The Dangers of Precipitous Adjustment Speed of Recovery Price to the Closed-loop Supply Chain and Improvement Measure

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Abstract: This paper analyzes the recycling price game in waste household appliance market in China. Based on the analysis of recycling behavior game of two retailers and a manufacturer in closed-loop supply chain, we propose that precipitous adjustment speed of recovery price will lead to the system go into a chaotic state. In this state, the recycling price of each recovery party will enter the chaotic state, and it will be significantly affected by the initial price, the profit of each corporate will be apparent fluctuations, and thus the chaotic state caused by the precipitous speed of price adjustment will make competition be invalid and vicious. In this regard, this paper introduces delay decision-making to control the chaotic state and gives the Economics of chaos control.

Key–Words: supply chain, recycling household appliance, Nash equilibrium, chaos, delay decision

1 Introduction

Reuse of resources, energy conservation, are important measures of the sustainable development strategy. The sustainable development strategy has become a hot topic. Today, the traditional supply chain adds the reverse logistics process, to become a closed-loop supply chain. The reverse logistics process is a process for product recycling. There are many things can be recycled by reverse logistics, such as used batteries, used bottles, scrap metal and used household appliances. Now in China, there has been the company recycling waste household appliances, such as TCL AOBO. Consumers can sell the used household appliances recycling companies to get some compensation. The recycling companies recycle the waste household from the market while recycle it from retailers. Appliance recycling vendors can use reusable parts for reproduction, in order to reduce production costs. They can also sell the useful parts to scrap material recycling market to get some benefits. Retailers play a role as recyclers in the recovery process, they earn the difference of the price between manufacturer and themselves. This paper will study the game complexity of appliance recycling market gradually developed in China.

Some people have done some research in the area of appliance recycling. Adam D. Read [1] found the way to improve people's awareness of recycling by studying door-to-door household waste recycling in Chelsea and other UK regions. Frans Melissen [2] analysed the Dutch consumer several waste recycling behaviour and appliance recycling pilot program, and draw some suitable measures to improve the recovery of waste household appliances. Carsten Nagel [3] advocated the use of reverse logistics to recycle IT products. Carsten Nagel and Peter Meyer[4] established an EOL network model to analyze an electronics recycling company collection network in Germany. H. R. Krikke et al. [5] proposed a stochastic dynamic programming model for the largest net gain of product recovery. Zsolt Istvan et al. [6] studied reverse logistics management system at the end electronic products. Anna Nagurney and Fuminori Toyasaki [7] used genetic algorithm to analyze production and price of logistics. Jae-chun Lee et al. [8] described the status of the recovery of waste electrical and electronic in Korea.

The enterprises which participate in the process of reverse logistics, not only can contribute to environmental protection, but also can benefit from the recycling process. Therefore, there are more and more studies about recycling channels and recycling strategy under different conditions, to get an optimal recovery strategies and reasonable contract. Fleischmann [9] summarized reverse logistics activities from the reverse distribution, inventory control and product planning. Fleischmann [10] also investigated the product recovery reverse logistics network design of different sectors, summed up the composition of typical reverse logistics network. Shih [11] studied reverse logistics network model of Taiwan discarded appliances and computer, in a variety of recovery rates and recovery produce conditions, determine the optimal system by the mixed integer linear programming (MILP), to determine the number and position of members involved in recycling network , and the service area of each facility. Sheu [12] considered the recovery rate of waste products and the corresponding availability of subsidies from government organizations, constructed multi-objective programming model. Sun Zhihui et al. [13] analyzed price game in Chinese cold rolled steel market and showed that how relevant parameters cause chaos to occur and how to change the relevant parameters to control the chaotic state. Guo Yuehong [14] et al. established a collecting price game model for a closed-loop supply chain system with a manufacturer and a retailer, analyzed complex dynamic phenomena and the influences of the system parameters. Ma, J et al.[15] considered a three-species symbiosis Lotka - Volterra model with discrete delays and analyzed the influence of delays to local stability.

The structure of this paper is: The second part describes the assumptions of this study, and a number of symbolic variables explanation, as well as the structure of the model; the third part introduces the analysis methods of the model; the fourth part describes the occurrence of the chaos state and the adverse effects of the chaotic state by numerical simulation; the fifth part describes the method to control chaos; and the sixed part summarized.

2 Model Description

2.1 Model Framework

There are three main game parties in the appliance recycling market in the paper: appliance manufacturers, appliance retailers Gome and Suning. Retailers act as recyclers in the reverse logistics process of a closed loop supply chain. Gome and Suning are the two largest home appliance retailers in China. Recovery amount they can obtain in the recycling process are so great that other small retailers cannot match. Therefore, the recovery process can be approximated as a kind of oligopoly game. Manufacturers and retailers recover the waste products in the market as competitors. Manufacturer recovers waste products that recovered by the retailers and itself. The closed-loop supply chain's structure is shown in Fig.1 (solid line represents the forward supply chain, while dotted line represents reverse supply chain).

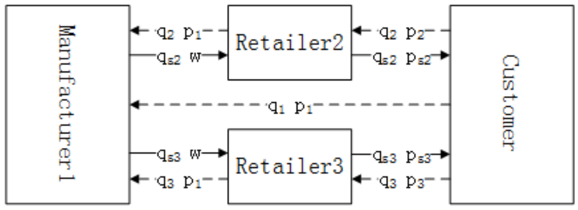

Figure 1: MR model closed-loop supply chain

2.2 Symbols and assumptions

First, give a description of symbols related in this paper:

i=1,2,3 represent manufacturer 1, retailer 2 and retailers 3, respectively.

q_i: recovery amount obtained by recycling party i in the market;

p_i: recycling price of recycling party i;

π_i: profit of party i;

γ, α and β: ratio that consumer willing to give the waste products back to manufacturer 1, retailer 2 and retailer 3, this variable is influenced by corporate image of recycling party;

b_i: price impact factor of the recycling amount, that is, the price elasticity of recycling;

c_i and d_i: competitive factor, that is, the degree of influence by other parties recycling prices;

ρ: recovery rate;

C_m: the cost of a new product;

C_r: remanufacturing costs of a recycling product;

C_b: disposal costs of a waste product;

q_{s2} and q_{s3}: sales volume of retailer 2 and retailer 3;

Q_s: sales volume of manufacturer, Q_s=q_{s2}+q_{s3};

w: wholesale price of manufacturer 1;

p_{s2} and p_{s3}: sale price of retailer 2 and retailer 3.

