Complexity and Control of a Cournot Duopoly Game in Exploitation of a Renewable Resource with Bounded Rationality Players

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Abstract: Based on the related literatures, a new dynamic Cournot duopoly game model in exploitation of a renewable resource with bounded rationality players is built up. The local stable region of Nash equilibrium point is obtained through using the theory of bifurcations of dynamical systems. It is found that increasing the output adjustment speed parameters of the system can affect the stability of Nash equilibrium point and lead chaos to occur. Its complex dynamics is demonstrated by the way of plotting the bifurcation diagrams, computing and plotting the Lyapunov exponents, plotting phase portraits and calculating the fractal dimension. Furthermore, the chaos can be respectively controlled by making use of the straight-line stabilization method, parameters adjustment method and time-delayed feedback method. The derived results have important theoretical and practical significance to the exploitation of renewable resource.

Key–Words: Complexity, Bifurcation, Chaos, Chaos control, Cournot duopoly game

1 Introduction

In 1963, chaos was first discovered by Lorenz [1] when he made numerical simulation of atmospheric convection. He found that the deterministic Lorenz equations appear aperiodic and chaotic solutions which are sensitive dependence on the initial datum. Lorenz called this kind of solution as chaotic solution. His work opened a prelude to research the chaos. However, the foundation of chaos was in 1975 when Li and Yorke [2] published their paper titled ”Period three implies chaos” in the journal of ”American Mathematical Monthly”. From then on, chaos has become a new science, which reveals the evolution from order to disorder. Till now, there has been no common definition of chaos in the mathematics. Li and Yorke [2], and Devaney [3] respectively gave two different definitions of chaos, which are the usual definition of chaos. Computation of the Lyapunove exponent is an important quantitative indicators to measure system dynamics, which represents the system in the phase space between the adjacent tracks of convergence or divergence in the average exponent rate. Wolf etc. [4] provided a method to calculate the Lyapunov exponent of dynamic systems. Refs. [5, 6, 7] are the recent research papers on theory of chaos. Benedicks and Viana [5] proved that there are no ”holes” in the basin of attraction: stable manifolds of points in the basin is generic for the SRB (Sinai-Ruelle Bowen) measure of the attractor. Stewart [6] convinced that there is no doubt of the existence of the Lorenz attractor. Tucker [7] gave an algorithm which is based on a partitioning process and used of interval arithmetic with directed rounding to compute rigorous solutions for a large class of ordinary differential equations. As a example, he proved that the Lorenz equations support a strange and robust attractor. Base on a combination of normal form theory and rigorous computations, he confirmed the flow of the equations admits a unique SRB measure, whose support coincides with the attractor.

In 1980s, chaos theory was firstly introduced into the economic research. Chaotic economists make use of the basic mathematic theory of chaos to improve the existing models of economic phenomena. The economic system is whether a chaotic system is a very hot topic in the economic field. Bifurcation theory based on difference equation has been applied in all branches of area [8]. In recent years, a considerable amount of research has been done. H. N. Agiza [9] and Michael Kopel [10] have considered bounded rationality and established duopoly Cournot model with linear cost functions. From then on, the model has been extended to multi-oligopolistic market. Gian Italo Bischi et al.[11] suppose that firms determine their output based on the reaction functions, that is, all the players take adaptive expectation. H. N. Agiza and A. A. Elsadany [12] have improved the model which contains two-types of heterogeneous players: boundedly rational player and adaptive expectation player. Jixiang Zhang et al. [13] have further improved the model with nonlinear cost functions. A. A. Elsadany [14] has studied a triopoly game with 0.5-th power inverse demand function and bounded rational players. Hongxing Yao et al. [15] have analyzed a triopoly game with isoelastic demand function and fully heterogeneous players: bounded rational player, adaptive player and naive player. Akio Matsumoto and Yasuo Nonaka [16] have researched the complexity of Cournot model with linear cost functions on complementary goods. Junhai Ma and Weizhuo Ji [17] have reported and considered a Cournot model in electric power triopoly with nonlinear inverse demand and cost functions. Junhai Ma and Xiaosong Pu [18] have considered a triopoly market which has quadratic inverse demand function, and where the firms have cubic total cost functions. Guanhui Wang and Junhai Ma [19] have studied the complexity of multi-enterprise cournot game model in the supply chain. Fang Chen et al. [20] have used Bertrand triopoly model with linear demand functions to study the competition in Chinese telecommunications market. Zhihui Sun and Junhai Ma [21] have introduced a Bertrand triopoly model with nonlinear demand functions in Chinese cold rolled steel market, and researched the complexity and the control of the model. Junling Zhang and Junhai Ma [22] have investigated a Bertrand game model with four oligarchs and different decision rules. Refs. [23, 24, 25] have researched delayed nonlinear Bertrand game models in the insurance market. M. T. Yassen and H. N. Agiza [26] have considered a Cournot duopoly game and the model with delayed bounded rationality. In these literatures, adjustment speed or other parameters are taken as bifurcation parameters, and complex results such as period doubling bifurcation, unstable period orbits, chaos are found. The subject has been extensively explored in other areas such as Refs. [27, 28, 29].

