Isogeometric Method for Least Squares Problem

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Abstract: In the framework of this work, we used the isogeometric method to solve the least squares problem in one dimension, on a curve of $\mathbb{R}^d, d = 1, 2$, including a semicircle. For this purpose, we presented the isogeometric method and the tools necessary for the description of this method, namely, the b splines basis, the parameterization of the $\mathbb{R}^d, d = 1, 2$ curve. We formulated the least squares problem which is a minimization problem. This problem was solved by using the Discontinuous Galerkin (DG) and the b splines basis as the approximation basis. The numerical method was validated by evaluating the error. For this purpose, an inverse inequality was therefore used.

Key-Words: -Isogeometric method, Least squares problem, B-spline basis, Parameterization, Discontinuous Galerkin, Inverse inequality.

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1 Introduction

The approximation of a function in the sense of least squares is a tool mathematics important in computer-aided design (CAD) because the engineer uses it to approximate a function. The least squares problem is used in biology for problems of mathematical modeling, [1].
In the framework of this work, we use isogeometric method to solve the least squares problem.
Isogeometric method has been introduced in 2005, [2], [3], [4]. The objectives of Isogeometric Analysis are to generalize and improve upon Finite Element Analysis (FEA) in the following ways :

1. To provide more accurate modeling of complex geometries and to exactly represent common engineering shapes such as circles, cylinders, spheres, ellipsoids, etc.

2. To fix exact geometries at the coarsest level of discretization and eliminate geometrical errors.

3. To vastly simplify mesh refinement of complex industrial geometries by eliminating the necessity to communicate with the CAD description of geometry.

4. To provide refinement procedures, including classical h- and p- refinements analogues, and to develop a new refinement procedure called k-refinement, [5].

The idea of isogeometric method is to build a geometry model and, rather than develop a finite element model approximating the geometry, directly use the functions describing the geometry in analysis, [6]. These functions are b-splines.

Isogeometric Analysis is approached, using continuous or discontinuous Galerkin method. In the framework of the isogeometric method, we use a parametrization to describe the domain on which, we solve our problem. This parametrization is used for numerical integration, [1],[7].

The least squares problem is addressed in one dimesnsion on a segment in, [1]. The particularity of our work is to solve our problem in one dimension on a curve of $\mathbb{R}^2$.

2 The least squares problem formulation

The isogeometric method uses b-splines as basis functions to construct the numerical approximations.

Definition 1. *Let $x_1 \leq x_2 \leq \cdots \leq x_m$ be an increasing sequence of reals, b-splines functions of degree k are defined by Cox-de Boor-Mansfield recursion formula, [8] :*

$$\begin{cases} For & 1 \leq i \leq m-1 \\ N_{i,0}(t) & = 1 \qquad if \quad t \in [x_i, x_{i+1}[\\ N_{i,0}(t) & = 0 \qquad otherwise \end{cases} \tag{1}$$

$$\begin{cases} For & k \geq 1 \quad and \quad 1 \leq i \leq m-k-1 \\ N_{i,k}(t) & = \dfrac{t - x_i}{x_{i+k} - x_i} N_{i,k-1}(t) + \\ & \dfrac{x_{i+k+1} - t}{x_{i+k+1} - x_{i+1}} N_{i+1,k-1}(t), \end{cases} \tag{2}$$

with the convention $\frac{x}{0} := 0$ for all real number x

The set $(x_i)_{i=1}^m \quad (1 \leq i \leq m)$ is called knots vector.

Definition 2. *Let $(x_i)_{i=1}^m \quad (1 \leq i \leq m)$ be a knots vector. Let n be a non-zero natural integer et let $(P_i)_{i=0}^n$ be a sequence of points of IR^d ($d = 1, 2$). We call b-spline curve of degree k (d'ordre $k+1$) and of control points P_i, $i = 0, \ldots, n$, the function P defined from the interval $[x_1 , x_m]$ into IR^d by :*

$$P(t) = \sum_{i=0}^{n} P_i N_{i,k}(t), \quad x_1 \leq t \leq x_m, \ 0 \leq k \leq n$$

The set of points $(P_i)_{i=0}^n$ are the vertices of a polygon called control polygon of the curve P.

