Novel robust stability of a class of Lur'e systems of neutral type with mixed interval time-varying delays

WAJAREE WEERA
University of Phayao
Department of Mathematics
19 Moo 2, Maeka, Muang, Phayao
THAILAND
wajaree.we@up.ac.th

THONGCHAI BOTMART
Khon Kaen University
Department of Mathematics
123 Moo 16, Nai-Muang, Muang, Khon Kaen
THAILAND
Corresponding author: thongbo@kku.ac.th

Abstract: This paper is considered with the problem of robust absolute stability of neutral type Lur'e systems with mixed time-varying delays. By constructing an new augmented Lyapunov-Krasovskii functional and combining integral inequality with approach to estimate the derivative of the Lyapunov-Krasovskii functional, which estimated some integral terms by Wirtinger's inequality, a matrix-based quadratic convex technique is used to design an LMI-based sufficient conditions. New stability condition is much less conservative and more general than some existing results. New stability criteria is given in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the results.

Key–Words: robust absolute stability; neutral type; Lur'e system; mixed time-varying delays; LMI approach

1 Introduction

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives. Heat exchanges, distributed networks containing lossless transmission lines and population ecology are examples of neutral systems. Because of its wider application, several researchers have studied neutral systems and provided sufficient conditions to guarantee the stability of neutral time delay systems, see [1, 8, 19] and references cited therein.

It is well know that nonlinearities may cause instability and poor performance of practical systems, [5, 8, 14, 20, 25]. Many nonlinear control systems can be modeled as a feedback connection of a linear neutral system and a nonlinear element. One of the important classes of nonlinear systems is the Lur'e system whose nonlinear element satisfies certain sector constraints. Absolute stability of Lur'e systems with sector bounded nonlinearities has attracted several researcher [5, 7, 9, 15].

It is well known that the existence of time delay in a system may cause instability and oscillations. Examples of time delay systems are chemical engineering systems, biological modeling, electrical networks, physical networks and many others, [11, 12, 17]. The stability criteria for system with time delays can be classified into two categories: delay-independent and delay-dependent. Delay-independent criteria does not employ any information on the size of the delay; while delay-dependent criteria makes use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the delay is small. In most of the existing results, the range of time-varying delay considered varies form 0 to an upper bound. In practice, the range of delay may vary in a range for which the lower bound is not restricted to be 0, i.e., interval time-varying delay. A typical example with interval time delay is the networked control system, which has been widely studied in the recent literature (see, e.g., [2, 11, 24]).

Recently, there are many research studies on the absolute stability of a class of neutral type Lur'e dynamical systems with time delay, see for examples [14, 16, 20, 22, 25]. The problems have been dealt with delay-dependent absolute and robust stability for time-delay Lur'e system [14]. Improved delay-dependent robust stability criteria for a class of uncertain mixed neutral and Lur'e dynamical systems with interval time-varying delays and sector-bounded nonlinearity were studied in [22]. On delay-dependent robust stability of a class of uncertain mixed neutral and Lur'e dynamical systems with interval time-varying delays were investigated in [25]. In [**?**], the authors considered the problem of global asymptotically stability analysis for delayed neural networks. By using a matrix-based quadratic convex approach to derive a sufficient condition, the positive definiteness of chosen LKF can be ensured. As a result the constraint $P > 0$ in both Kim(2011) and Zhang et al.(2013) is removed. However, it is worth pointing out that, even though these results were elegant, there still exist some points waiting for the improvement. Firstly, most of the works above [5, 14], the augmented Lyapunov matrix P must be positive definite. So, for removing this restriction by assuming that P are only real matrices in Lur'e systems. Secondly, By introducing new augmented Lyapunov-Kravoskii functional which have not been considered yet in stability analysis of Lur'e systems. Thirdly, by taking the time derivative of $\int_{t-h_1}^{t} h_1(h_1 - t + s)\dot{x}^T(s)W_1\dot{x}(s), \int_{t-h_1}^{t}(h_1 - t + s)^2\dot{x}^T(s)W_2\dot{x}(s)ds$, $\int_{t-h_2}^{t-h_1} h_{21}(h_2 - t + s)\dot{x}^T(s)R_1\dot{x}(s), \int_{t-h_2}^{t-h_1}(h_2 - t + s)^2\dot{x}^T(s)R_2\dot{x}(s)ds$, it is found that the integral terms $2\int_{t-h_2}^{t-h_1}(h_2 - t + s)\dot{x}^T(s)R_2\dot{x}(s)ds$, $2\int_{t-h_1}^{t}(h_1 - t + s)\dot{x}^T(s)W_2\dot{x}(s)ds$, $h_{21}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_1\dot{x}(s)ds$, $-h_1\int_{t-h_1}^{t}\dot{x}^T(s)W_1\dot{x}(s)ds$, appear. For estimating these terms, techniques in [21, 27] are applied in this paper, called matrix-based quadradic convex optimization approach combined with some improved bounding techniques for integral terms such as Wirtinger-based integral inequality; as a result we obtain inequality encompassing the Jensen one and also goes to tractable LMI criteria to futher reduce the conservatism over the existing results [14, 16, 20, 22, 25]. Fourthly, most of the previous works did not consider the lower bound of the time-varying delay and its time-derivative. Factually, the lower bound can play an important role in reducing the conservatism when it can be available and fully tackled in [16, 20, 22, 25].

Based on the above discussions, we consider the problem of delay-dependent absolute stability of Lur'e systems of neutral type with time-varying delays, matrix-based quadratic convex approach will be used. The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time varying delay are available. Based on the construction of improved Lyapunov-Krasovskii functionals combined with a quadratic convex approach, some new cross terms will be introduced which enhance the feasible stability criterion. New delay-dependent sufficient conditions for the neutral type Lur'e dynamical systems are established in terms of LMIs. The new stability condition is much less conservative and more general than some existing results. Numerical examples are given to illustrate the effectiveness of our theoretical results.

2 Problem statements and preliminaries

The following notation will be used in this paper: $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the $n-$dimensional space and the vector norm $\| \ . \ \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\text{Re}\lambda; \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-h, 0]\}, \| \ x_t \ \| = \sup_{s\in[-h,0]} \| \ x(t+s) \ \|$; $C([0,t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n-$valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $x^TAx \geq 0$, for all $x \in \mathbb{R}^n; A$ is positive definite $(A > 0)$ if $x^TAx > 0$ for all $x \neq 0; A > B$ means $A - B > 0$; diag$(c_1, c_2, ..., c_m)$ denotes block diagonal matrix with diagonal elements $c_i, i = 1, 2, ..., m$. The symmetric term in a matrix is denoted by $*$.