On the basis of a dynamical multi-team Cournot game in exploitation of a renewable resource [30] writen by S. S. Asker, we establish a new dynamic Cournot duopoly game model in exploitation of a renewable resource with bounded rationality players. This duopoly model is closer to the economic reality is worth promoting in the oligopoly market. Suppose the inverse demand function is linear, and the cost functions are nonlinear. In this model, the bounded rational players regulate output speed according to marginal profit, and decide their output. We obtain the stable region about the output adjustment speed parameters by way of theoretical analysis and numerical simulation. It is found that the output adjustment speed causes the chaos to happen at a definite range. It has an important theoretical and applied significance to further research the complexity of new nonlinear dynamical system.

The structure of this paper is as follows. In Section 2, a new nonlinear dynamic Cournot duopoly game model in exploitation of a renewable resource with bounded rational players is described. In Section 3, we investigate the stability and dynamic characteristics of the model. We analyze the existence of the Nash equilibrium point, local stability and bifurcation of the equilibrium points. Its dynamics of complexity is described via computing and plotting the Lyapunov exponents, phase portraits, sensitive dependence on initial conditions by numerical simulations, and computing the fractal dimension. In Section 4, chaos control of the model is considered with the straight-line stabilization method, parameters adjustment method and Time-delayed feedback method. Finally, the results are summarized.

2 The Model

Suppose that there are two representative oligopoly enterprises X_1 , X_2 in exploitation of a renewable resource. The enterprise $X_i(i = 1, 2)$ makes the optimal output decision, and suppose the t-output is $q_i(t)(i = 1, 2)$. At each period t, the price $P(t)$ is determined by the total output $Q_T(t) = q_1(t) + q_2(t)$.

According to Ref. [30], the linear inverse demand function is

$$P(t) = a - bQ_T(t) \tag{1}$$

The cost function of the enterprise $X_i(i = 1, 2)$ is as follows

$$C_i(t) = c_i + \frac{d_i q_i^2(t)}{Q_T(t)} \tag{2}$$

The profit of the enterprise $X_i(i = 1, 2)$ is

$$\pi_i(t) = [a - bQ_T(t)]q_i(t) - c_i - \frac{d_i q_i^2(t)}{Q_T(t)} \tag{3}$$

We propose the enterprise $X_i(i = 1, 2)$ takes bounded rational strategy. The game between the enterprises is a continuous and long-term repeated dynamic process, the dynamic adjustment of the output of the player $X_i(i = 1, 2)$ in this duopoly game model can be expressed as follows:

$$q_i(t+1) = q_i(t) + \alpha_i q_i(t)\frac{\partial \pi_i(t)}{\partial q_i(t)} \quad (4)$$

Where $\alpha_i(i = 1, 2)$ is the output adjustment speed parameter.

