Parameter Estimation of Linear and Nonlinear Systems Connected in Parallel

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Abstract: - Presently, the problem of parameter estimation is addressed for a more general nonlinear model. In this respect, the proposed nonlinear system is composed of the parallel connection of linear and nonlinear blocks. The interconnected structure of linear and nonlinear blocks systems leads to a highly nonlinear problem involving several unknown parameters and inaccessible internal signals. In this parameter estimation method, the linear and nonlinear block parameters are estimated in one stage by using simple a sine signal or multi-cosine wave.

Key-Words: - Linear and nonlinear systems; Parameter estimation; Sine signals.

Received: September 28, 2022. Revised: April 15, 2023. Accepted: May 8, 2023. Published: May 22, 2023.

1 Introduction

The problem of nonlinear system parameter estimation has been given a great deal of interest, [1], [2], [3], [4], [5], [6]. Most available parameter estimation approaches are focused on the cascading link of linear and nonlinear subsystems (Wiener and Hammerstein systems), [7], [8], [9], [10], [11], [12]. The latter are obtained using the series connection of linear and nonlinear subsystems. The Wiener and Hammerstein nonlinear systems are very popular models and can model several practical systems, [13], [14], [15], [16], [17].
There exist many parameter estimation methods in the literature to deal with the problem of nonlinear systems composed by the series connections of linear and nonlinear blocks, [18], [19], [20], [21], [22], [23].

To increase the modelling capacity, the parallel connection of linear and nonlinear subsystems can be very efficient, [24], [25]. The parallel connection of linear and nonlinear subsystems is more general than the series connection [26], [27].

Presently, the parameter estimation of a nonlinear system composed of the parallel connection of linear and nonlinear subsystems is presented. This nonlinear system is shown in Fig. 1.

Note that several real systems can be captured using the parallel connection of linear and nonlinear subsystems, [14], [28]. We propose a solution to estimate the parameters of linear subsystem $G(s)$ and those of nonlinear subsystem $f(.)$. This approach is based on sine signals. In this respect, note that when linear and nonlinear systems are excited by sine signal, their output is thus periodic of the same frequency as the input, [29], [30], [31].
This means that the output is also periodic but not necessarily a sine signal.
Then, it is quite interesting to note that very few works have treated this problem. The most of existing parameter estimation problems concern the classical Wiener and Hammerstein systems.

At this stage, it is worth emphasizing that this nonlinear system can describe several nonlinear structures.

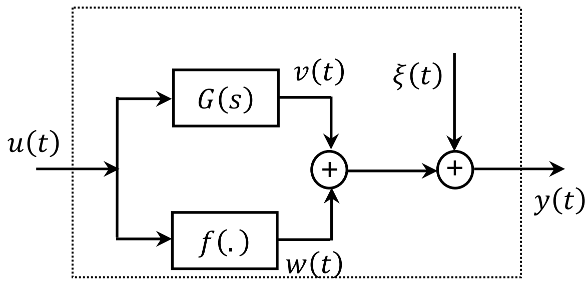

Fig. 1: Nonlinear system composed by the parallel connection of linear and nonlinear subsystems

For convenience, the rest of this paper is organized as follows. Firstly, the problem statement in this study is introduced in Section 2. Then, the parameter estimation method is described in Section 3. Finally, the performances of the parameter estimation method are presented in Section 4.

2 Problem Statement

This paper aims to estimate the parameters of the considered nonlinear system. The latter is obtained by the parallel link of linear and nonlinear subsystems (Fig. 1).

Firstly, the nonlinear part can be described as follows:

$$w(t) = f\big(u(t)\big) = \sum_{k=0}^{n} \alpha_k u^k \quad (1)$$

where n and $(\alpha_1, \ldots, \alpha_n)$ are the degree and coefficient vector of the nonlinearity $f(.)$.

Then, the linear part is analytically described as follows:

$$v(t) = u(t) * g(t) \quad (2)$$

The Equation (2) can be rewritten in the Laplace domain as:

$$V(s) = U(s)G(s) \quad (3)$$

where $U(s)$ and $V(s)$ are the Laplace transforms of the input $u(t)$ an output $v(t)$ of the linear subsystem, respectively.