Based on the recycling amount model Guo Yuehong [14] proposed, the recycling amount model in this paper can be written as model (1). The hypothetical scenario of this paper is that, the manufacturer and the two retailers recover the waste, manufacturer recovers from retailers at a market recycling price. There is a price game between manufacturer and retailers in the recycling market. Recycling amount is influence by recycling price and corporate image, so the recycling amount consists of three parts: the amount that the consumer willing to give back, the amount influenced by its own price, the amount influenced by the other parties prices. In addition, the consumers have the same preference for the old products and the new products.

$$\begin{cases} q_1 = \gamma\left(q_{\mathrm{s}2} + q_{\mathrm{s}3}\right) + b_1 p_1 - c_1 p_2 - d_1 p_3 \\ q_2 = \alpha\left(q_{\mathrm{s}2} + q_{\mathrm{s}3}\right) + b_2 p_2 - c_2 p_3 - d_2 p_1 \\ q_3 = \beta\left(q_{\mathrm{s}2} + q_{\mathrm{s}3}\right) + b_3 p_3 - c_3 p_1 - d_3 p_2 \end{cases} \quad (1)$$

So the total amount on the chain is the sum of three sides recycling amount, that is $Q = q_1 + q_2 + q_3$.

The profits of three vendors are shown in function (2). Manufacturer's profit is expressed as selling new products and the remanufacturing product, minus the cost of production of new manufactured products, minus available remanufacturing products processing costs, minus the payment cost of recovery process, minus the treatment cost of the recycled product which can not be reused.

Retailer's profit consists of two parts, sales profits and the price difference earned in the recycling process.

$$\begin{cases} \pi_1 = wQ_{\mathrm{s}} - C_{\mathrm{m}}\left(Q_{\mathrm{s}} - \rho Q\right) - C_{\mathrm{r}}\rho Q - p_1 Q \\ -C_{\mathrm{b}}\left(1-\rho\right)Q \\ \pi_2 = \left(p_{\mathrm{s}2} - w\right)q_{\mathrm{s}2} + \left(p_1 - p_2\right)q_2 \\ \pi_3 = \left(p_{\mathrm{s}3} - w\right)q_{\mathrm{s}3} + (p_1 - p_3)q_3 \end{cases} \quad (2)$$

This paper studies the impact of recycling prices on the corporate profits, therefore, it is necessary to consider the marginal profitability of recycling price, that is, how many changes per unit change of recycling price can impact on the corporate profits. It is shown by function (3)

$$\begin{cases} \frac{\partial\pi_1}{\partial p_1} = -\mathrm{C_b}\left(b_1 - c_3 - d_2\right)\left(1-\rho\right) \\ -\rho(\left(2p_1 - D\right)\left(b_1 - c_3 - d_2\right) + \left(b_2 - c_1 - d_3\right)p_2 \\ + \left(b_3 - c_2 - d_1\right)\mathrm{p}_3 + \left(\alpha+\beta+\gamma\right)Q_{\mathrm{s}}) \\ \frac{\partial\pi_2}{\partial p_2} = -\alpha Q_{\mathrm{s}} + p_1\left(d_2 + b_2\right) - 2p_2 b_2 + c_3 p_3 \\ \frac{\partial\pi_3}{\partial p_3} = -\beta Q_{\mathrm{s}} + p_1\left(c_3 + b_3\right) - 2p_3 b_3 + d_3 p_3 \end{cases} \quad (3)$$

Model (4) is a cournot model, represents a price adjustment model.

$$\begin{cases} p_1\left(t+1\right) = p_1\left(t\right) + g_1 p_1(t)\frac{\partial\pi_1(t)}{\partial p_1(t)} \\ p_2\left(t+1\right) = p_2\left(t\right) + g_2 p_2(t)\frac{\partial\pi_2(t)}{\partial p_2(t)} \\ p_3\left(t+1\right) = p_3\left(t\right) + g_3 p_3(t)\frac{\partial\pi_3(t)}{\partial p_3(t)} \end{cases} \quad (4)$$

Three parties will determine the next phase of the recycling price based on their current marginal profits. When the marginal profit is positive, the higher price will bring an increase in profits, so the corporate will raise price. On the other hand, negative marginal profit may lead to the fact that the lower price will bring an increase in profit, so the corporate will reduce price. We introduce the concept of price adjustment factors: g_1, g_2 and g_3. Price adjustment factor indicates the pace of price adjustment.

3 Model analysis

To further analyze the nature of the price adjustment model to understand the game process of three parties recycling prices, we should find the equilibrium point of the system and analyze the nature of the equilibrium point.

3.1 Solving the equilibrium point

Under the condition of price stability, the three vendors' prices will never change, so system have achieved stable. In this state, the price will never change as time goes on. So we can use the following equation to express this state: $p_{\mathrm{i}}\left(t+1\right) = p_{\mathrm{i}}\left(t\right)$.

The three parties profits for the second derivative of the price function should be less than zero, in order to ensure the highest profits are exist.