Consider the following Lur'e system of neutral type with interval time-varying delay:

$$\begin{aligned}\dot{x}(t) &= A_1\dot{x}(t-\tau(t)) + Ax(t) \\ &\quad + Bx(t-h(t)) + Cf(\omega(t)) + Dh(\sigma(t)),\end{aligned} \tag{1}$$

$$\begin{aligned}\omega(t) &= Ex(t) = [E_1 \ \ E_2 \ \ ... \ \ E_{k_1}]^T x(t), \\ &\quad \forall t \geq 0,\end{aligned} \tag{2}$$

$$\begin{aligned}\sigma(t) &= Fx(t-h(t)) = [F_1 \ \ F_2 \ \ ... \ \ F_{k_2}]^T \\ &\quad \times x(t-h(t)), \ \ \forall t \geq 0,\end{aligned} \tag{3}$$

$$\begin{aligned}x(t+s) &= \phi(t+s), \ \dot{x}(t+s) = \varphi(t+s), \\ &\quad s \in [-m, 0], \ m = \max\{h_2, \tau_2\},\end{aligned}$$

where $x(t) \in \mathbb{R}^n$, $\omega(t) \in \mathbb{R}^{k_1}$ and $\sigma(t) \in \mathbb{R}^{k_2}$ denote the state vector and output ones of the system, respectively; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{n\times k_1}$, $A_1 \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times k_2}$ are constant known matrices; $f(Ex(\cdot)) = [f_1(E_1^Tx(\cdot)), ..., f_{k_1}(E_{k_1}^Tx(\cdot))]^T$, $h(Fx(\cdot)) = [h_1(F_1^Tx(\cdot)), ..., h_{k_2}(F_{k_2}^Tx(\cdot))]^T$ are the nonlinear elements.

Assumption 1. *The delays $\tau(t)$ and $h(t)$ are time-varying continuous functions that satisfying*

$$0 \leq h_1 \leq h(t) \leq h_2, \quad \mu_1 \leq \dot{h}(t) \leq \mu_2, \tag{4}$$

$$0 \leq \tau(t) \leq \tau_2, \qquad \dot{\tau}(t) \leq \delta < 1, \tag{5}$$

in which $h_1, h_2, \tau_2, \mu_1, \mu_2$ and δ are constants.

Assumption 2. *For any $\epsilon_1, \epsilon_2 \in \mathbb{R}$, the nonlinear function $f_i(\cdot)$ and $h_j(\cdot)$ satisfy $f_i(0) = h_j(0) = 0$,* *and*

$$\sigma_i^- \le \frac{f_i(\epsilon_1)-f_i(\epsilon_2)}{\epsilon_1-\epsilon_2} \le \sigma_i^+,$$
$$\delta_j^- \le \frac{h_j(\epsilon_1)-h_j(\epsilon_2)}{\epsilon_1-\epsilon_2} \le \delta_j^+,$$
$$\epsilon_1 \ne \epsilon_2, i=1,...,k_1; j=1,...,k_2,$$

where $\sigma_i^+, \sigma_i^-, \delta_j^+$, *and* δ_j^- *are given constants. Here, we give*

$$\begin{aligned}
\Upsilon_1 &= diag(\sigma_1^+\sigma_1^-,...,\sigma_{k_1}^+\sigma_{k_1}^-),\\
\Upsilon_2 &= diag(\frac{\sigma_1^+ + \sigma_1^-}{2},...,\frac{\sigma_{k_1}^+ + \sigma_{k_1}^-}{2}),\\
\Upsilon_3 &= diag(\delta_1^+\delta_1^-,...,\delta_{k_2}^+\delta_{k_2}^-),\\
\Upsilon_4 &= diag(\frac{\delta_1^+ + \delta_1^-}{2},...,\frac{\delta_{k_2}^+ + \delta_{k_2}^-}{2}),\\
\overline{\Upsilon}_1 &= diag(\sigma_1^+,...,\sigma_{k_1}^+),\\
\overline{\Upsilon}_2 &= diag(\sigma_1^-,...,\sigma_{k_1}^-),\\
\overline{\Upsilon}_3 &= diag(\delta_1^+,...,\delta_{k_2}^+),\\
\overline{\Upsilon}_4 &= diag(\delta_1^-,...,\delta_{k_2}^-). \qquad (6)
\end{aligned}$$

Assumption 3. *All the eigenvalues of matrix* A_1 *are inside the unit circle.*

We introduce the following technical well-known propositions and Definition, which will be used in the proof of our results.

Lemma 4. *[21] For a given matrix* $R>0$*, the following inequality holds for all continuously differentiable function* ω *in* $[a,b] \to \mathbb{R}^n$*:*

$$\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du \ge \frac{1}{b-a}(\omega(b)-\omega(a))^T R \times(\omega(b)-\omega(a)) + \frac{3}{b-a}\tilde{\Omega}^T R\tilde{\Omega} \qquad (7)$$

where $\tilde{\Omega} = \omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b \omega(u)du.$

Remark 5. *Clearly, the inequality (7) contains a tighter lower bound for* $\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du$ *than Jensen's inequality. As a result, this technique is applied in this paper that this resulting encompasses the Jensen one and also goes to tractable LMI criteria to futher reduce the conservatism over the existing results [14,16,20,22,25].*

Lemma 6. *[27] Let* $h(t)$ *be a continuous function satisfying* $0 \le h_1 \le h(t) \le h_2$*. For any* $n \times n$ *real matrix* $R_1 > 0$ *and a vector* $\dot{x} : [-h_2, 0] \to \mathbb{R}^n$ *such that the integration concerned below is well defined, the following inequality holds for any* $2n \times 2n$ *real matrices* S_1 *satisfying* $\begin{bmatrix} \tilde{R}_1 & S_1 \\ S_1^T & \tilde{R}_1 \end{bmatrix} \ge 0$

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_1\dot{x}(s)ds\\
=&2\varphi_{11}^T S\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21}, \qquad (8)
\end{aligned}$$

where $\tilde{R}_1 \triangleq \mathrm{diag}\{R_1, 3R_1\}$ *and*

$$\varphi_{11} \triangleq \begin{bmatrix} x(t-h(t))-x(t-h_2) \\ x(t-h(t))+x(t-h_2)-2\omega_1(t) \end{bmatrix},$$
$$\varphi_{21} \triangleq \begin{bmatrix} x(t-h_1)-x(t-h(t)) \\ x(t-h_1)+x(t-h(t))-2\omega_2(t) \end{bmatrix},$$

where

$$\begin{aligned}
\omega_1 &\triangleq \frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)} x(s)ds,\\
\omega_2 &\triangleq \frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1} x(s)ds. \qquad (9)
\end{aligned}$$

Lemma 7. *[27] Let* $h(t)$ *be a continuous function satisfying* $0 \le h_1 \le h(t) \le h_2$*. For any* $n \times n$ *real matrix* $R_2 > 0$ *and a vector* $\dot{x} : [-h_2, 0] \to \mathbb{R}^n$ *such that the integration concerned below is well defined, the following inequality holds for any* $\phi_{i1} \in \mathbb{R}^q$ *and real matrices* $Z_i \in \mathbb{R}^{q\times q}, N_i \in \mathbb{R}^{q\times n}$ *satisfying* $\begin{bmatrix} Z_i & N_i \\ N_i^T & R_2 \end{bmatrix} \ge 0 \ (i=1,2)$

$$\begin{aligned}
&-\int_{t-h_2}^{t-h_1}(h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds\\
\le&\frac{1}{2}(h_2-h(t))^2\phi_{11}^T Z_1\phi_{11} + 2(h_2-h(t))\phi_{11}^T N_1\phi_{12}\\
&+\frac{1}{2}[(h_2-h_1)^2-(h_2-h(t))^2]\phi_{21}^T Z_2\phi_{21}\\
&+2\phi_{21}^T N_2[(h_2-h(t))\phi_{22}+(h(t)-h_1)\phi_{23}],
\end{aligned}$$

where

$$\begin{aligned}
\phi_{12} &\triangleq x(t-h(t))-\omega_1(t),\\
\phi_{22} &\triangleq x(t-h_1)-x(t-h(t)),\\
\phi_{23} &\triangleq x(t-h_1)-\omega_2(t).
\end{aligned}$$