Univariate B-spline basis functions are piecewise polynomial. They form a partition of unity, have local support, and are non-negative, [9].

Theorem 1 (existence and uniqueness of solution)**.** *Let C be a non-empty closed convex set of a Hilbert space H. Then for all $f \in H$, there exists a unique $u \in C$ such that :*

$$\|f - u\| = \min_{v \in C} \|f - v\| \tag{3}$$

We note $u = P_C(f)$, the projection of f on C. u is therefore the solution of the minimization problem.

The parametrization F of the physical domain is defined by :

$$F : \widehat{\Omega} \subset \text{IR} \rightarrow \Omega \subset \text{IR}^d, d = 1, 2$$

$$\varepsilon \longmapsto x = F(\varepsilon) = \sum_{i=0}^{n} C_i \widehat{N}_{i,k}(\varepsilon)$$

where $\widehat{\Omega}$ is the parametric domain, the $(C_i)_{i=0}^n$ are control points and $n+1$ is the number of basis functions.

We use the standard notation of Sobolev spaces $H^l(\Omega)$. The norm and semi-norm will be denoted respectively by $\|.\|_{H^l(\Omega)}$ and $|.|_{H^l(\Omega)}$.
When $l = 0, H^l(\Omega) = L^2(\Omega)$.
We use also the inner product on $L^2(\Omega)$ denoted by $(.,.)_{L^2(\Omega)}$.
Let us consider the function $t \overset{F}{\longmapsto} F(t) \overset{g}{\longmapsto} g(F(t))$, Where F is the parametrization of the domain $\Omega \subset \mathbb{R}^d$, $d = 1, 2$.

$X = (X_i)_{i=1}^m$ is an open knots vector, with m the total number of knots.

$X_d = (\chi_i)_{i=1}^{N_d}$ is the set of distinct knots of X.

Let be $\Omega_i = \{x \in \text{IR}^d; x = F([\chi_i, \chi_{i+1}]), \}$, $1 \leq i \leq N_d - 1$

Thus $\exists!$ $q(i) \in \{1, \ldots, m\}$ such as $\chi_i = X_{q(i)}$ where $q(i) = \sum_{k=1}^{i} n_k$,
And n_i is the multiplicity of χ_i in X.

Let be $\Omega = \bigcup_{i=1}^{N_d - 1} \Omega_i$, with $\Omega_i \cap \Omega_s = \emptyset, \forall i \neq s$.
$(N_{j,p})_{j \in I_i}$ is a b-spline basis of degree p,

Where $I_i = \{k \in \{0,\dots,n\}; SuppN_{k,p} \cap \Omega_i \neq \emptyset\}$,

$SuppN_{j,p} = F([\chi_j;\chi_{j+p+1}])$.

$SuppN_{j,p} \cap F([\chi_i;\chi_{i+1}]) \neq \emptyset \iff j \in \{q(i)-p,\dots,q(i)\}$.

$$\forall i \in \{1,\dots,N_d-1\}, N^i_{j,p}(x) = \begin{cases} N_{j,p}(x) \text{ if } x \in F([\chi_i;\chi_{i+1}]) \\ 0 \text{ otherwise} \end{cases}$$

We want to approximate a function g by a b-spline curve $\tilde{g}$ on a domain $\Omega \subset \mathbb{R}^d, d = 1,2$ in the least squares sense

$$\text{Where} \quad \tilde{g}(F(t)) = \sum_{i=1}^{N_d-1}\sum_{j\in I_i} P^i_j N^i_{j,p}(F(t)), [7].$$

We are therefore looking for the coefficients (P^i_j), knowing that :

$$\int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF =$$

$$\int_{\Omega_i}\sum_{i=1}^{N_d-1}\sum_{j\in I_i} P^i_j N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF$$

Solving this approximation problem amounts to minimizing $\|g-\tilde{g}\|_{L^2(\Omega)}$.
Before proposing an isogeometric formulation of our problem, we give a definition and a theorem.

