

For magnetometer measurements, sensor noise is characterized by zero mean Gaussian white noise with standard deviation of σ_m = 0.006. Magnetometers are calibrated in-flight by using biases estimated by EKF. Sun sensors are calibrated against sensor biases. Sensor noise is also characterized by zero means Gaussian white noise with standard deviation of σ_s = 0.002. Gyroscope measurements are also calibrated in-flight. Gyroscope noise standard deviation is σ_g = 0.005.

Simulations are conducted for 5910 seconds with sampling time of 1 sec. Body angles, angular velocities and sensor biases are estimated. Estimated biases are fed back to sensor measurements for in-flight calibration. Hence, calibrated sensor measurements are used in both TRIAD and EKF. In Figures 1, Figure 2, Figure 3 and Figure 4, estimation results are given.

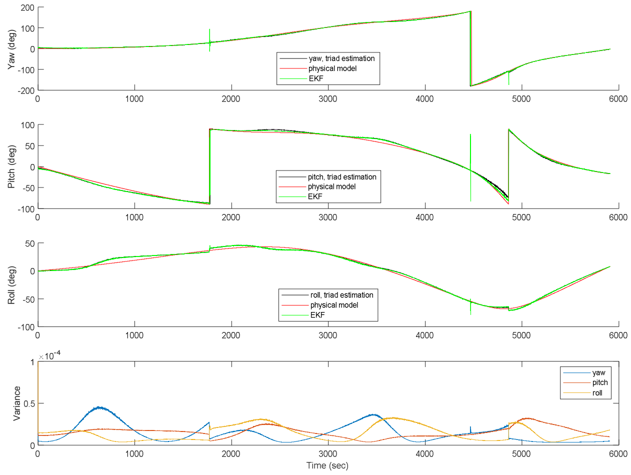

Figure 1: Body angle estimations

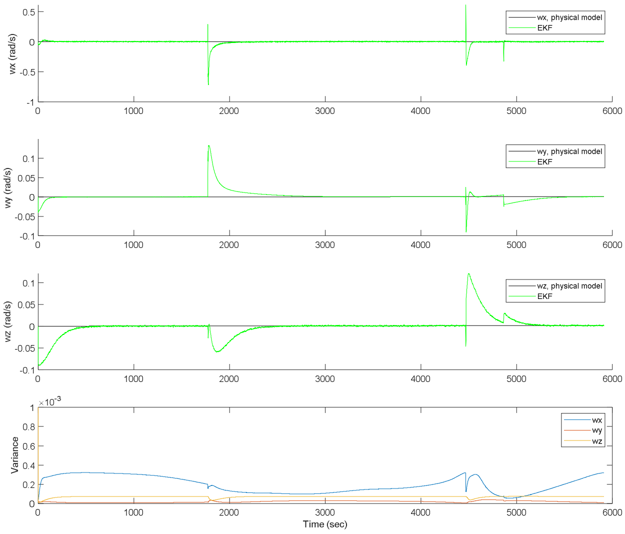

Figure 2: Angular velocity estimations

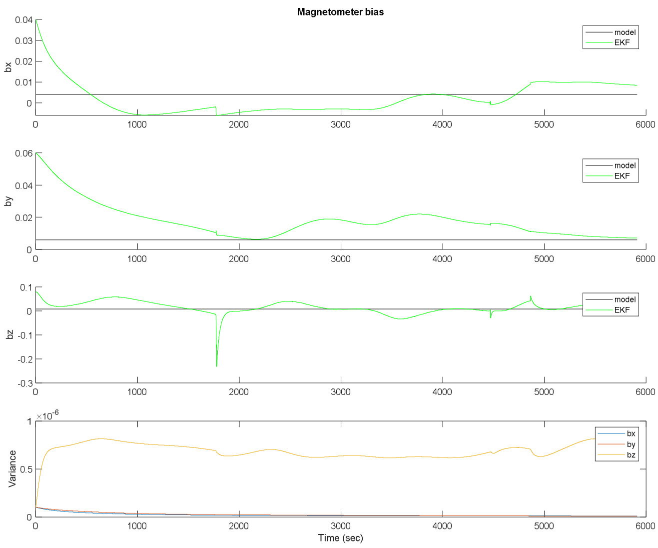

Figure 3: Magnetometer bias estimations

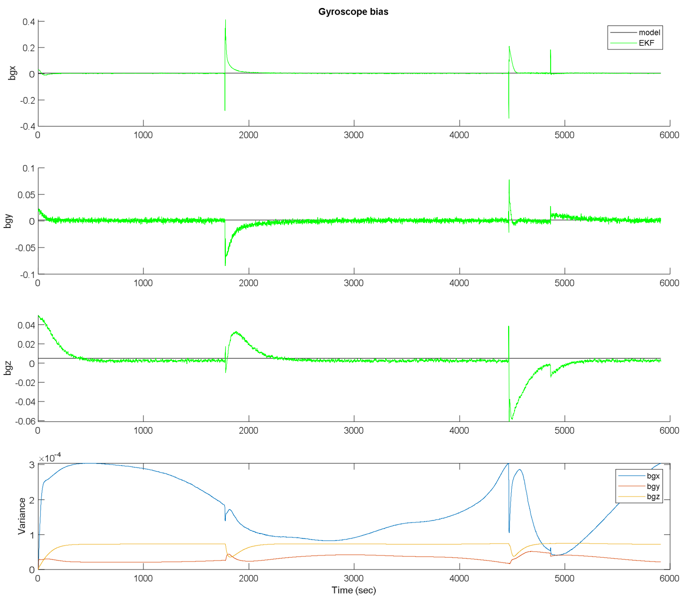

Figure 4: Gyroscope bias estimations

As can be seen in all of the graphs, there are some spikes and sudden fluctuations approximately in the same places. These anomalies are caused by high body angles. When yaw, pitch or roll angle goes to their respective boundaries, 180 degrees for yaw and roll, 90 degrees for pitch angle, The TRIAD algorithm starts to fail and it leads EKF to the same behaviour. In roll angle estimation, it can be seen that when the angle is not close to its limits, both TRIAD and EKF estimate decently.

6 Conclusion

In this study, magnetometer and sun sensor measurements are fused together with the TRIAD algorithm to produce body angle estimation. Combining with gyroscopes measurements, an Extended Kalman Filter is used to estimate body angles, angular velocities, gyroscope and magnetometer biases.