Lemma 8. *[27] Let* ξ_0, ξ_1 *and* ξ_2 *be* $m \times m$ *real symmetric matrices and a continuous function* h *satisfy* $h_1 \le h \le h_2$*, where* h_1 *and* h_2 *are constants satisfying* $0 \le h_1 \le h_2$*. If* $\xi_0 \ge 0$*, then*

$$\begin{aligned}
&h^2\xi_0 + h\xi_1 + \xi_2 < 0(\le 0),\ \forall h \in [h_1,h_2],\\
\leftrightarrow\ &h_i^2\xi_0 + h_i\xi_1 + \xi_2 < 0(\le 0),\ (i=1,2), \qquad (10)
\end{aligned}$$

or

$$\begin{aligned}
&h^2\xi_0 + h\xi_1 + \xi_2 > 0(\ge 0),\ \forall h \in [h_1,h_2],\\
\leftrightarrow\ &h_i^2\xi_0 + h_i\xi_1 + \xi_2 > 0(\ge 0),\ (i=1,2). \qquad (11)
\end{aligned}$$

3 Main results

Now we present a Lyapunov-Krasovskii functional for the Lur'e system (1) satisfying the conditions (2), (3) with interval time-varying delay

$$V(t,x_t,\dot{x}_t)=\sum_{i=1}^{4}V_i(t), \tag{12}$$

where

$$\begin{aligned}
V_1(t) &\triangleq \eta^T(t)P\eta(t)+\int_{t-h_1}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds \\
&\quad +\int_{t-\tau(t)}^{t}\dot{x}^T(s)J\dot{x}(s)ds \\
V_2(t) &\triangleq \int_{t-h_1}^{t}[x^T(t)\ x^T(s)]Q_1[x^T(t)\ x^T(s)]^Tds \\
&\quad +\int_{t-h(t)}^{t-h_1}[x^T(t)\ \ x^T(s)]Q_2[x^T(t)\ \ x^T(s)]^Tds \\
&\quad +\int_{t-h_2}^{t-h(t)}[x^T(t)\ \ x^T(s)]Q_3[x^T(t)\ \ x^T(s)]^Tds \\
V_3(t) &\triangleq \int_{t-h_1}^{t}\Big\{h_1(h_1-t+s)\dot{x}^T(s)W_1\dot{x}(s) \\
&\quad +(h_1-t+s)^2\dot{x}^T(s)W_2\dot{x}(s)\Big\}\,ds \\
&\quad +\int_{t-h_2}^{t-h_1}\Big\{h_{21}(h_2-t+s)\dot{x}^T(s)R_1\dot{x}(s) \\
&\quad +(h_2-t+s)^2\dot{x}^T(s)R_2\dot{x}(s)\Big\}\,ds \\
V_4(t) &\triangleq 2\sum_{i=1}^{n}\int_0^{E_i^Tx}[k_i[f_i(s)-\sigma_i^-(s)] \\
&\quad +l_i[\sigma_i^+(s)-f_i(s)]]ds \\
&\quad 2\sum_{i=1}^{n}\int_0^{F_i^Tx}[g_i[h_i(s)-\delta_i^-(s)] \\
&\quad +t_i[\delta_i^+(s)-h_i(s)]]ds
\end{aligned} \tag{13}$$

where P are real matrices, $Q_0>0, Q_j>0, W_q>0, R_q>0, J>0 (j=1,2,3; q=1,2)$, $K=\text{diag}(k_1,...,k_n)>0, L=\text{diag}(l_1,...,l_n)>0, G=\text{diag}(g_1,...,g_n)>0, T=\text{diag}(t_1,...,t_n)>0$; and $h_{21}\triangleq h_2-h_1$,
$\eta(t) \triangleq \text{col}\{x(t),\ x(t-h_1),\ \int_{t-h_2}^{t-h(t)}x(s)ds,$
$\int_{t-h(t)}^{t-h_1}x(s)ds,$
$\int_{t-h_1}^{t}x(s)ds\ \}$.

Remark 9. *This of [14], previous work only focused on some the augment vectors but our work includes not only on $x(t), \int_{t-\tau_0}^{t}x(s)ds$ but also $x(t), \int_{t-h_2}^{t-h(t)}x(s)ds, x(t-h_1), \int_{t-h(t)}^{t-h_1}x(s)ds$. We can see that the adoption of new augmented variables, cross terms of variables and more multiple integral terms may reduce the conservatism.*

Remark 10. *Those of [5,14] previous works, the augmented Lyapunov matrix P still need $P>0$, but for our work does not need to be positive definite, which can be seen in Lemma 11. So we can see that the introduction of the vector $\Theta(t)$ plays a important key role in deriving a quadratic convex combination $\Sigma(h(t),\dot{h}(t))$. Hence, a matrix-based quadratic convex technique can be applied to design an LMI-based sufficient conditions.*

For simplicity of presentation, we set in the following
ω_1,ω_2 are defined in (9) and $\omega_3=\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds$. Denote by $\tilde{e}_i (i=1,\dots,5)$ the block-row vectors of the $5n\times 5n$ identity matrix. Then we have the following result.

Lemma 11. *[27] For the LKF (13), there exist scalars $\epsilon_1>0$ and $\epsilon_2>0$ such that*

$$\epsilon_1\|x\|^2\le V(t,x_t,\dot{x}_t)\le\epsilon_2\|x_t\|_W^2 \tag{14}$$

if the following LMIs are satisfied

$$\begin{aligned}
&\tilde{e}_1P\tilde{e}_1^T>0,\ \ P_0\ge 0,\ \ \Lambda_1(h_1)+\Lambda_2(h_1)\ge 0, \\
&\Lambda_1(h_2)+\Lambda_2(h_2)\ge 0,
\end{aligned} \tag{15}$$

where

$$\Lambda_1(h(t))\triangleq\begin{cases}\Delta, & h_1=0\\ \Delta+\frac{1}{h_1}\Gamma_2^T diag\{Q_0,3Q_0\}\Gamma_2, & h_1\ne 0\end{cases} \tag{16}$$

$$\begin{aligned}
\Lambda_2(h(t)) &\triangleq h_1[\tilde{e}_1^T\ \tilde{e}_5^T]Q_1[\tilde{e}_1^T\ \tilde{e}_5^T]^T+(h(t)-h_1) \\
&\quad\times[\tilde{e}_1^T\ \tilde{e}_4^T]Q_2[\tilde{e}_1^T\ \tilde{e}_4^T]^T+(h_2-h(t)) \\
&\quad\times[\tilde{e}_1^T\ \tilde{e}_3^T]Q_3[\tilde{e}_1^T\ \tilde{e}_3^T]^T
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\Gamma_1 &= col\{\tilde{e}_1,\tilde{e}_2,(h_2-h(t))\tilde{e}_3,(h(t)-h_1)\tilde{e}_4,h_1\tilde{e}_5\}, \\
\Gamma_2 &= col\{\tilde{e}_1-\tilde{e}_2,\tilde{e}_1+\tilde{e}_2-2\tilde{e}_5\}, \\
P_0 &= (\tilde{e}_4^T\tilde{e}_4-\tilde{e}_3^T\tilde{e}_3)P(\tilde{e}_4^T\tilde{e}_4-\tilde{e}_3^T\tilde{e}_3), \\
\Delta &= \Gamma_1^TP\Gamma_1-\tilde{e}_1^T\tilde{e}_1P\tilde{e}_1^T\tilde{e}_1.
\end{aligned}$$