Solving equations (4), we can get eight group solutions

$$\begin{aligned} &E_1(0,0,0), E_2(0,0,-\frac{\beta Q_s}{2b_3}), E_3(0,-\frac{\alpha Q_s}{2b_2},0), \\ &E_4(-\frac{(C_b(1-\rho)-d\rho)m_1 + \eta Q_s\rho}{2m_1\rho},0,0), \\ &E_5(0,-\frac{(2\alpha b_3+\beta c_2)Q_s}{4b_2b_3-c_2d_3},-\frac{(2\beta b_2+\alpha d_3)Q_s}{4b_2b_3-c_2d_3}), \\ &E_6(e_{61},0,e_{63}), E_7(e_{71},e_{72},0), E_8(e_{81},e_{82},e_{83}). \end{aligned}$$

where

$$\begin{aligned} &e_{61} = [\beta m_3 Q_s + 2b_3(-C_b m_1 - Q_s\eta + m_1\rho C_v)]/ \\ &[(b_3+c_3)m_3 + 4b_3m_1] \\ &e_{63} = -[(b_3+c_3)(-\beta m_3Q_s - 2b_3(-C_bm_1 \\ &-Q_s\eta m_1\rho C_v))]/[2b_3((b_3+c_3)m_3 + 4b_3m_1)] \\ &-\beta Q_s/2b_3 \\ &e_{71} = [\alpha m2Q_s + 2b2(-C_bm_1 - Q_s\eta + m_1\rho C_v)]/ \\ &[4b_2m_1 + (b_2+d_2)m_2] \\ &e_{72} = -[(b_2+d_2)(-\alpha m_2Q_s - 2b_2(-C_bm_1 \\ &-Q_s\eta m_1\rho C_v))]/[2b2(4b_2m_1 + (b_2+d_2)m_2)] \\ &-[\alpha Q_s]/[2b_2] \\ &e_{81} = [(2b_2m_3 + c_2m_2)(2\alpha b_3 + \beta c_2)Q_s - (4b_2b_3 \\ &-c_2d_3)(\alpha m_3Q_s + c_2(m_1C_b + Q_s(\alpha+\beta+\gamma) \\ &-m_1\rho(C_b + C_m - C_r)))]/[(m_3(b_2+d_2) \\ &-2c_2m_1)(-4b_2b_3 + c_2d_3) + (c_2(b_3+c_3) \\ &+2b_3(b_2+d_2))(2b_2m_3 + c_2m_2)] \\ &e_{82} = [(2\alpha b_3 + \beta c_2)Q_s]/[-4b_2b_3 + c_2d_3] \\ &+[c_2(b_3+c_3) + 2b_3(b_2+d_2))((2b_2m_3 \\ &+c_2m_2)(-2\alpha b_3 - \beta c_2)Q_s + (4b_2b_3 - c_2d_3) \\ &(\alpha m_3Q_s + c_2(m_1C_b + Q_s(\alpha+\beta+\gamma) \\ &-m_1\rho(C_b + C_m - C_r)))]/[(-4b_2b_3 + c_2d_3) \end{aligned}$$

$((m_3(b_2+d_2)-2c_2m_1)(-4b_2b_3+c_2d_3)$
$+(c_2(b_3+c_3)+2b_3(b_2+d_2))(2b_2m_3+c_2m_2))]$
$e_{83}=[(2b_2\beta+\alpha d_3)Q_s]/[-4b_2b_3+c_2d_3]$
$+[2b_2(b_3+c_3)+d_3(b_2+d_2))(2b_2m_3$
$-c_2m_2(2\alpha b_3+\beta c_2)Q_s+(4b_2b_3$
$-c_2d_3)(-\alpha m_3Q_s+c_2(-m_1C_b-Q_s(\alpha+\beta+\gamma)$
$+m_1\rho(C_b+C_m-C_r)))]/[(-4b_2b_3$
$+c_2d_3)((m_3(b_2+d_2)-2c_2m_1)(-4b_2b_3+c_2d_3)$
$-(c_2(b_3+c_3)-2b_3(b_2+d_2))(2b_2m_3+c_2m_2)))]$

To demonstrate this 8 groups of solution more clearly, we make

$$\begin{cases} m_1=b_1-c_3-d_2\\ m_2=b_2-c_1-d_3\\ m_3=b_3-c_2-d_1\end{cases} \tag{5}$$

The above 8 solutions are the 8 equilibrium points of the system.

3.2 Stability Analysis

Nash equilibrium provides neither side will be able to take the initiative to change in order to increase revenue. When the profit margin is zero, increasing or decreasing the recovery price will result in loss of profits. Therefore, when each of the three vendors profit margin for the price is zero, the Nash equilibrium point is obtained. E_1 to E_7 are the fixed points which cant guarantee the three parties profit margin is zero, only E_8 is obtained when three parties Margin is zero. Therefore, only E_8 is the Nash equilibrium point.

Function (6) is the Jacobian matrix of the model (4), it is shown as follows:

$$J=\begin{vmatrix} j_1,j_2,j_3\\ j_4,j_5,j_6\\ j_7,j_8,j_9\end{vmatrix}=\begin{vmatrix} j_1,-m_2g_1p_1,-m_3g_1p_1\\ (b_2+d_2)\,g_2p_2,j_5,c_2g_2p_2\\ (b_3+d_3)g_3p_3,d_3g_3p_3,j_9\end{vmatrix} \tag{6}$$

where

$$\begin{cases} j_1=1-g_1((4p_1+C_b-(C_m-C_r+C_b)\,\rho)\\ (b_1-c_3-d_2)\,(b_2-c_1-d_3)\,p_2+\\ (b_3-c_2-d_1)\,p_3+(\alpha+\beta+\gamma)\,Q_s)\\ j_5=1-2b_2g_2p_2+g_2((b_2+d_2)\,p_1\\ -2b_2p_2+c_2p_3-\alpha Q_s)\\ j_9=1-2b_3g_3p_3+g_3((b_3+d_3)\,p_1\\ -2b_3p_3+d_3p_2-\beta Q_s)\end{cases} \tag{7}$$

Function (8) is the corresponding characteristic equation of function (6), is shown as follows

$$F(\lambda)=|\lambda E-J|=A+A_1\lambda+A_2\lambda^2+A_3\lambda^3 \tag{8}$$

where

$$\begin{cases} A_0=j_1j_6j_8+j_2j_4j_9+j_3j_5j_7-j_1j_5j_9\\ -j_3j_4j_8-j_2j_6j_7\\ A_1=j_1j_5+j_1j_9+j_5j_9-j_3j_7-j_2j_4-j_6j_8\\ A_2=-j_1-j_5-j_9\\ A_3=1\end{cases} \tag{9}$$

After getting a unique Nash equilibrium point E_8, it needs to determine local stability. Routh-Hurwitz stability criterion proposes that the necessary and sufficient condition for system fixed points asymptotically stable is all zero points of its characteristic polynomial are inside the unit circle in the complex plane, it should meet the following condition (10)

$$\begin{cases} F(1)=A_0+A_1+A_2+A_3>0\\ F(-1)=A_0-A_1+A_2-A_3<0\\ A_0^2-A_3<0\\ (A_3-A_0^2)^2-(A_1-A_2A_0)^2>0\end{cases} \tag{10}$$

Based on the condition (10), we can work out the stable region composed by g_1, g_2 and g_3. And then we can know when the system is stable and when it will be chaos. Through numerical simulations, it can be more visually to observe system status.