Then, one immediately gets using Fig. 1 that:

$$y(t) = v(t) + w(t) + \xi(t) \quad (4)$$

Finally, it follows from (1)-(2) and (4) that the output of the nonlinear system can be expressed as:

$$y(t) = u(t) * g(t) + \sum_{k=0}^{n} \alpha_k u^k + \xi(t) \quad (5)$$

For convenience, let us consider the signal $x(t)$ defined as:

$$x(t) = v(t) + w(t) \quad (6)$$

Then, the signal $x(t)$ can be rewritten as follows:

$$x(t) = u(t) * g(t) + \sum_{k=0}^{n} \alpha_k u^k \quad (7)$$

In this study, we propose an input of the form:

$$u(t) = U\, sin(\omega_l t) \quad (8)$$

where $\omega_l \in \{\omega_1, \ldots, \omega_P\}$ is any arbitrarily chosen set. It follows (8) that the signal $u(t)$ is of period $T_l = 2\pi/\omega_l$. Then, it is shown that the output of a linear subsystem is of period $T_l = 2\pi/\omega_l$ (e.g., see, [1]).

In this respect, it is also shown that the output of a nonlinear subsystem is not necessarily a sine signal, but of period $T_l = 2\pi/\omega_l$ (e.g., see, [1], [24]). Specifically, using (8) the signal $w(t)$ in Fig. 1 can be expressed as follows:

$$w(t) = f(U\, sin(\omega_l t)) = \sum_{k=0}^{n} \alpha_k U^k sin^k(\omega_l t) \quad (9)$$

3 Determination of Linear and Nonlinear Parameters

This section aims to present an estimation solution to give $\big(|G(j\omega_l)|, Arg\big(G(j\omega_l)\big)\big)$ and $(\alpha_0, \ldots, \alpha_n)$.

Firstly, it is readily seen that the signal $\boldsymbol{v}(t)$ for $u(t) = U\, sin(\omega_l t)$ can be expressed as follows:

$$v(t) = U|G(j\omega_l)|\, sin\Big(\omega_l t + Arg\big(G(j\omega_l)\big)\Big) \quad (10)$$

where the set $\big(|G(j\omega_l)|, Arg\big(G(j\omega_l)\big)\big)$ are the parameters of the linear subsystem to be estimated.

Then, using (9) one immediately gets that the signal $w(t)$ can be expressed as (for more details see, [1], [24]):

$$w(t) = \sum_{k=0}^{n} A_k(U, \alpha_0, \ldots, \alpha_k)\, cos(k\omega_l t + \theta_k) \quad (11)$$

where θ_k, for $k = 0 \ldots n$, is known real constant, and the expression $A_k(.)$ according to the variables $\alpha_0 \ldots \alpha_k$ is already determined, but $A_k(.)$ is not known. Finally, the output $y(t)$ can be expressed as follows:

$$y(t) = U|G(j\omega_l)|\, sin\Big(\omega_l t + Arg\big(G(j\omega_l)\big)\Big) + \sum_{k=0}^{n} A_k(U, \alpha_0, \ldots, \alpha_k)\, cos(k\omega_l t + \theta_k) + \xi(t) \quad (12)$$

Accordingly, it is seen that the signal $\boldsymbol{x}(t)$ defined by (6) is related to the output by the expression:

$$y(t) = x(t) + \xi(t) \quad (13)$$

Furthermore, knowing that the signals $\boldsymbol{v}(t)$ and $w(t)$ are of period $T_l = 2\pi/\omega_l$, using (6) we conclude that the signal $\boldsymbol{x}(t)$ is of period $T_l = 2\pi/\omega_l$. This result is very interesting, it allows to filter the output $y(t)$. Specifically, the filtered output noted $y_f(t)$, can be given, for any large integer N, by:

$$y_f(t) = \frac{1}{N}\sum_{k=0}^{N-1} y(t + kT_l) \ \text{ for } \ 0 \le t < T_l \quad (14a)$$

$$y_f(t) = y_f(t - T_l) \ \text{ otherwise} \quad (14b)$$

It is readily seen that the filtered output $y_f(t)$ converges to the signal $\boldsymbol{x}(t)$, which is given by:

$$x(t) = U|G(j\omega_l)|\, sin\Big(\omega_l t + Arg\big(G(j\omega_l)\big)\Big) + \sum_{k=0}^{n} A_k(U, \alpha_0, \ldots, \alpha_k)\, cos(k\omega_l t + \theta_k) \quad (15)$$