Theorem 12. *The system (1) satisfying the sector condition (2), (3), for given scalars h_1,h_2,μ_1,μ_2 and δ is absolutely stable if there exist P are real matrices to be determined, symmetric positive definite matrices $Q_0>0, Q_j>0, W_q>0, R_q>0, J>$*

$0 (j = 1, 2, 3; q = 1, 2)$*, and* $n \times n$ *diagonal matrices* $K > 0, L > 0, G > 0, T > 0, U > 0, V > 0$ *such that (15) and the following LMI holds:*

$$\sum_{i=1,h(t)=h_1,\mu(t)=\mu_1}^{4} \Sigma_i < 0, \quad \sum_{i=1,h(t)=h_1,\mu(t)=\mu_2}^{4} \Sigma_i < 0,$$
$$\sum_{i=1,h(t)=h_2,\mu(t)=\mu_1}^{4} \Sigma_i < 0, \quad \sum_{i=1,h(t)=h_2,\mu(t)=\mu_2}^{4} \Sigma_i < 0, \tag{18}$$

$$\begin{bmatrix} Z_i & N_i \\ N_i^T & R_2 \end{bmatrix} \geq 0, (i = 1, 2) \begin{bmatrix} Z_3 & N_2 \\ N_3^T & W_2 \end{bmatrix} \geq 0, \tag{19}$$

$$\begin{bmatrix} \tilde{R}_1 & S_1 \\ S_1^T & \tilde{R}_1 \end{bmatrix} \geq 0, Z_1 \geq Z_2, \tag{20}$$

where $\tilde{R}_1 = diag\{R_1, 3R_1\}$*;and*

$$\begin{aligned}
\Sigma_1(h(t), \dot{h}(t)) \triangleq{}& \Delta_1^T P \Delta_2 + \Delta_2^T P \Delta_1 + \Delta_0^T Q_0 \Delta_0 \\
&- e_8^T Q_0 e_8 + \Delta_0^T J \Delta_0 \\
&- (1 - \dot{\tau}(t)) e_{11}^T J e_{11} \\
\Sigma_2(h(t), \dot{h}(t)) \triangleq{}& \Psi_{20} + [h(t) - h_1]\Psi_{21} + [h_2 - d(t)]\Psi_{22} \\
\Sigma_3(h(t)) \triangleq{}& \tilde{\varphi}_1^T S_1 \tilde{\varphi}_2 + \tilde{\varphi}_2^T S_1^T \tilde{\varphi}_1 - \tilde{\varphi}_1^T \tilde{R}_1 \tilde{\varphi}_1 \\
&+ (h_2 - h(t))^2 (Z_1 - Z_2) \\
&+ (h_2 - h(t))\Psi_{31} + (h(t) - h_1)\Psi_{32} \\
&+ h_{21}^2 Z_2 - \tilde{\varphi}_2^T \tilde{R}_1 \tilde{\varphi}_2 \\
\Sigma_4 \triangleq{}& -\tilde{\varphi}_3^T \tilde{W}_1 \tilde{\varphi}_3 + \Delta_0^T (h_1^2 W_1 + h_1^2 W_2) \Delta_0 \\
&+ 2h_1 N_3 (e_1 - e_7) + e_8^T (h_{21}^2 R_1 \\
&+ h_{21}^2 R_2) e_8 + 2h_1 (e_1 - e_7)^T N_3^T + h_1^2 Z_3 \\
&+ e_{10}^T [K - L] E \Delta_0 + \Delta_0^T E^T [K - L]^T e_{10} \\
&+ e_1^T E^T [\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K] E \Delta_0 \\
&+ \Delta_0^T E^T [\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]^T E e_1 \\
&+ e_9^T [G - T] F \Delta_0 + \Delta_0^T F^T [G - T]^T e_9 \\
&+ e_1^T F^T [\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T] F \Delta_0 \\
&+ \Delta_0^T F^T [\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]^T F e_1 \\
&- [e_1^T E^T U \Upsilon_1 E e_1 - 2e_1^T E^T U \Upsilon_2 e_{10} \\
&+ e_{10}^T U e_{10}] - [e_1^T F^T V \Upsilon_3 F e_1 \\
&- 2e_1^T F^T V \Upsilon_4 e_9 + e_9^T V e_9]
\end{aligned} \tag{21}$$

with $e_i (i = 1, 2, \ldots, 11)$ *denoting the i-th row-block vector of the* $11n \times 11n$ *identity matrix* $\tilde{W}_1 = diag\{W_1, 3W_1\}$*; and*

$$\begin{aligned}
\Psi_{20} \triangleq{}& [e_1^T \ e_3^T](Q_2 - Q_1)[e_1^T \ e_3^T]^T \\
&+ h_1[\Delta_0^T \ 0]Q_1[e_1^T \ e_7^T]^T + h_1[e_1^T \ e_7^T]Q_1[\Delta_0^T \ 0]^T \\
&- (1 - \dot{h}(t))[e_1^T \ e_2^T](Q_2 - Q_3)[e_1^T \ e_2^T]^T \\
&- [e_1^T \ e_4^T]Q_3[e_1^T \ e_4^T]^T + [e_1^T \ e_1^T]Q_1[e_1^T \ e_1^T]^T \\
\Psi_{21} \triangleq{}& [e_1^T \ e_6^T]Q_2[\Delta_0^T \ 0]^T + [\Delta_0^T \ 0]Q_2[e_1^T \ e_6^T]^T \\
\Psi_{22} \triangleq{}& [e_1^T \ e_5^T]Q_3[\Delta_0^T \ 0]^T + [\Delta_0^T \ 0]Q_3[e_1^T \ e_5^T]^T \\
\Psi_{31} \triangleq{}& 2N_1(e_2 - e_5) + 2N_2(e_3 - e_2) \\
&+ 2(e_3 - e_2)^T N_2^T + 2(e_2 - e_5)^T N_1^T \\
\Psi_{32} \triangleq{}& 2N_2(e_3 - e_6) + 2(e_3 - e_6)^T N_2^T \\
\tilde{\varphi}_1 \triangleq{}& col\{e_2 - e_4, e_2 + e_4 - 2e_5\} \\
\tilde{\varphi}_2 \triangleq{}& col\{e_3 - e_2, e_3 + e_2 - 2e_6\} \\
\tilde{\varphi}_3 \triangleq{}& col\{e_1 - e_3, e_1 + e_3 - 2e_7\} \\
\Delta_1 \triangleq{}& col\{e_1, e_3, (h_2 - h(t))e_5, (h(t) - h_1)e_6, h_1 e_7\} \\
\Delta_2 \triangleq{}& col\{\Delta_0, e_8, (1 - \dot{h}(t))e_2 - e_4, e_3 \\
&- (1 - \dot{h}(t))e_2, e_1 - e_3\}.
\end{aligned}$$

For simplicity of presentation, we denote $\Theta \triangleq col\{x(t), x(t - h(t)), x(t - h_1), x(t - h_2), \omega_1(t), \omega_2(t), \omega_3(t), \dot{x}(t - h_1), h(\sigma(t)), f(\omega(t)), \dot{x}(t - \tau(t))\}$, $\dot{x}(t) = \Delta_0 \Theta(t), \Delta_0 \triangleq A_1 e_{11} + A e_1 + B e_2 + C e_{10} + D e_9$.