Definition 3. *Let π_h be a map which is such that :*

$$\forall v \in L^2(\Omega), \pi_h v \in L^2(\Omega) \text{ with}$$

$$(\pi_h v, y_h)_{L^2(\Omega)} = (v, y_h)_{L^2(\Omega)},$$

$$\forall y_h \in L^2(\Omega). \tag{4}$$

Theorem 2 (isogeometric inverse inequality)**.** *Given the integers l and s such that $0 \le l \le s \le p+1$ and a function $u \in H^s(\Omega)$, then:*

$$\sum_{K\in\tau_h} |u-\pi_h u|^2_{H^l(K)} \le Ch^{2(s-l)}\|u\|^2_{H^s(\Omega)}, \qquad [7],[10]$$
(5)
where C is independent of h.

Minimizing $\|g-\tilde{g}\|_{L^2(\Omega)}$ is equivalent to writing the following equalities :

$$\int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF =$$
$$\int_{\Omega_i}\sum_{j\in I_i} P^i_j N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF$$

$$\int_{\Omega_i}\sum_{j\in I_i} P^i_j N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF =$$
$$\int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF$$

$$\sum_{j\in I_i} P^i_j \int_{\Omega_i} N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF =$$
$$\int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF$$

$$\sum_{j\in I_i} M^i_{k,j}P^i_j = S^i_k, \forall k \in I_i. \tag{6}$$

$$M^iP^i = S^i, \forall i \in \{1,\dots,N_d-1\}, [7]. \tag{7}$$

Where

$$M^i = (M^i_{kj})_{\substack{k,j\in I_i \\ 1\le i\le N_d-1}},$$

$$P^i = (P^i_j)_{\substack{j\in I_i \\ 1\le i\le N_d-1}},$$

$$S^i = (S^i_k)_{\substack{k\in I_i \\ 1\le i\le N_d-1}},$$

With $M^i_{k,j} = \int_{\Omega_i} N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF$ and

$$S^i_k = \int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF.$$

To calculate the P^i, we will be interested in studying the properties of the mass matrix M^i.

Property 1. *M^i is a square matrix of order $(p+1)$ which is symmetric, [7].*

Proof. Knowing that $M^i = (M^i_{kj})_{\substack{k,j\in I_i \\ 1\le i\le N_d-1}}$, and that $card(I_i) = p+1$, M^i is a square matrix of order $(p+1)$. Moreover, M^i is a symmetric matrix because $M^i_{kj} = M^i_{jk}, \forall 1 \le i \le N_d-1, \forall k \in I_i$ and $\forall j \in I_i$. □

Property 2. *M^i is an invertible matrix, [7].*

We will show that M^i is a positive definite matrix.

Proof. Let $L^i = (L^i_j)_{j\in I_i}$ be a column vector.

$$M^iL^i = (\lambda^i_k)_{k\in I_i} \text{ with } \lambda^i_k = \sum_{j\in I_i} M^i_{kj}L^i_j$$

So we get :

$$\begin{aligned}(L^i)^tM^iL^i &= \sum_{k\in I_i} \lambda^i_k L^i_k \\ &= \sum_{k\in I_i}\left(\sum_{j\in I_i} M^i_{kj}L^i_j\right)L^i_k \\ &= \sum_{k\in I_i}\sum_{j\in I_i}\int_{\Omega_i}\left(N^i_{k,p}(x)N^i_{j,p}(x)dx\right)L^i_jL^i_k \\ &= \int_{\Omega_i}\left(\sum_{j\in I_i} N^i_{j,p}(x)L^i_j\right)^2 dx \\ &= \left\|\sum_{j\in I_i} N^i_{j,p}(x)L^i_j\right\|^2_{L^2(\Omega_i)}\end{aligned}$$

So $(L^i)^tM^iL^i \geq 0$.

$$\begin{aligned}(L^i)^tM^iL^i = 0 &\Longrightarrow \sum_{j\in I_i} N^i_{j,p}(x)L^i_j = 0 \qquad (8)\\ &\Longrightarrow L^i_j = 0 \text{ because the} \\ &N^i_{j,p} \text{ form a basis of } \mathbb{P}_{(p+1)(N_d-1)}. \\ &\Longrightarrow L^i_j = 0, \forall j \in I_i\end{aligned}$$