The TRIAD method, even though it is an aging algorithm, can estimate satellite altitude well. The sun sensor and magnetometer are selected for inputs to the TRIAD algorithm because of their wide usage in the space industry. Many different filtering algorithms are presented to this day but proven algorithms are still getting attention from engineers. EKF proves itself on many missions. Satellite dynamic and kinematic equations show that all of the states are connected. The simulation results show that, the proposed TRIAD-aided EKF algorithm estimates the attitude, attitude rate and magnetometer and gyroscope biases decently

References

[1] J. Wertz, Spacecraft Attitude Determination and Control. Kluwer Academic Publishers, Dordrecht.
[2] H.D. Black. "A passive system for determining the attitude of a satellite" AIAA Journal 7 (1964): 1350-1351
[3] C. Hajiyev and M. Bahar, "Increase of accuracy of the small satellite attitude determination using redundancy techniques," *Acta Astronaut.*, vol. 50, no. 11, pp. 673–679, 2002.
[4] F.L. Markley and J.L. Crassidis, Fundamentals of spacecraft attitude determination and control, Springer New York, New York, NY, 2014.
[5] D. C. Guler and C. Hajiyev, Gyroless Attitude Determination of Nanosatellites: Single-Frame Methods and Kalman Filtering Approach. LAP LAMBERT Academic Publishing, 2017.
[6] P. Bretagnon and G. Francou. "Planetary Theories in rectangular and spherical variables: VSOP87 solution" Astronomy and Astrophysics 202 (1988): 309-315.
[7] H.E. Soken and S. Sakai, TRIAD+Filtering approach for complete magnetometer calibration, in: Proceedings of 9th International Conference on Recent Advances in Space Technologies, RAST 2019, Istanbul, Turkey, pp.703-708
[8] Gokcay, M.A. and Hajiyev, C. (2022). Comparison of TRIAD+EKF and TRIAD+UKF Algorithms for Nanosatellite Attitude Estimation. WSEAS Transactions on Systems and Control, vol. 17, pp. 201-206.DOI: 10.37394/23203.2022.17.23
[9] Kinatas, H., Hajiyev, C. (2022). QUEST Aided EKF for Attitude and Rate Estimation Using Modified Rodrigues Parameters. WSEAS Transactions on Systems and Control, vol. 17, pp. 250-261.DOI: 10.37394/23203.2022.17.29

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The author(s) contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The author(s) declare no potential conflicts of interest concerning the research, authorship, or publication of this article.