Proof. Taking the derivative of V along the solution of system(1), we can be obtains as

$$\begin{aligned}
\dot{V}_1(t) ={}& 2\eta^T(t) P \dot{\eta}(t) + \dot{x}^T(t) Q_0 \dot{x}(t) - \dot{x}^T(t - h_1) Q_0 \\
&\times \dot{x}(t - h_1) + \dot{x}^T(t) J \dot{x}(t) \\
&- (1 - \dot{\tau}(t)) \dot{x}^T(t - \tau(t)) J \dot{x}(t - \tau(t)) \\
\dot{V}_2(t) ={}& [x^T(t) \ x^T(t)]Q_1[x^T(t) \ x^T(t)]^T \\
&- [x^T(t) \ x^T(t - h_1)]Q_1[x^T(t) \ x^T(t - h_1)]^T \\
&+ 2\int_{t-h_1}^{t} [x^T(t) \ x^T(s)]Q_1[\dot{x}(t)^T \ 0]^T ds \\
&+ [x^T(t) \ x^T(t - h_1)]Q_2[x^T(t) \ x^T(t - h_1)]^T \\
&- (1 - \dot{h}(t))[x^T(t) \ x^T(t - h(t))]Q_2[x^T(t) \\
&x^T(t - h(t))]^T + 2\int_{t-h(t)}^{t-h_1} [x^T(t) \ x^T(s)]Q_2 \\
&\times [\dot{x}^T(t) \ 0]^T ds + [x^T(t) \ x^T(t - h_2)] \\
&\times Q_3[x^T(t) \ x^T(t - h_2)]^T \\
&- (1 - \dot{h}(t))[x^T(t) \ x^T(t - h(t))]Q_3
\end{aligned} \tag{22}$$

$$
\begin{aligned}
&- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_3 \\
&\times [x^T(t)\ x^T(t-h(t))]^T \\
&+ 2\int_{t-h_2}^{t-h(t)} [x^T(t)\ x^T(s)]Q_3[\dot{x}^T(t)\ 0]^T ds \\
\dot{V}_3(t) =& \Theta^T(t)(h_1^2\Delta_0^T W_1\Delta_0 + h_1^2\Delta_0^T W_2\Delta_0)\Theta(t) \\
&- h_1\int_{t-h_1}^{t} \dot{x}^T(s)W_1\dot{x}(s)ds \\
&- 2\int_{t-h_1}^{t} (h_1-t+s)\dot{x}^T(s)W_2\dot{x}(s)ds \\
&+ h_{21}^2\dot{x}^T(t-h_1)R_1\dot{x}(t-h_1) \\
&+ h_{21}^2\dot{x}^T(t-h_1)R_2\dot{x}(t-h_1) \\
&- h_{21}\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&- 2\int_{t-h_2}^{t-h_1} (h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds. \\
\dot{V}_4(t) =& 2f^T(Ex(t))[K-L]E\dot{x}(t) + 2x^T(t)E^T[\bar{\Upsilon}_1 L \\
&- \bar{\Upsilon}_2 K]E\dot{x} + 2h^T(Fx(t))[G-T]F\dot{x}(t) \\
&+ 2x^T(t)F^T[\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]F\dot{x}(t). \qquad (23)
\end{aligned}
$$

On the condition (6) and diagonal matrices $U > 0, V > 0$, then we have

$$
\begin{aligned}
-[x^T(t)E^T U\Upsilon_1 Ex(t) - 2x^T(t)E^T U\Upsilon_2 f(Ex(t)) \\
+f^T(Ex(t))Uf(Ex(t))] - [x^T(t)F^T V\Upsilon_3 Fx(t) \\
-2x^T(t)F^T V\Upsilon_4 h(Fx(t)) + h^T(Fx(t))Vh(Fx(t))] \\
\geq 0.
\end{aligned}
$$

With the consideration of some term of $\dot{V}_2(t), \dot{V}_3(t)$, we obtained the following equality and inequality:

$$
\begin{aligned}
&\int_{t-h_1}^{t} [x^T(t)\ x^T(s)]Q_1[\dot{x}(t)^T\ 0]^T ds \\
&= [\int_{t-h_1}^{t} x^T(t)ds \quad \int_{t-h_1}^{t} x^T(s)ds]Q_1[\dot{x}^T(t)\ \ 0]^T \\
&= h_1[x^T(t)\ \ \omega_3^T]Q_1[\dot{x}^T(t)\ \ 0]^T, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
&\int_{t-h(t)}^{t-h_1} [x^T(t)\ x^T(s)]Q_2[\dot{x}(t)^T\ 0]^T ds \\
&= [\int_{t-h(t)}^{t-h_1} x^T(t)ds \quad \int_{t-h(t)}^{t-h_1} x^T(s)ds]Q_2[\dot{x}^T(t)\ \ 0]^T \\
&= (h(t)-h_1)[x^T(t)\ \ \omega_2^T]Q_2[\dot{x}^T(t)\ \ 0]^T, \qquad (25)
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{t-h_2}^{t-h(t)} [x^T(t)\ x^T(s)]Q_3[\dot{x}(t)^T\ 0]^T ds \\
&= [\int_{t-h_2}^{t-h(t)} x^T(t)ds \quad \int_{t-h_2}^{t-h(t)} x^T(s)ds] \\
&\times Q_3[\dot{x}^T(t)\ \ 0]^T \\
&= (h_2-h(t))[x^T(t)\ \ \omega_1^T]Q_3[\dot{x}^T(t)\ \ 0]^T. \qquad (26)
\end{aligned}
$$

By utilizing Lemma4, we can be estimated

$$
\begin{aligned}
&-\int_{t-h_1}^{t} \dot{x}^T(s)h_1 W_1\dot{x}(s)ds \\
&\leq -[x(t)-x(t-h_1)]^T W_1[x(t)-x(t-h_1)] \\
&-3\tilde{\Omega_1}^T W_1\tilde{\Omega}_1, \qquad (27)
\end{aligned}
$$

where

$$
\tilde{\Omega_1} = x(t) + x(t-h_1) - 2\omega_3.
$$

And applying [27], we obtained the following

$$
\begin{aligned}
&-2\int_{t-h_1}^{t} (h_1-t+s)\dot{x}^T(s)W_2\dot{x}(s)ds \\
&\leq h_1^2\Theta^T(t)Z_3\Theta(t) + 2h_1\Theta^T(t)N_3[x(t)-\omega_3] \\
&+ 2h_1[x(t)-\omega_3]^T N_3^T\Theta(t), \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)h_{21}R_1\dot{x}(s)ds \\
&\leq 2\varphi_{11}^T S_1\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21} \qquad (29)
\end{aligned}
$$

and

$$
\begin{aligned}
&-2\int_{t-h_2}^{t-h_1} (h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds \\
&\leq (h_2-h(t))^2\Theta^T(t)Z_1\Theta(t) \\
&+ 4(h_2-h(t))\Theta^T(t)N_1[x(t-h(t))-\omega_1] \\
&+ [(h_2-h_1)^2-(h_2-h(t))^2]\Theta^T(t)Z_2\Theta(t) \\
&+ 4\Theta^T(t)N_2[(h_2-h(t))[x(t-h_1)-x(t-h(t))] \\
&+ (h(t)-h_1)[x(t-h_1)-\omega_2(t)]]. \qquad (30)
\end{aligned}
$$