Combining Eqs. (3), (4), a new dynamic Cournot duopoly game in exploitation of a renewable resource with bounded rationality players is obtained. This model can be given as follows:

$$\begin{cases} q_1(t+1) = & q_1(t) + \alpha_1 q_1(t)[a - bQ_T(t) \\ & -bq_1(t) - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}], \\ q_2(t+1) = & q_2(t) + \alpha_2 q_2(t)[a - bQ_T(t) \\ & -bq_2(t) - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)}] \end{cases} \quad (5)$$

3 Dynamic features of system (5)

In this game model, the enterprises make the optimal output decision to gain the maximum profit, and regulate output based on their marginal profit of last period. If the marginal profit is positive, one firm can increase its output at the next period to increase the profit. On the contrary, when the marginal profit is negative, one firm is able to decrease its output at the next period to increase the profit. As the ability of the decision-makers is distinct, their output adjustment speed is different, which affects the results of competition. So, the output adjustment speed parameter $\alpha_i(i = 1, 2)$ plays a important role on the game results. We will analyze the effect of $\alpha_i(i = 1, 2)$ on system (5) in the following section.

3.1 The equilibrium point and stability region

In system (5), $\alpha_i(i = 1, 2)$ is taken as variable, and the other parameters are as follows: $a = 3.2, b = 0.96, d_1 = 0.23, d_2 = 0.27$.

We can calculate the partial differentiation of the profit and let it be equal to 0 to derive the Nash equilibrium. The fixed points of system (5) satisfy the following algebraic equations:

$$\begin{cases} q_1[a - bQ_T - bq_1 - \frac{2d_1 q_1}{Q_T} + \frac{d_1 q_1^2}{Q_T^2}] = 0, \\ q_2[a - bQ_T - bq_2 - \frac{2d_2 q_2}{Q_T} + \frac{d_2 q_2^2}{Q_T^2}] = 0 \end{cases} \quad (6)$$

We can see that the solutions of the algebraic equations are independent of parameter $\alpha_i(i = 1, 2)$. For models in economics, only non-negative equilibrium solution makes sense.

The Eqs. (6) are solved and three meaningful fixed points $p_1(1.059951, 1.032162), p_2(0, 1.526042)$, $p_3(1.546875, 0)$ are obtained. In this paper, we only consider the Nash equilibrium point $p_1(q_1^* = 1.059951, q_2^* = 1.032162)$, and denote $Q_T^* = q_1^* + q_2^*$.

The Jacobian matrix at Nash equilibrium point can be derived as follows:

$$J_1 = \begin{pmatrix} 1 + j_{11} & j_{12} \\ j_{21} & 1 + j_{22} \end{pmatrix}, \quad (7)$$

where

$$\begin{aligned} j_{11} &= \alpha_1 q_1^*(-2b - \tfrac{2d_1}{Q_T^*} + \tfrac{4d_1 q_1^*}{Q_T^{*2}} - \tfrac{2d_1 q_1^{*2}}{Q_T^{*3}}), \\ j_{12} &= \alpha_1 q_1^*(-b + \tfrac{2d_1 q_1^*}{Q_T^{*2}} - \tfrac{2d_1 q_1^{*2}}{Q_T^{*3}}), \\ j_{21} &= \alpha_2 q_2^*(-b + \tfrac{2d_2 q_2^*}{Q_T^{*2}} - \tfrac{2d_2 q_2^{*2}}{Q_T^{*3}}), \\ j_{22} &= \alpha_2 q_2^*(-2b - \tfrac{2d_2}{Q_T^*} + \tfrac{4d_2 q_2^*}{Q_T^{*2}} - \tfrac{2d_2 q_2^{*2}}{Q_T^{*3}}) \end{aligned} \quad (8)$$

Then, we can get the characteristic polynomial

$$\lambda^2 - (2 + j_{11} + j_{22})\lambda + (1 + j_{11})(1 + j_{22}) - j_{12}j_{21} = 0 \quad (9)$$

According to the Jury test [31], the necessary and sufficient condition of the local stability of Nash equilibrium is the following three conditions which are satisfied.

$$\begin{cases} 1 - (2 + j_{11} + j_{22}) + (1 + j_{11})(1 + j_{22}) - j_{12}j_{21} > 0, \\ 1 + (2 + j_{11} + j_{22}) + (1 + j_{11})(1 + j_{22}) - j_{12}j_{21} > 0, \\ (1 + j_{11})(1 + j_{22}) - j_{12}j_{21} < 1 \end{cases} \quad (10)$$