It is readily seen that the signal $\boldsymbol{x}(t)$ is also of the period $T_l = 2\pi/\omega_l$. Furthermore, using the filtered signal $y_f(t)$, the parameters of linear subsystem $\left(|G(j\omega_l)|, Arg\left(G(j\omega_l)\right)\right)$ and those of nonlinear subsystem $(\alpha_0, \ldots, \alpha_n)$ can be obtained.

4 Simulation

This section aims to present some examples of simulation to show the effectiveness of the obtained results. The studied system (Fig. 1) is described by the following linear subsystem:

$$G(s) = \frac{1}{(s+0.3)(s+0.6)} \quad (16)$$

and by the following nonlinear subsystem:

$$f(u) = -0.1u^3 + 0.5u^2 - 0.3u \quad (17)$$

The nonlinearity $f(u)$ is plotted in Fig. 2.

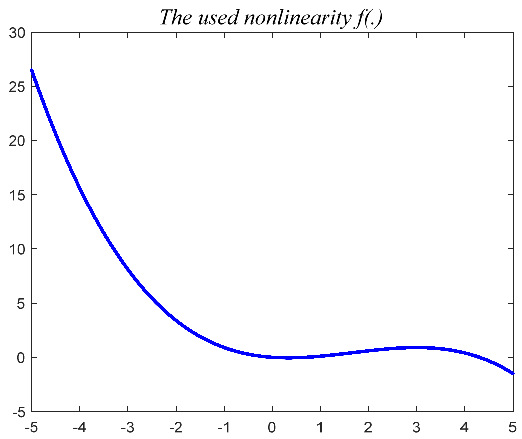

Fig. 2: The nonlinearity $f(.)$

Then, using the proposed method established in section 3, all variables can be estimated. Firstly, the system of Fig. 1 is excited by the signal (8) by choosing $\omega_l = 0.02\, rad/s$, where $\omega_l \in \{\omega_1, \ldots, \omega_P\}$ is the chosen set of applied frequencies. It is shown that the signal $\boldsymbol{x}(t)$ is of period $T_l = 2\pi/\omega_l = 314.16s$. Then, the output of this system is given in Fig. 3. This result shows that the signal $\boldsymbol{y}(t)$ is of period $T_l = 2\pi/\omega_l = 314.16s$, but affected by disturbances. Using the filtered signal $y_f(t)$, given by (14a-b), one gets the signal plotted in Fig. 4. This figure confirms that the signal $y_f(t)$ (or the estimate of $x(t)$) is also of period $T_l = 2\pi/\omega_l = 314.16s$.