Hence, according to (23)-(30) we get

$$\begin{aligned}
&\dot{V}(t,x_t,\dot{x}_t) \le 2\eta^T(t)P\dot{\eta}(t) + \dot{x}^T(t)Q_0\dot{x}(t)\\
&- \dot{x}^T(t-h_1)Q_0\dot{x}(t-h_1) + \dot{x}^T(t)J\dot{x}(t)\\
&- (1-\delta)\dot{x}^T(t-\tau(t))J\dot{x}(t-\tau(t))\\
&+ [x^T(t)\ x^T(t)]Q_1[x^T(t)\ x^T(t)]^T\\
&- [x^T(t)\ x^T(t-h_1)]Q_1[x^T(t)\ x^T(t-h_1)]^T\\
&+ 2h_1[x^T(t)\ \ \omega_3^T]Q_1[\dot{x}^T(t)\ \ 0]^T\\
&+ [x^T(t)\ x^T(t-h_1)]Q_2[x^T(t)\ x^T(t-h_1)]^T\\
&- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_2[x^T(t)\\
&x^T(t-h(t))]^T + 2(h(t)-h_1)[x^T(t)\ \ \omega_2^T]\\
&\times Q_2[\dot{x}^T(t)\ \ 0]^T + [x^T(t)\ x^T(t-h_2)]\\
&\times Q_3[x^T(t)\ x^T(t-h_2)]^T\\
&- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_3\\
&\times [x^T(t)\ x^T(t-h(t))]^T\\
&+ 2(h_2-h(t))[x^T(t)\ \ \omega_1^T]Q_3[\dot{x}^T(t)\ \ 0]^T\\
&+ \Theta^T(t)(h_1^2\Delta_0^TW_1\Delta_0 + h_1^2\Delta_0^TW_2\Delta_0)\Theta(t)\\
&- [x(t)-x(t-h_1)]^TW_1[x(t)-x(t-h_1)]\\
&- 3\tilde{\Omega_1}^TW_1\tilde{\Omega}_1 + h_1^2\Theta^T(t)Z_3\Theta(t)\\
&+ 2h_1\Theta^T(t)N_3[x(t)-\omega_3] + 2h_1[x(t)-\omega_3]^T\\
&\times N_3^T\Theta(t) + h_{21}^2\dot{x}^T(t-h_1)R_1\dot{x}(t-h_1)\\
&+ h_{21}^2\dot{x}^T(t-h_1)R_2\dot{x}(t-h_1)\\
&+ 2\varphi_{11}^TS_1\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21}\\
&+ (h_2-h(t))^2\Theta^T(t)Z_1\Theta(t)\\
&+ 4(h_2-h(t))\Theta^T(t)N_1[x(t-h(t))-\omega_1]\\
&+ [(h_2-h_1)^2-(h_2-h(t))^2]\Theta^T(t)Z_2\Theta(t)\\
&+ 4\Theta^T(t)N_2[(h_2-h(t))[x(t-h_1)-x(t-h(t))]\\
&+ (h(t)-h_1)[x(t-h_1)-\omega_2(t)]]\\
&+ 2f^T(Ex(t))[K-L]E\dot{x}(t) + 2x^T(t)E^T\\
&\times [\bar{\Upsilon}_1L-\bar{\Upsilon}_2K]E\dot{x} + 2h^T(Fx(t))[G-T]F\dot{x}(t)\\
&+ 2x^T(t)F^T[\bar{\Upsilon}_3G-\bar{\Upsilon}_4T]F\dot{x}(t)\\
&- [x^T(t)E^TU\Upsilon_1Ex(t) - 2x^T(t)E^TU\Upsilon_2f(Ex(t))\\
&+ f^T(Ex(t))Uf(Ex(t))] - [x^T(t)F^TV\Upsilon_3Fx(t)\\
&- 2x^T(t)F^TV\Upsilon_4h(Fx(t))\\
&+ h^T(Fx(t))Vh(Fx(t))]\\
&\dot{V}(t,x_t,\dot{x}_t) \le \Theta^T(t)\Sigma(h(t),\dot{h}(t))\Theta(t) \qquad (31)
\end{aligned}$$

where $\Sigma(h(t),\dot{h}(t)) \triangleq \sum_{i=1}^4\Sigma_i$. Clearly, $\Sigma(h(t),\dot{h}(t))$ can be rewritten as $\Sigma(h(t),\dot{h}(t)) = h^2(t)\Pi_0 + h(t)\Pi_1 + \Pi_2$ where $\Pi = Z_1 - Z_2$ and Π_1 and Π_2 are $h(t)-$ independent real metrices. Now together with (8) and if $Z_1 - Z_2 \ge 0$ and the inequalities in (18) hold, then $\Sigma(h(t),\dot{h}(t)) < 0, \forall h(t) \in [h_1,h_2], \forall\dot{h}(t) \in [\mu_1,\mu_2]$. Then $\dot{V}(t,x_t) \le -\lambda\|x(t)\|$ for some $\lambda > 0, \forall x(t) \ne 0$. Thus the system (1) satisfy conditions (2),(3) is absolutely stable. □

When $D = 0$, time-varying delay system in (1) reduces to

$$\begin{aligned}\dot{x}(t) &= A_1\dot{x}(t-\tau(t)) + Ax(t) + Bx(t-h(t))\\ &\quad +Cf(\omega(t)). \qquad (32)\end{aligned}$$

In the following, we present a stability criterion for $h_1 = 0$. We consider the following LKF candidate

$$\begin{aligned}
\hat{V}(t,x_t,\dot{x}_t) &= \bar{\eta}^T(t)P\bar{\eta}(t)\\
&+ \int_{t-\tau(t)}^t \dot{x}^T(s)J\dot{x}(s)ds\\
&+ \int_{t-h(t)}^t [x^T(t)\ \ x^T(s)]Q_2\\
&\times[x^T(t)\ \ x^T(s)]^Tds\\
&+ \int_{t-h_2}^{t-h(t)} [x^T(t)\ \ x^T(s)]Q_3\\
&\times[x^T(t)\ \ x^T(s)]^Tds\\
&+ \int_{t-h_2}^t \Big\{h_2(h_2-t+s)\dot{x}^T(s)\\
&\times R_1\dot{x}(s) + (h_2-t+s)^2\\
&\times\dot{x}^T(s)R_2\dot{x}(s)\Big\}\,ds\\
&+2\sum_{i=1}^n\int_0^{E_i^Tx}[k_i[f_i(s)-\sigma_i^-(s)]\\
&+l_i[\sigma_i^+(s)-f_i(s)]]ds, \qquad (33)
\end{aligned}$$

where $\bar{\eta} = [x(t)\ \int_{t-h_2}^{t-h(t)}x(s)ds\ \int_{t-h(t)}^t x(s)ds]$ and $-[x^T(t)E^TU\Upsilon_1Ex(t) - 2x^T(t)E^TU\Upsilon_2$ $f(Ex(t)) + f^T(Ex(t))Uf(Ex(t))] \ge 0$.