4 Numerical Simulation

In order to visualize the dynamic performance of the process, we do the numerical simulation of the model in this section. Recovery rate is impossible to achieve one hundred percent in the market. In the recovery process, the amount of recovery and its own recovery price are positive correlation, while the amount of recovery and competitors recovery price is negative correlation. In addition, ensure the price is greater than the wholesale price, cost saved by recovery process is greater than recycling prices. According to the content of the above analysis, we can make the following settings: $\alpha=0.12$, $\beta=0.06$, $\gamma=0.42$, $\rho=0.9$, $b_1=6$, $b_2=4$, $b_3=5$, $c_1=2$, $c_2=1$, $c_3=2$, $d_1=1$, $d_2=2$, $d_3=2$, $w=13$, $C_m=11$, $C_r=1$, $C_b=1$, $p_{s2}=16$, $p_{s3}=15$, $q_{s2}=8$, $q_{s3}=12$.

Through the above model, we can calculate the equilibrium price is $p_1=0.96, p_2=0.50, p_3=0.65$. Based on the condition (10) and the above numerical simulation, we can work out the stable region, it is shown as Fig.2.

When combination of g_1, g_2, g_3 is in the stable region, the system is in a steady state. It can also be observed through the evolution of the system as g_i changes. Assuming g_1 constant, when g_1 and g_2 simultaneously changes, the evolution of the system

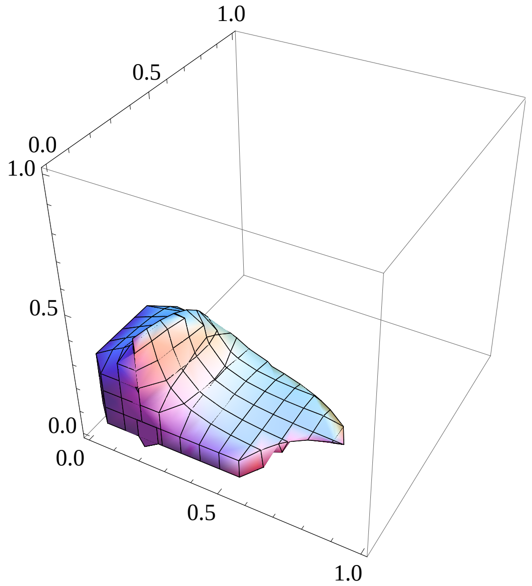

Figure 2: The stable region of the system

is shown as Fig.3. As can be seen, when g_i is larger and larger, the system will enter into chaos state from stable state. In order to analyze the impact of g_3 to the system expediently, we observe the impact of g_3 to the system by fixing g_1 and g_2. When $g_1 = 0.1$, $g_2 = 0.1$, we can observe how price adjustment factor g_3 of retailer 3 changes the three parties profits and prices. According to the Routh-Hurwitz criterion, we can precisely calculate stability domain of g_3 is $0 < g_3 < 0.340$.

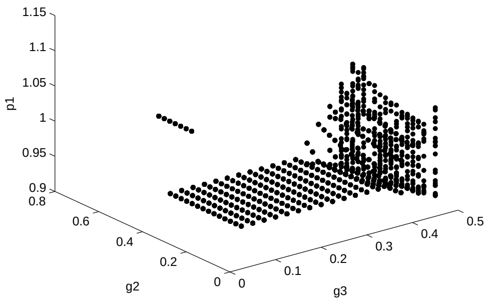

Figure 3: Dynamic Evolution of P1 as g_2, g_2 changes with g_1=0.1

4.1 Lyapunov exponent and the dynamic evolution of the system

In order to demonstrate the influence of retailer 3 price adjustment speed to the market, under the condition of $g_1 = 0.1$, $g_2 = 0.1$, we can make the largest Lyapunov exponent and profits bifurcation diagram with the change of g_3. The results are shown in Fig.4 and Fig.5.