By solving the above equations, local stable region of Nash equilibrium point can be got. It is bounded in the region of hyperbolic plane with positive (α_1, α_2) as shown in Fig. 1. The meaning of the stable region is that whatever initial output are chosen by two companies in the local stable region, they will eventually arrive at Nash equilibrium output after a finite games. It is worth studying that the enterprises accelerate the output adjustment speed parameters to increase their profits. Output adjustment speed parameters do not change the Nash equilibrium point, but once one party is adjusting output speed too fast and pushing $\alpha_i(i = 1, 2)$ out of the stable region, the system will become unstable and fall into chaos. We use numerical simulation method to analyze the characteristics of nonlinear dynamical system with the change

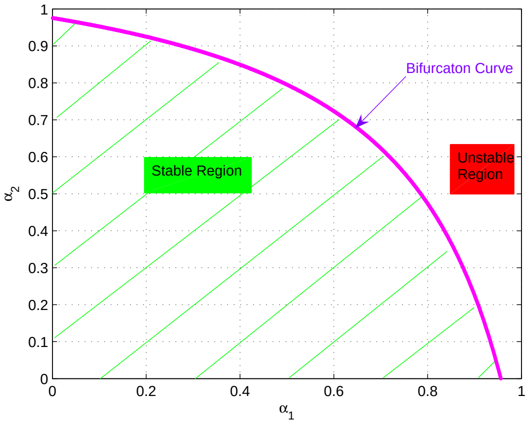

Figure 1: The stable region of system (5) at Nash equilibrium point

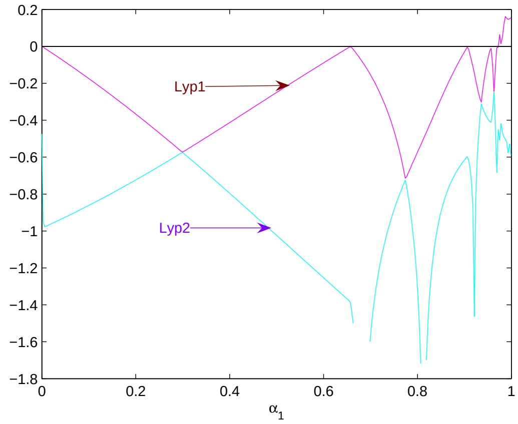

Figure 3: The Lyapunov exponents of system (5) with $\alpha_1 \in [0, 1]$ when $\alpha_2 = 0.67$

of $\alpha_i (i = 1, 2)$. To better knowing of the dynamic characters of the system, numerical simulations such as the bifurcation diagrams, strange attractors, Lyapunov exponents, sensitive dependence on initial conditions and fractal structure will be investigated.

3.2 The effect of output adjustment speed

The stability of Nash equilibrium point will change if company X_1 accelerates output adjustment speed and pushes out of the stable region. Fig. 2 shows a one-parameter bifurcation diagram with respect to α_1 when $\alpha_2 = 0.67$. With output adjustment speed parameter α_1 increasing, the output evolution of duopoly starts with equilibrium state, through period doubling, and ends with chaotic state. We can see that

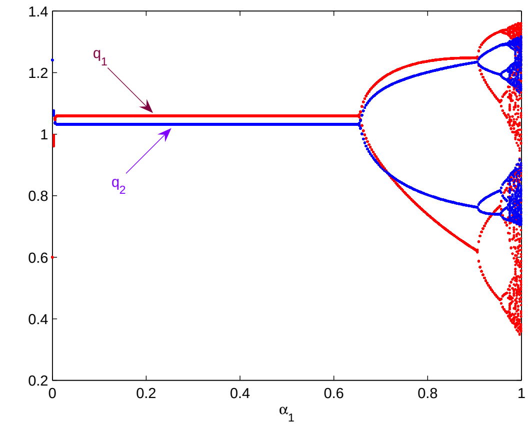

Figure 2: Bifurcation diagram of system (5) with $\alpha_1 \in [0, 1]$ when $\alpha_2 = 0.67$