Corollary 13. *The system (32) satisfying the sector condition (2), for given scalars h_2,μ_1,μ_2 and δ is absolutely stable if there exist $P = P^T$ with $P_{11} > 0$ symmetric positive definite matrices $R_q > 0, J > 0 (q = 1,2)$, and $n\times n$ diagonal matrices $K > 0, L > 0, U > 0$ such that (15) and the following LMIs hold:*

$$\begin{aligned}&F(h(t),\dot{h}(t)) + h_2\Omega_1 + (1-\dot{h}(t))\Omega_3 + \Omega_4 < 0,\\ &h(t) = 0, \dot{h}(t) = \mu_1,\mu_2. \qquad (34)\end{aligned}$$

$$\mathcal{F}(h(t),\dot{h}(t)) + h_2\Omega_2 + (1-\dot{h}(t))\Omega_3 + \Omega_4 < 0,$$
$$h(t) = h_2, \dot{h}(t) = \mu_1, \mu_2. \quad (35)$$

$$\begin{bmatrix} \tilde{R}_1 & S_1 \\ S_1^T & \tilde{R}_1 \end{bmatrix} \geq 0, \begin{bmatrix} P_{22} & P_{23} \\ * & P_{33} \end{bmatrix} \geq 0, Z_1 \geq Z_2, \quad (36)$$

$$\begin{bmatrix} Z_i & N_i \\ N_i^T & R_2 \end{bmatrix} \geq 0, (i = 1, 2), \quad (37)$$

$$\begin{bmatrix} h_2 Q_{21} & h_2(P_{13} + Q_{22}) \\ * & h_2^2 P_{33} + h_2 Q_{23} \end{bmatrix} \geq 0, \quad (38)$$

$$Q_j = \begin{bmatrix} Q_{j1} & Q_{j2} \\ * & Q_{j3} \end{bmatrix} \geq 0, (i = 2, 3), \quad (39)$$

$$\begin{bmatrix} h_2 Q_{11} & h_2(P_{12} + Q_{12}) \\ * & h_2^2 P_{22} + h_2 Q_{13} \end{bmatrix} \geq 0, \quad (40)$$

where $\tilde{R}_1 \triangleq diag\{R_1, 3R_1\}$

$$\begin{aligned}
\mathcal{F}(h(t),\dot{h}(t)) &\triangleq \mathcal{F}_1^T(h(t))P\mathcal{F}_2(\dot{h}(t)) \\
&\quad +\mathcal{F}_2^T(\dot{h}(t))P\mathcal{F}_1(h(t)) \\
\Omega_1 &\triangleq [\hat{e}_1^T\ \hat{e}_4^T]Q_3\emptyset_0 + \emptyset_0^T Q_3[\hat{e}_1^T \hat{e}_4^T]^T \\
&\quad +h_2(Z_1 - Z_2) + 2N_1[\hat{e}_2 - \hat{e}_4] \\
&\quad +2[\hat{e}_2 - \hat{e}_4]^T N_1^T + 2N_2[\hat{e}_1 - \hat{e}_2] \\
&\quad +2[\hat{e}_1 - \hat{e}_2]^T N_2^T \\
\Omega_2 &\triangleq [\hat{e}_1^T\ \hat{e}_5^T]Q_2\emptyset_0 + \emptyset_0^T Q_2[\hat{e}_1^T \hat{e}_5^T]^T \\
&\quad +2N_2[\hat{e}_1 - \hat{e}_5] + 2[\hat{e}_1 - \hat{e}_5]^T N_2^T \\
\Omega_3 &\triangleq [\hat{e}_1^T\ \hat{e}_2^T](Q_3 - Q_2)[\hat{e}_1^T\ \hat{e}_2^T]^T \\
\Omega_4 &\triangleq \emptyset_0^T J\emptyset_0 - (1-\delta)\hat{e}_7^T J\hat{e}_7 \\
&\quad +[\hat{e}_1^T\ \hat{e}_1^T]Q_2[\hat{e}_1^T\ \hat{e}_1^T]^T \\
&\quad -[\hat{e}_1^T\ \hat{e}_3^T]Q_3[\hat{e}_1^T\ \hat{e}_3^T]^T \\
&\quad +h_2^2\emptyset_0^T(R_1 + R_2)\emptyset_0 + \emptyset_1^T S_1\emptyset_2 \\
&\quad +\emptyset_2^T S_1^T\emptyset_1 - \emptyset_1^T\tilde{R}_1\emptyset_1 - \emptyset_2^T\tilde{R}_1\emptyset_2 \\
&\quad +h_2^2 Z_2 + \hat{e}_6^T[K - L]E\emptyset_0 \\
&\quad +\emptyset_0^T E^T[K - L]^T\hat{e}_6 \\
&\quad +\hat{e}_1 E^T[\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]E\emptyset_0 \\
&\quad +\emptyset_0^T E^T[\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]^T E\hat{e}_1^T \\
&\quad -[\hat{e}_1^T E^T U\Upsilon_1 E\hat{e}_1 - \hat{e}_1^T U\Upsilon_2\hat{e}_6 \\
&\quad -\hat{e}_6^T\Upsilon_2^T U^T\hat{e}_1 + \hat{e}_6^T U\hat{e}_6],
\end{aligned}$$

with $\hat{e}_1 = [I\ 0\ 0\ 0\ 0\ 0\ 0], ..., \hat{e}_7 = [0\ 0\ 0\ 0\ 0\ 0\ I]$*; and*

$$\begin{aligned}
\mathcal{F}_1(h(t)) &\triangleq col\{\hat{e}_1, (h_2 - h(t))\hat{e}_4, h(t)\hat{e}_5\} \\
\mathcal{F}_2(\dot{h}(t)) &\triangleq col\{\emptyset_0, (1-\dot{h}(t))\hat{e}_2 - \hat{e}_3, \hat{e}_1 \\
&\qquad -(1-\dot{h}(t))\hat{e}_2\} \\
\emptyset_0 &\triangleq A_1\hat{e}_7 + A\hat{e}_1 + B\hat{e}_2 + C\hat{e}_6 \\
\emptyset_1 &\triangleq col\{\hat{e}_2 - \hat{e}_3, \hat{e}_2 + \hat{e}_3 - 2\hat{e}_4\} \\
\emptyset_2 &\triangleq col\{\hat{e}_1 - \hat{e}_2, \hat{e}_1 + \hat{e}_2 - 2\hat{e}_5\}. \quad (41)
\end{aligned}$$

4 Numerical Example

In this section, we provide numerical examples to show the effectiveness of our theoretical results.
In this section, we provide numerical examples to show the effectiveness of our theoretical results.
Example 4.1 Consider the following Lur'e system with time-varying delays which is studied in [25], [22], [20], [14]:

$$\begin{aligned}
\dot{x}(t) &= A_1\dot{x}(t-\tau(t)) + Ax(t) + Bx(t-h(t)) \\
&\quad +Cf(\omega(t)), \quad (42)
\end{aligned}$$

with the following parameters:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, \\
B &= \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix}, C = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}, \\
E &= \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}.
\end{aligned}$$

Applying Corollary13, The maximum allowable value of h_2 is given in Table I when $h_1 = 0$, $-\mu_1 = \mu_2 = \mu$ and values of $\delta = 0.9$. The constraint on the augmented Lyapunov matrix $P > 0$ is not required, so our conditions are generally less conservative than [14].

Table I: Upper bounds of interval time-varying delays with $h_1 = 0, \delta = 0.9$ and different values of μ for Example 4.1.