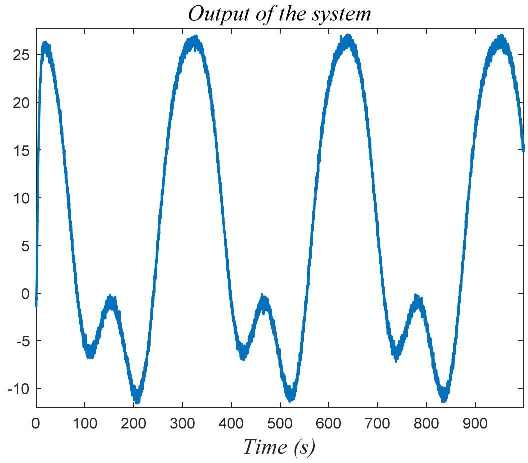

Fig. 3: The output $y(t)$ of the system

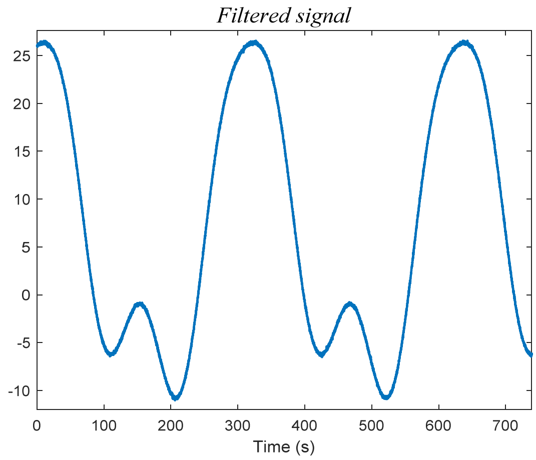

Fig. 4: The filtered signal $y_f(t)$

This test is repeated for other periods $T_l = 2\pi/\omega_l$, where $\omega_l = 0.1\, rad/s$. Then, use the signal (8) in the input of the system. Then, the output $\boldsymbol{y}(t)$ of this system is given in Fig. 5. This result shows that the signal $\boldsymbol{y}(t)$ (but affected by disturbances) is of period $T_l = 2\pi/\omega_l = 62.83s$. Then, the filtered signal $y_f(t)$, obtained using (14a-b), is given in Fig. 6.

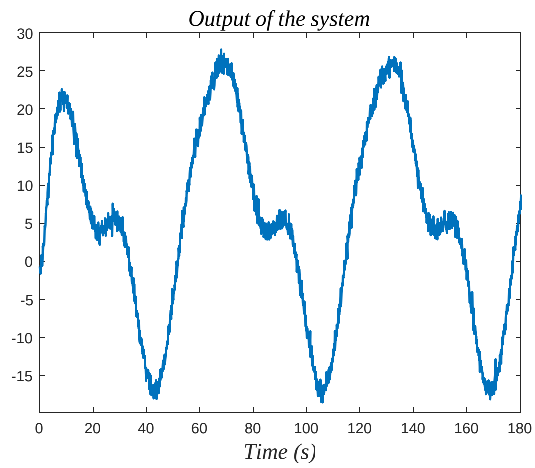

Fig. 5: The output $y(t)$ of the system

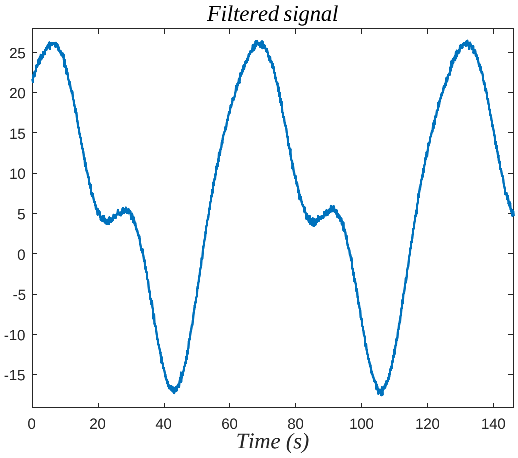

Fig. 6: *The filtered signal* $y_f(t)$

Finally, using the filtered signals $y_f(t)$, obtained using (14a-b) and represented by Fig. 4 and Fig. 6, the parameters of linear subsystem $\left(|G(j\omega_l)|, Arg\left(G(j\omega_l)\right)\right)$ and those of nonlinear subsystem $(\alpha_0, \ldots, \alpha_n)$ can be estimated (using (15)).

5 Conclusion

The problem of system parameter estimation is addressed for a more general nonlinear model. In this respect, the proposed nonlinear system is composed of the parallel connection of linear and nonlinear blocks. This solution is easy and more general.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Adil Brouri: Conceptualization, formal analysis, investigation, project administration, supervision.
-Hafid Oubouaddi: Methodology, software, validation.
-Fatima Ezzahra El Mansouri: Methodology, software, validation

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflict of interest to declare.