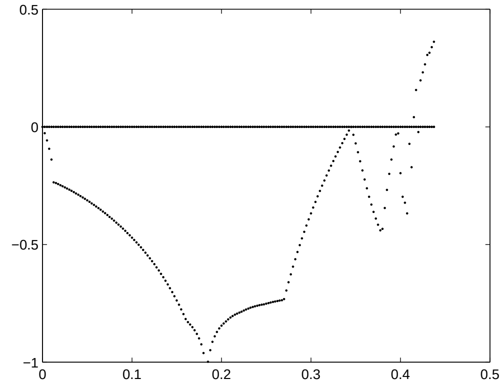

Figure 4: The largest Lyapunov exponent

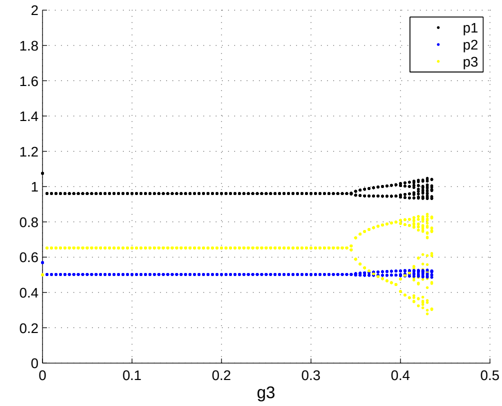

Figure 5: Dynamic Evolution of P1 as g_3 changes with g_1=g_3=0.1

Lyapunov Exponent represents the rate of index of the average convergence or divergence in similar orbital in phase space. If there is at least one Lyapunov exponent which is greater than zero in system, then the system is chaotic, and the larger the Lyapunov exponent, the stronger the chaotic state. A positive Lyapunov exponent indicates in the system phase space, regardless of however small the initial two-rail line spacing is, the distance will increases in the form of exponential rate with the increasing of the number of iterations, and ultimately achieve an unpredictable state, that is, chaos state. From largest Lyapunov exponent, it can be seen that, in the interval $0 < g_3 < 0.340$, the maximum Lyapunov exponent is less than zero, according to the definition above, its corresponding price is in a steady state, in the interval $0.34 < g_3 < 0.40$, the corresponding system is period-doubling bifurcation state, when $g_3 = 0.40$, the system reaches a second bifurcation, and so on. Until the largest Lyapunov exponent of system is greater than 0 for the first time, from this point on, the system goes into chaos.

From system equilibrium dynamic evolution chart of the recovery prices, it also easy to see that when $0 < g_3 < 0.340$, the three parties equilibrium price is stable. But when $g_3 > 0.34$, the recovery price is beginning to go into the state of period-doubling bifurcation, until to the chaos. Meanwhile, using Matlab software, we can easily calculate the system two cycle bifurcation points are: (1.01,0.52,0.80), (0.95,0.50,0.44).

Similar as analysis of the equilibrium price, it can also be observed that the influence of retailer 3 price adjustment coefficient to three parties profits when $g_1 = 0.1$ and $g_2 = 0.1$. The results are as shown in Fig.6 and Fig.7.

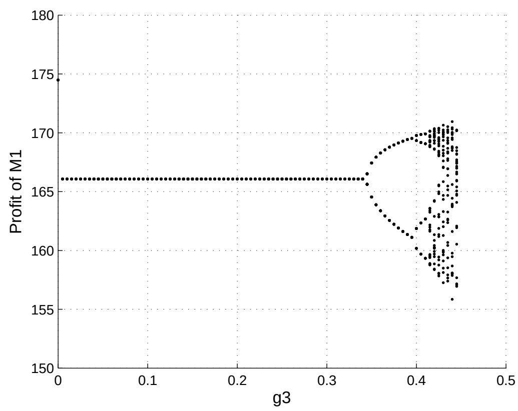

Figure 6: Dynamic Evolution of profit of manufacturer as g_3 changes with $g_1 = g_2 = 0.1$

It is obvious that, the bifurcation of the profit begins to appear when $g_3 = 0.34$, the system goes into a chaotic state. At this point, the market appears invalid vicious competition state.

4.2 Changes of recycling price

In order to understand the recycle price changes over time in the stability and chaos state, the following simulates the changes in price. According to the above

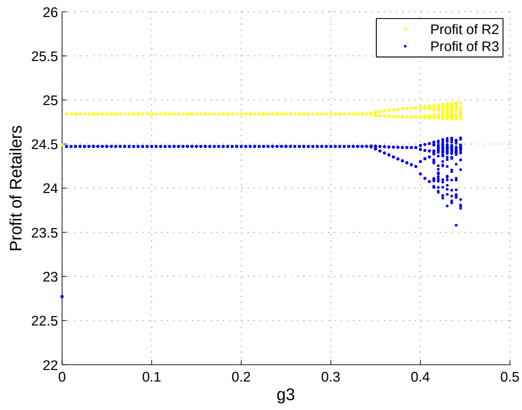

Figure 7: Dynamic Evolution of profit of retailers as g_3 changes with g_1=g_2=0.1

analysis, it is easy to know the price adjustment factor $g_1 = 0.1$, $g_2 = 0.1$, $g_3 = 0.3$ may represent a stable state. At this point, the price of three vendors change as shown in Fig.8.

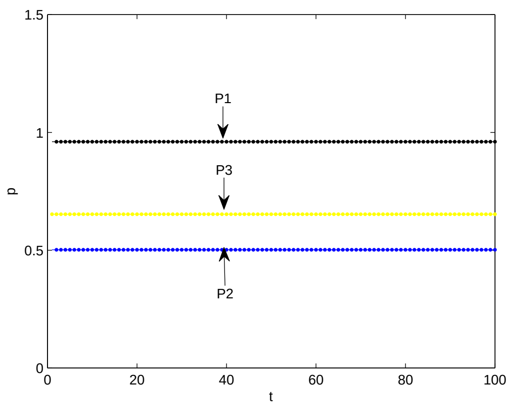

Figure 8: Three vendors price changes when the system is in stable state

Similarly, when the system is in a chaotic state with $g_1 = 0.1$, $g_2 = 0.1$, $g_3 = 0.43$, the time series of three parties equilibrium price are as shown in Fig.9. It is obvious that when the system is in steady state, the three parties price will go into the stable state after a brief recovery fluctuations; when the system is in a chaotic state, the three parties price fluctuates all the time, can not go into a stable state.

When the game runs after a certain period in a stable system, it will get a set of stable equilibrium. When a sudden change occurs in one of the decision-making for some reason, it will break this equilibrium state. But a stable system can make such mutation of

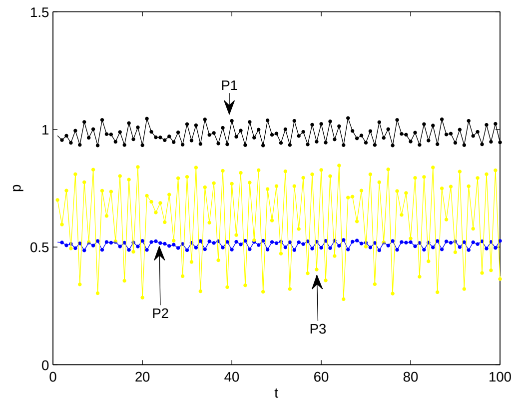

Figure 9: Three vendors price changes when the system is in chaos state

decision variables gradually stabilized, so that the system can go back to its original equilibrium state. As shown in Fig.10, when the game goes to the 20 cycles and 60 cycles, changes in party decision variables will make the system enter into a short unstable state, but when the game runs enough times, the system will be stable again.