δ	μ	0.2	0.4
	[25]	1.841	1.315
	[22]	2.154	1.704
0.5	[20]	2.456	1.801
	[14]	2.462	1.810
	our	> 10000	> 10000
	[25]	0.124	0.108
	[22]	0.112	0.109
0.9	[20]	0.113	0.110
	[14]	0.125	0.113
0.9	Corollary13	0.510	0.451

δ	μ	0.6	0.8
	[25]	0.790	0.632
	[22]	1.383	1.009
0.5	[20]	1.482	1.116
	[14]	1.490	1.122
	our	2.8080	2.1934
	[25]	0.099	0.098
	[22]	0.109	0.109
0.9	[20]	0.110	0.110
	[14]	0.113	0.112
0.9	Corollary13	0.414	0.388

Example 4.2 Consider the following neutral system with time-varying delays which is studied in [16]:

$$\dot{x}(t) = A_1\dot{x}(t-\tau(t)) + Ax(t) + Bx(t-h(t))$$

with the following parameters:

$$A_1 = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$
$$B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

By applying our proposed Theorem 12 to the above system, one can obtain maximum delay bounds as listed in Table II. It can be found that the maximum upper bounds on the allowable sizes to be $h(t) = \tau(t) = 4.2365$, which is larger than in [16]. This means that the proposed ideas in theorem 12 is effective in reducing the conservatism of stability criterion.

Table II: Upper bounds of interval time-varying delays with $h_1 = 0$ and $\tau(t) = h(t)$ for Example 4.2.

Methods	$h_2[\tau(t) = h(t)]$
[16]	0.985
Theorem 12	4.2365

5 Conclusion

In this paper, we have investigated with the problem of robust absolute stability of neutral type Lur'e systems with mixed time-varying delays. By constructing an new augmented Lyapunov-Krasovskii functional and combining integral inequality with approach to estimate the derivative of the Lyapunov-Krasovskii functional, which estimated some integral terms by Wirtinger's inequality, a matrix-based quadratic convex technique is used to design an LMI-based sufficient conditions. New stability condition is much less conservative and more general than some existing results. New stability criteria is given in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the theoretic results which show that our results are much less conservative than some existing results in the literature.

Acknowledgements: The first author was financial supported by University of PhaYao, PhaYao, Thailand. The second author was financial supported by the Thailand Research Fund (TRF), Khon Kaen University (grant number : MRG5880009) and National Research Council of Thailand and Khon Kaen University 2017 (grant number : 600061).

References:

[1] I. Amri, D. Soudani and M. Benrejeb, Delay–dependent robust exponentially stability criteria for perturbed and uncertain neutral systems with time-varying delays, *Stud. Inform. Contol.*, 19, 2010, pp. 135–144.

[2] T. Botmart, P. Niamsup and V. N. Phat, Delay–dependent exponential stabilization for uncertain linear systems with interval non–differentiable time-varying delays,*Appl. Math. Comput.*, 217, 2011, pp. 8236–8247.

[3] J. Cao, G. Chen and P. Li, Synchronization for Coupled Neural Networks With Interval Delay A Novel Augmented Lyapunov-Krasovskii Functional Method, *IEEE Trans. Neural Netw. Learn. Syst.*, 22, 2013, pp. 58–70.

[4] J. Cao, L. Li, Cluster synchronization in an array of hybrid coupled neural networks with delay, *Neural Netw.*, 22, 2009, pp. 335–342.

[5] Y. Chen, W. Bi and W. Li,New delay–dependent absolute stability criteria for Lur'e systems with time–varying delay, *Internat. J. Systems Sci.* , 42, 2011, pp. 1105–1113.

[6] K. Gu, V. L. Kharitonov and J. Chen, *Stability of time–delay system.*, Boston: Birkhauser; 2003.

[7] J. F. Gao, H. P. Pan and X. F. Ji, A new delay–dependent absolute stability criterion for Lurie systems with time–varying delay, *IActa Automat. Sinica.*, 36, 2010, pp. 845–850.

[8] Q. L. Han, A. Xue, S. Liu and X. Yu, Robust absolute stability criteria for uncertain Lur'e systems of neutral type, *Internat. J. Robust Nonlinear Control*, 18, 2008, pp. 278–295.

[9] Q. L. Han, D. Yue, Absolute stability of Lure systems with time–varying delay, *IIET Control Theory Appl.*, 1, 2007, pp. 854–859.

[10] H. K. Khalil, Nonlinear Systems, Prentice–Hall, Upper Saddle River, NJ, 1996.

[11] O. M. Kwon, J. H. Park, Exponential stability for time–delay systems with interval time–varying delays and nonlinear perturbations, *J. Optim. Theory Appl.*, 13, 2008, pp. 277–293.

[12] O. M. Kwon, J. H. Park and S. M. Lee, On robust stability criterion for dynamic systems with time–varying delays and nonlinear perturbations, *Appl. Math. Comput.*, 203, 2008, pp. 937–942.

[13] X. X. Liao, *Absolute Stability of Nonlinear Control Systems*, Science Press, Beijing, 1993.

[14] T. Li, W. Qian, T. Wang and S. Fei, Further results on delay–dependent absolute and robust stability for time–delay Lur'e system, *Internat. J. Robust Nonlinear Control*, 24, 2014, pp. 3300-3316.

[15] X. Liu, J. Z. Wang and Z. S. Duan, New absolute stability criteria for time–delay Lure systems with sector–bounded nonlinearity, *Internat. J. Robust Nonlinear Control*, 20, 2010, pp. 659–672.

[16] R. Lun, H. Wu and J. Bai, New delay–dependent robust stability criteria for uncertain neutral systems with mixed delays, *J. Franklin Inst.*, 351, 2014, pp. 1386–1399.

[17] J. H. Park, Novel robust stability criterion for a class of neutral systems with mixed delays and nonlinear perturbations, *Appl. Math. Comput.*, 161, 2005, pp. 413–421, 2005.

[18] V. M. Popov, *Hyperstability of Control Systems*, Springer, New York, 1973.

[19] F. Qiu, B. Cui and Y.Ji, Further results on robust stability of neutral system with mixed time–varying delays and nonlinear perturbations, *Nonlinear Anal.*, 11, 2010, pp. 895–906.

[20] K. Ramakrishnan, G. Ray, An improved delay–dependent stability criterion for a class of Lure systems of neutral type, *J. Dyn. Syst.*, 134, 2012, pp. 011008.

[21] A. Seuret, F. Gouaisbaut, Wirtinger–based integral inequality: Application to time–delay systems," *Automat.*, 49, 2013, pp. 2860–2866.

[22] Y. T. Wang, X. Zhang and Y. He. Chen, Improved delay–dependent robust stability criteria for a class of uncertain mixed neutral and Lure dynamical systems with interval time–varying delays and sector–bounded nonlinearity, *J. Franklin Inst.*, 347, 2010, pp. 1623–1642.

[23] W. Weera, P. Niamsup, Robust Stability of a Class of Uncertain Lur'e Systems of Neutral Type, *Abstr. Appl. Anal.*, 2012, 2012, pp. 1–18.

[24] K. Y. Yu, C. H. Lien, Stability criterion for uncertain neutral systems with interval time–varying delays, *Chaos Solitons Fractals*, 38, 2008, pp. 650–657.

[25] C. Yin, S. M. Zhong and W. F. Chen, On delay–dependent robust stability of a class of uncertain mixed neutral and Lure dynamical systems with interval time–varying delays, *Nonlinear Anal: Real World Appli*, 13, 2012, pp. 2188–2194.

[26] H. Zhang, D. Gong, B. Chen and Z. Liu, Global Synchronization in an Array of Delayed Neural Networks With Hybrid Coupling, *IEEE trans systems.*, 38, 2008, pp. 488–498.

[27] X. M. Zhang, Q. L. Han, New atability criterion using a matrix–based quadratic convex approach and some novel integral inequalities," *IET Control Theory Appl.*, 8, 2014, pp. 1054–1061.