M^i is therefore positive definite, hence M^i is invertible. □

Property 3. *$invM^i$ the inverse of M^i, is a square matrix of order $(p+1)$ which is symmetric, [7].*

Proof. M^i being a square matrix of order $(p+1)$, its inverse is also a square matrix of order $(p+1)$. M^i being symmetric, we have : $(M^i)^t = M^i$.

$$\begin{aligned}(M^i)^t = M^i &\Longrightarrow inv(M^i)^t = invM^i \\ &\Longrightarrow (invM^i)^t = invM^i \text{ because} \\ &M^i \text{ is an invertible square matrix}\end{aligned}$$

So, $invM^i$ is a symmetric matrix.

□

The properties of the mass matrix having been enumerated, we obtain from the relation 7 that $P^i = (invM^i)S^i$.

Computing an integral over Ω_i amounts to computing this integral over $\widehat{\Omega}_i$ by means of the parameterization F, [11]. This integral over the parametric domain is then brought back to the interval $[-1;1]$, using a transformation. So, we get :

$$\begin{aligned}M^i_{k,j} &= \int_{\Omega_i} N^i_{j,p}(F(t))N^i_{k,p}(F(t))dF \\ &= \int_{\widehat{\Omega_i}} N^i_{j,p}(F(\varepsilon))N^i_{k,p}(F(\varepsilon))Jac(F(\varepsilon))d\varepsilon \\ &= \int_{\widehat{\Omega_i}} \widehat{N}^i_{j,p}(\varepsilon)\widehat{N}^i_{k,p}(\varepsilon)\widehat{Jac}(\varepsilon)d\varepsilon\end{aligned}$$

because $N^i_{j,p}\circ F = \widehat{N}^i_{j,p}$ and $Jac\circ F = \widehat{Jac}$

$$M^i_{k,j} = \sum_{r=pos1(i)}^{pos2(i)} l^i_r \widehat{N}^i_{j,p}(\varepsilon^i_r)\widehat{N}^i_{k,p}(\varepsilon^i_r)\widehat{Jac}(\varepsilon^i_r)$$

With $l^i_r = \frac{\chi_{i+1}-\chi_i}{2}\omega^i_r$ then $pos1(i) = (i-1)Npgs+1$, $pos2(i) = iNpgs$ and $1 \leq i \leq N_d - 1$.
$Npgs$ is the number of Gaussian points per segment, the ω^i_r and the ε^i_r are respectively the Gaussian weights and knots.

$$S^i_k = \int_{\Omega_i} g(F(t))N^i_{k,p}(F(t))dF \qquad (9)$$

$$S^i_k = \sum_{r=pos1(i)}^{pos2(i)} l^i_r g(F(\varepsilon^i_r))\widehat{N}^i_{k,p}(\varepsilon^i_r)\widehat{Jac}(\varepsilon^i_r), [7]$$

3 Numerical solution

In the previous section, we approximated a function g by a b-spline curve of degree p in the sense of least squares on a domain Ω. Subsequently, we want to validate the approximation in the sense of least squares by verifying the inverse inequality 5, for $l = 0$ and $s = 1$.
For $l = 0$ and $s = 1$, inverse inequality 5 becomes :

$$\|u-\pi_h u\|_{L^2(\Omega)} \leq Ch\|u\|_{H^1(\Omega)}, \forall u \in H^1(\Omega), \quad (10)$$

Where C is independent of h.
Therefore, we put in an array, the space step h, $log(h)$ and $\log\left(\frac{\|error\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$. Then, we determine the slope of the curve of the $\log\left(\frac{\|error\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$ as a function of $\log(h)$. Then, we construct this curve in each case.
Numerical tests are performed using **Fortran** and **Gnuplot.**

Experience 1. $g(x) = e^x, x \in \Omega_1 =]-1;1[$
$p = 2$, $\|u\|_{H^1(\Omega)} = \sqrt{e^2 - e^{-2}}$.
The parametrization of Ω_1 *is given by :*

1. *The knots vector :*
   $X = [0000.1250.250.3750.50.6250.750.875$ $111]$
2. *The control points :* $A_1(-1;-0.75)$, $A_2(-0.5;-0.25)$, $A_3(0;0)$, $A_4(0.25;0.5)$ *and* $A_5(0.75;1)$.