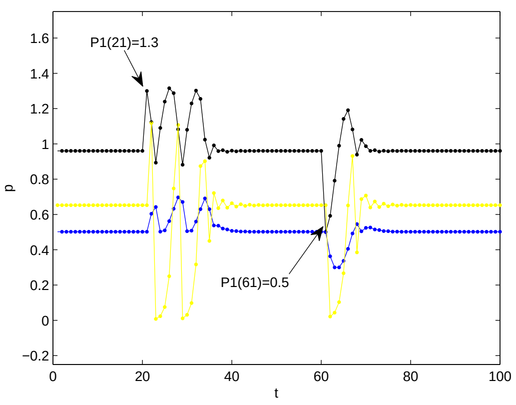

Figure 10: The influence of sudden changes in price on the stability of system

4.3 Chaotic attractor and initial sensitivity

$g_1 = 0.1$, $g_2 = 0.1$, $g_3 = 0.43$ means a chaotic state. In this chaotic state and initial recovery price of p_1=0.5, p_2=0.5, p_3=0.5, the system's chaotic attractor is shown in Fig.11.

A distinctive feature of chaotic attractor is that the exponential segregation of attractor neighboring points, which indicates that chaotic systems have sensitive dependence on the initial conditions.

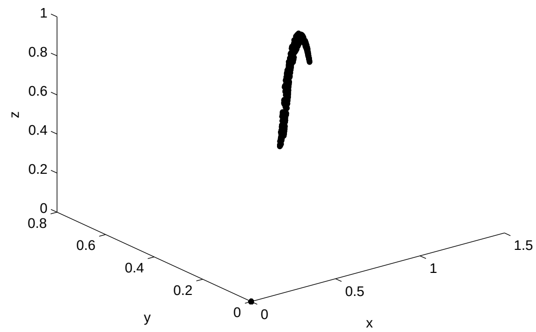

Figure 11: The chaos attractor in a chaotic state with g_1=0.1, g_2=0.1, g_3=0.43

The system in chaotic state has a certain sensitivity to initial values, just as the butterfly effect, small changes in initial may bring enormous changes in adjacent tracks. In the chaotic state g_1=0.1, $g_2 = 0.1$, $g_3 = 0.43$, the small changes in initial, from $p_1 = 0.5$, $p_2 = 0.5$, $p_3 = 0.5$ to $p_1 = 0.5$, $p_2 = 0.5$, $p_3 = 0.5001$, observe three parties recycling prices sensitivity to initial. Fig.12 and Fig.13 are analysis on three recycling prices sensitivity to initial.

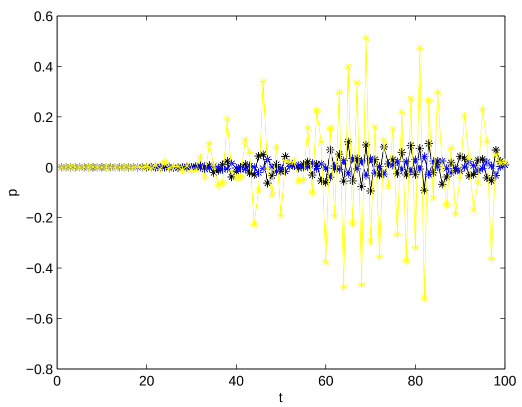

Figure 12: The initial value sensitivity of three parties recycling prices with g_1=0.1, g_2=0.1, g_3=0.43

It is obvious in Fig.12 that, in the case that the original recovery price changes small, the difference of three parties recycling prices is not obvious between the two original prices conditions until $t = 20$. But as time goes on, the difference gradually emerged, and the maximum is more than 0.5. This difference appears in retailer 3 whose price changes too fast.

On the other hand, the stable system is not af-

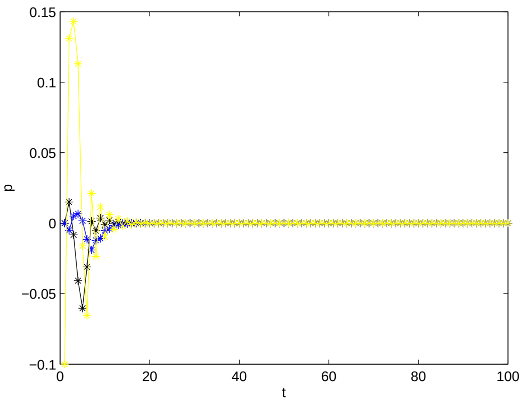

Figure 13: The initial value sensitivity of three parties recycling prices with g_1=0.1, g_2=0.1, g_3=0.3

fected by changes in the initial value. Fig.13 shows that the price difference when the initial value changes from $p_1 = 0.5$, $p_2 = 0.5$, $p_3 = 0.5$ to $p_1 = 0.5$, $p_2 = 0.5$, $p_3 = 0.6$.

It is not difficult to see the stable system can play a role in stabilizing prices. But when the market is in a chaotic state, a small recycling price changes may have a huge impact on the others' recovery price. What is worse, the current price will have a huge impact on the next period recovery prices. In this case, the manufacturer is difficult to make accurate judgments on the market, and difficult to draw the best recycle price.

In summary, it is necessary to control the unstable state.

5 Chaos control

In a recovery market, the price equilibrium is short-term, temporary. The factors such as price adjustment speed, will broke equilibrium into the chaotic state. Once the market goes into the chaotic state, it will be difficult for companies to adjust price to a suitable state, to obtain a higher stable income. Therefore, a method of controlling chaos is needed. There are many ways of chaos control, such as the OGY method, continuous feedback control method, adaptive control method, etc. In order to reflect the economic characteristics of the model, we use delayed feedback control method, which belong to the continuous feedback control method.