Table 1. Results of experiment 1

Step h	$log(h)$	$\log\left(\frac{\|erreur\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$	Rate
$h = 0.125$	-2.079	-7.556	-
$\frac{h}{2}$	-2.773	-9.270	2.47
$\frac{h}{4}$	-3.466	-10.996	2.48
$\frac{h}{8}$	-4.159	-12.727	2.49

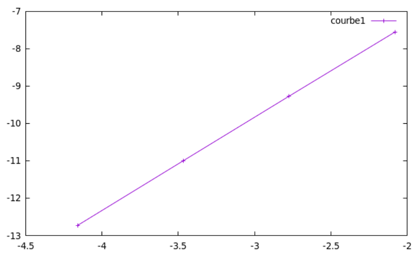

Fig. 1: line of experiment 1

Experience 2. $g(x) = x^3, x \in \Omega_1 =]-1;1[$
$p = 2$, $\|u\|_{H^1(\Omega)} = \sqrt{\frac{136}{35}}$.

Table 2. Results of experiment 2

Step h	$log(h)$	$\log\left(\frac{\|erreur\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$	Rate
$h = 0.125$	-2.079	-6.518	-
$\frac{h}{2}$	-2.773	-8.249	2.498
$\frac{h}{4}$	-3.466	-9.982	2.498
$\frac{h}{8}$	-4.159	-11.713	2.498

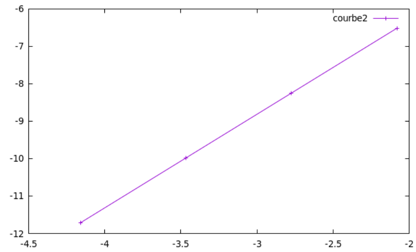

Fig. 2: line of experiment 2

Experience 3. $g(x,y) = x^2 + y^2, (x,y) \in \Omega_2$, *the half-circle with center* $(0,0)$ *and radius* 1.

$$p = 2,\ \|u\|_{H^1(\Omega)} = \sqrt{\pi + \frac{8}{3}}$$

The parameterization of Ω_2 *is given by :*
-The knots vector :
X=[0 0 0 0.125 0.25 0.375 0.5 0.625 0.75 0.875 1 1 1]
-The control points :
$A_1(1;0)$, $A_2(1;0.414)$, $A_3(0.707;0.707)$, $A_4(0.414;1)$, $A_5(0;1)$, $A_6(0;1)$, $A_7(-0.414;1)$, $A_8(-0.707;0.707)$, $A_9(-1;0.414)$, $A_{10}(-1;0)$, [7].

Table 3. Results of experiment 3

Step h	$log(h)$	$\log\left(\frac{\|erreur\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$	Rate
$h = 0.5$	-0.693	-3.497	-
$\frac{h}{2}$	-1.386	-5.020	2.19
$\frac{h}{4}$	-2.079	-6.555	2.21
$\frac{h}{8}$	-2.772	-8.386	2.64

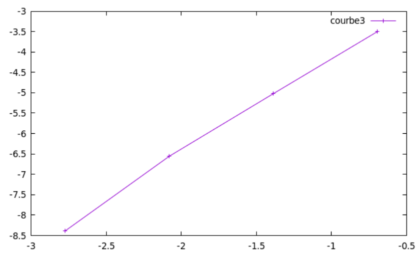

Fig. 3: line of experiment 3

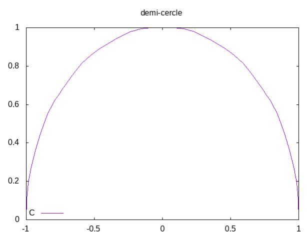

Fig. 4: Half-circle with center $(0; \quad 0)$ and radius 1

Three numerical tests were carried out on domains of IR^d, $d = 1, 2$. The tables $1, 2$ and 3 permit us to determine the slope of $\log\left(\frac{\|error\|_{L^2(\Omega)}}{\|u\|_{H^1(\Omega)}}\right)$ as function of $\log(h)$. In each case, the slope is quadratic. This slope is observed thanks to Figures 1,Figure 2 and Figure 3.
The inverse inequality 5 was verified for each of the three experiments, after performing a mesh refinement. Moreover, with regard to the experiment 3, it should be noted that the domain Ω_2 is a domain used for isogeometric method for $1D$ problems and not in finite elements. The Figure 4was represented thanks to domain Ω_2. To get Ω_2, we have built a parametrization of our domain. This parametrization is used only in the framework of the isogeometric method. This shows that isogeometric method allows us to correct the shortcomings of the finite element method.

4 Conclusion

The isogeometric method approximates a function by a b-spline curve on a domain $\mathrm{IR}^d, d = 1, 2$. Numerical tests have been done to validate numerically an isogeometric inverse inequality and show the necessity of using the isogeometric method, to the detriment of the finite element method, in one dimension. In perspective, we can use the isogeometric method with NURBS as the basis of approximation, to solve the least squares problems in two and three dimensions. We can use this approach for modeling problems. This project is currently ongoing.

References:

[1] Gdhami Asma, Isogeometric method for hyperbolic partial differential equations, Doctoral thesis, 2018. `https://theses.hal.science/tel-02272817`, Last Accessed Date: $2024-05-02$

[2] Cottrell J. A., Hughes T. J. R., Bazilevs Y., Isogeometric Analysis: Toward Integration of CAD and FEA,2009. DOI:10.1002/9780470749081 `https://doi.org/10.1002/9780470749081`

[3] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Computer Methods in Applied Mechanics and Engineering, Vol 194, (39-41), 2005, pp.4135-4195. DOI:10.1016/j.cma.2004.10.008 `https://doi.org/10.1016/j.cma.2004.10.008`

[4] L. Beirao da Veiga, A. Buffa, G. Sangalli, R. Vazquez, Mathematical analysis of variational isogeometric methods, Cambridge University Press, 2014. DOI:10.1017/S096249291400004X `https://doi.org/10.1017/S096249291400004X`

[5] Y. Bazilevs, L.Beirao da Veiga, A. Cottrell, Thomas Hughes, G.Sangalli, Isogeometric analysis : approximation, stability and error estimates for h-refined meshes, Mathematical Models and Methods in Applied Sciences, vol.16, issue.07, pp.1031-1090, 2006. DOI:10.1142/S0218202506001455 `https://doi.org/10.1142/S0218202506001455`

[6] T.J.R.Hughes, A.Reali, G.Sangalli, Efficient quadrature for NURBS based isogeometric analysis, Elsevier, Volume 199, Issues 5–8, Pages 301-313, 2010. DOI:10.1016/j.cma.2008.12.004 `https://doi.org/10.1016/j.cma.2008.12.004`

[7] Aguemon Uriel, Solving some partial differential equations with isogeometric method, Doctoral thesis, 2021. `https://hal.science/tel-04509591/document`, Last Accessed Date: $2024-05-02$

[8] Gerald Farin, curves and surfaces for computer aided geometric design, a pratical Guide, Fifth edition, Academic press,Library of Congress Control Number: 2001094373 ISBN:1-55860-737-4 2002. Last Accessed Date: $2024-05-02$

[9] Les Piegl, Wayne Tiller, The Nurbs book, second edition, Springer 1995. DOI:10.1007/978-3-642-59223-2_7 `https://doi.org/10.1007/978-3-642-59223-2_12`

[10] John Evans, Thomas Hughes, Explicit trace inequalities for isogeometric analysis and parametric hexahedral finite elements, The Institute for computational engineering and sciences report, May 2011. Last Accessed Date: $2024 - 05 - 02$
https://www.oden.utexas.edu/media/reports/2011/1117.pdf

[11] Gernot Beer, Stephane Bordas, Isogeometric methods for numerical simulation, CISM International Centre for Mechanical Sciences,Courses and Lectures, Springer, 2015.
DOI:10.1007/978-3-7091-1843-6
https://doi.org/10.1007/978-3-7091-1843-6

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