The core idea of the delay control method is to use part of the information of the output signal feedback to the system after a time delay, as an alternative to an external input, and ultimately achieve stabilization of unstable periodic orbits of chaotic attractor. Response signal functions is the main difference between the delay control method and force feedback control method. Response signal function of delayed feedback control method is:

$$f(t) = k\left[y(t-T) - y(t)\right] = kD(t) \tag{11}$$

Function (11) is response signal function. Where, $f(t)$ represents the control signal, k represents chaos control factor, T is the delay time, in order to facilitate the analysis of the economic characteristics of the model, delay time the paper selected is 1. Thus the control signal may be expressed by function (12):

$$f(t) = k\left[p_i(t) - p_i(t+1)\right] \tag{12}$$

As this study is about the influence of retailer 3 price adjustment factor to the chaotic phenomena, Therefore, we should exert control to retailer 3. So system joined the chaos control signal can be expressed as

$$\begin{cases} p_1(t+1) = p_1(t) + g_1 p_1(t)\frac{\partial \pi_1(t)}{\partial p_1(t)} \\ p_2(t+1) = p_2(t) + g_2 p_2(t)\frac{\partial \pi_2(t)}{\partial p_2(t)} \\ p_3(t+1) = p_3(t) + g_3 p_3(t)\frac{\partial \pi_3(t)}{\partial p_3(t)} + k[p_3(t) \\ -p_3(t+1)] \end{cases} \tag{13}$$

After adjustment, the model (13) can be written as

$$\begin{cases} p_1(t+1) = p_1(t) + g_1 p_1(t)\frac{\partial \pi_1(t)}{\partial p_1(t)} \\ p_2(t+1) = p_2(t) + g_2 p_2(t)\frac{\partial \pi_2(t)}{\partial p_2(t)} \\ p_3(t+1) = p_3(t) + \frac{g_3}{1+k} p_3(t)\frac{\partial \pi_3(t)}{\partial p_3(t)} \end{cases} \tag{14}$$

Also in chaotic state of g_1=0.1, g_2=0.1, g_3=0.43, we consider the influence of chaos control factor k to the system equilibrium prices.

It is not difficult to see from the Fig.14, with the chaos control factor k increases, the system gradually goes into the period-doubling bifurcation state from a chaotic state, when k=0.26, the system begins to enter the steady state. That is, when chaos control factor k is greater than 0.26, the chaotic state is controlled. Similarly, we can change the state of chaos to observe the changes of k.

It is obvious that different chaotic state needs different k to be controlled, with the retailer 3's price adjustment speed increases, the chaotic state needs a greater k to be controlled.

Similarity, we can also get the changes of 3 parties profits.

On condition that Price adjustment speed is g_1=0.1, g_2=0.1, g_3=0.43 and Initial Price is p_1=0.5,

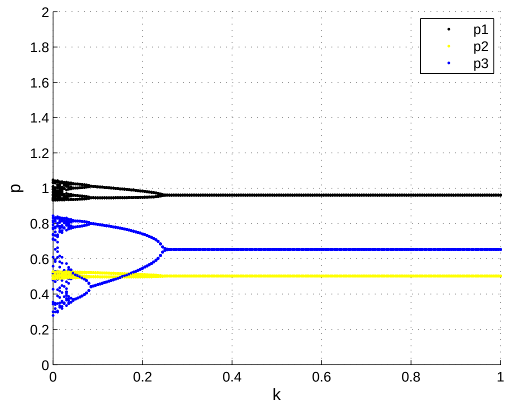

Figure 14: Control of Dynamic of Evolution of the system by Retailer 3

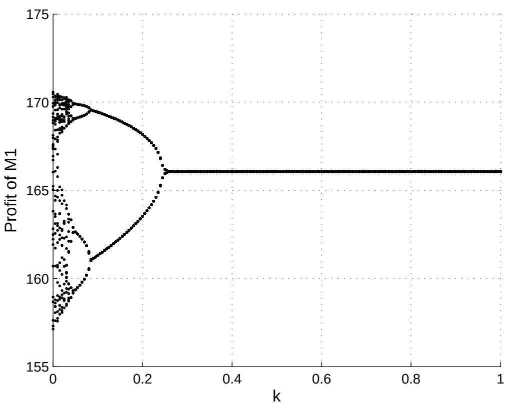

Figure 15: Control of Dynamic of Evolution of the profit of manufacturer

p_2=0.5, p_3=0.5. Three parties price adjustment is shown in Fig.9. The price is up and down and unable to reach a stable state. Remain other conditions unchanged and introduce chaos control factor k=0.5, the adjustment of three parties prices is shown in Fig.17. It is clear that after the introduction of chaos control factor, three parties prices gradually go into a stable state after a period of time (less than 10 cycles).

The size of the price adjustment factor is relative, three factors come together to form the system stability region. Thus in essence, chaotic phenomena is caused by the three parties. So considering three parties adding chaos control, the model can evolve from

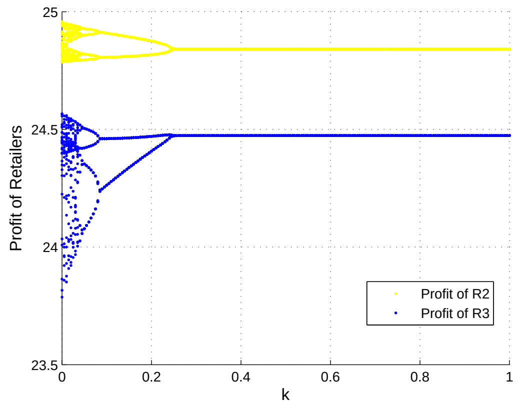

Figure 16: Control of Dynamic of Evolution of the profit of retailers

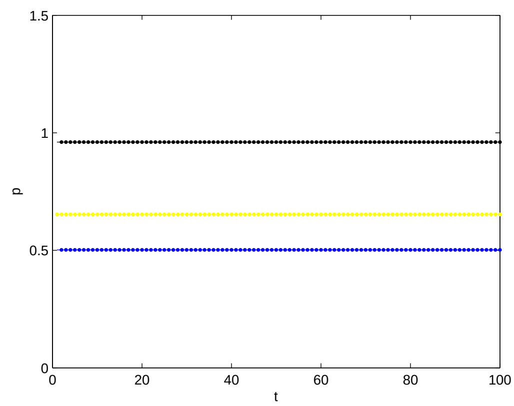

Figure 17: Control of price time array in a chaotic state

model (13) to model (15), as follows:

$$\begin{cases} p_1(t+1) = p_1(t) + g_1 p_1(t) \frac{\partial \pi_1(t)}{\partial p_1(t)} + k_1[p_1(t) \\ -p_1(t+1)] \\ p_2(t+1) = p_2(t) + g_2 p_2(t) \frac{\partial \pi_2(t)}{\partial p_2(t)} + k_2[p_2(t) \\ -p_2(t+1)] \\ p_3(t+1) = p_3(t) + g_3 p_3(t) \frac{\partial \pi_3(t)}{\partial p_3(t)} + k_3[p_3(t) \\ -p_3(t+1)] \end{cases} \tag{15}$$

When $k_1 = k_2 = k_3 = k$, it can get the control of chaotic systems by control factor k, as shown in Fig.18.

Fig.19 shows that the different amount of the vendors involved in the control can get a different control effect. It is not that more vendors use control method can get a better control effect. The specific case needs a specific analysis.

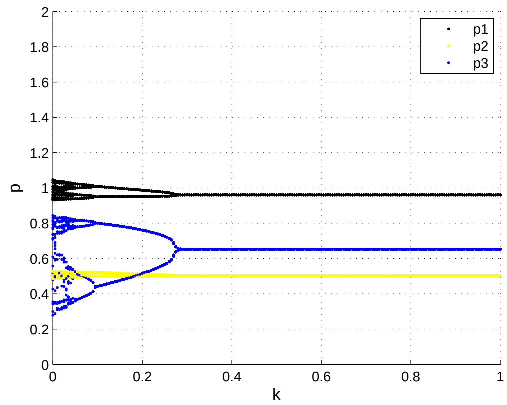

Figure 18: Control of Dynamic of Evolution of the system by three vendors

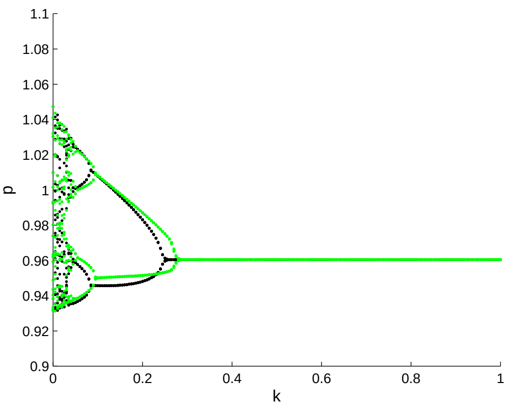

Figure 19: The different control effect of the amount of vendors use control method

Based on function (11), we can change the cournot model (4) to function model (14). By replacing $\frac{\partial \pi_3}{\partial p_3}$, we can get the function (16):

$$p_3\left(t+1\right) = p_3\left(t\right) + \tfrac{g_3}{1+k} p_3\left(t\right)\left(-\beta Q_s + p_1\left(c_3+b_3\right) - 2p_3 b_3 + d_3 p_2\right) \tag{16}$$

In order to investigate the economic characteristics of the model, and use the economic argument to explain the model, we transform chaos control function (16) to function (17):

$$p_3\left(t+1\right) = p_3\left(t\right) + g_3 p_3\left(t\right)\left(-\tfrac{2b_3}{1+k} p_3 + \tfrac{c_3+b_3}{1+k} p_1 + \tfrac{d_3}{1+k} p_2 + \tfrac{-\beta}{1+k} Q_s\right) \tag{17}$$

From function (16), we can easily to see, the delay control method we used in this case can reduce the original price adjustment factor g_3 to $\frac{g_3}{1+k}$. It can be seen as reducing the speed of the original price adjustment. From function (17), we can see that in order to avoid recycling prices going into chaotic state, several other parameters can be changed, in addition to controlling the price adjustment speed.

By means of integral, we can integrate the part of the function (17), and get the following formula (18):

$$(p_1 - p_3)\left(\frac{b_3}{1+k} p_3 - \frac{c_3}{1+k} p_1 - \frac{d_3}{1+k} p_2 + \frac{\beta}{1+k} Q_s\right) \tag{18}$$

Adding chaos control factor k is equal to making the recycling amount function of retailer 3 become

$$q_3 = \frac{\beta}{1+k} Q_s + \frac{b_3}{1+k} p_3 - \frac{c_3}{1+k} p_1 - \frac{d_3}{1+k} p_2 \tag{19}$$

Chaos control factor k increases mean coefficient β, b_3, c_3, d_3 reduce respectively, and under the conditions of $k > 0.26$, Four coefficients will also have a certain degree of reduced.

It is obvious that it is an effective way to control the chaotic state by reducing the price elasticity, competitive factors, as well as the proportion of voluntary. That is, if one party price changes too fast, it should reduce voluntary recycling amount it can get, reduce the influence of its price to recovery amount, reduce the influence of the others price to recovery amount.

To achieve this purpose, the vendors should pay attention to the recycling process and recycling services, in order to avoid consumers choosing the recovery channel only by comparing recycling prices. In conclusion, the vendors should control their price adjustment factors in a low degree. Or they should do the following improvement to keep the system in a stable state:

- To improve the quality of recycling service.
- To provide differentiated recovery services.
- To promote the importance of environmental protection to the consumers.

6 Conclusion

The paper analyzes recovery price of three parties in waste household appliance market in China, proposes that if price adjustments speed is too fast, the system will go into a chaotic state, and describes this adverse outcome of the chaotic state. Finally, we give a chaos control method, and explain its economic significance.

To simplify the study, we consider only one manufacturer in the model, and we do not consider other factors such as government subsidies. Therefore, in future studies, we can increase the number of manufacturers, and introduce manufacturer game process in the system. We can also add other factors to make the model more perfect.

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