Nash equilibrium point is stable for $0 < \alpha_1 < 0.6503$, that is, output of the three firms is in the equilibrium state. With α_1 increasing, the stability of equilibrium point changes, output go through period-doubling bifurcation and eventually come into chaos. Computing and plotting the Lyapunov exponents is one of the most efficient way to analyze the quantitative character of the dynamic system. The largest Lyapunov exponent is positive when the system is in a chaotic state. In addition, the positive Lyapunov exponent is larger, the system is more obviously in a chaotic state. Fig. 3 shows the corresponding Lyapunov exponents. $0.6503 < \alpha_1 < 0.9021$ is a range of 2-cycle output orbit as shown in Fig. 4. For $\alpha_1 > 0.9021$, output doubling occurs again. $0.9021 < \alpha_1 < 0.9510$ is a range of 4-cycle output orbit as shown in Fig. 5. $0.9510 < \alpha_1 < 0.9560$ is a range of 8-cycle output orbit as shown in Fig. 6. While $0.9560 < \alpha_1 < 1$, system (5) falls into chaos. Fig. 7 shows chaos attractor at initial point $(q_{1_0} = 0.6, q_{2_0} = 0.9)$ when $(\alpha_1 = 0.987, \alpha_2 = 0.67)$.

Similarly, Fig. 8 shows a one-parameter bifurcation diagram with respect to α_2 when $\alpha_1 = 0.72$, and Fig. 9 shows the corresponding Lyapunov exponents. We can see that Nash equilibrium point is asymptotically stable for $0 < \alpha_2 < 0.5944$. $0.5944 < \alpha_2 < 0.8741$ is a domain of 2-cycle output orbit as shown in Fig. 10. $0.8741 < \alpha_2 < 0.9301$ is a domain of 4-cycle area of output orbit as shown in Fig. 11. $0.9301 < \alpha_2 < 0.9441$ is a domain of 8-cycle output orbit as shown in Fig. 12. For $0.9441 < \alpha_2 < 1$, system (5) goes into a chaotic state. Fig. 13 illustrates chaos attractor at initial point $(q_{1_0} = 0.6, q_{2_0} = 0.9)$ when $(\alpha_1 = 0.72, \alpha_2 = 0.97)$.

The system is sensitive dependence on initial data when it is in a chaotic state, that is to say, a slight difference between initial values can lead to a great effect on the game results. Figs. 14, 15 show

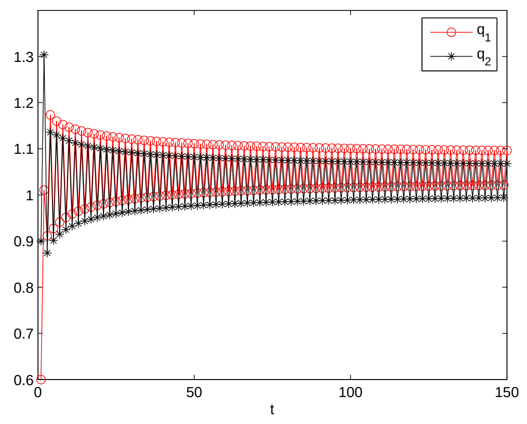

Figure 4: Phase portrait of system (5) for ($\alpha_1 = 0.66, \alpha_2 = 0.67$)

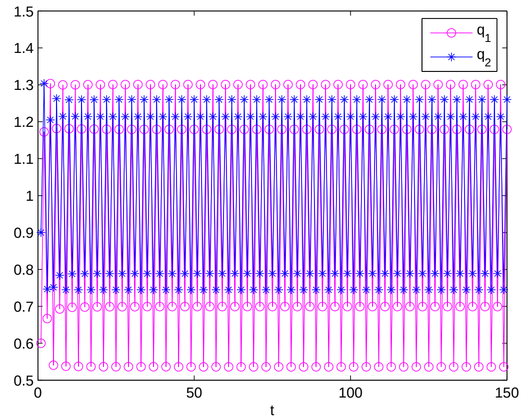

Figure 5: Phase portrait of system (5) for ($\alpha_1 = 0.92, \alpha_2 = 0.67$)

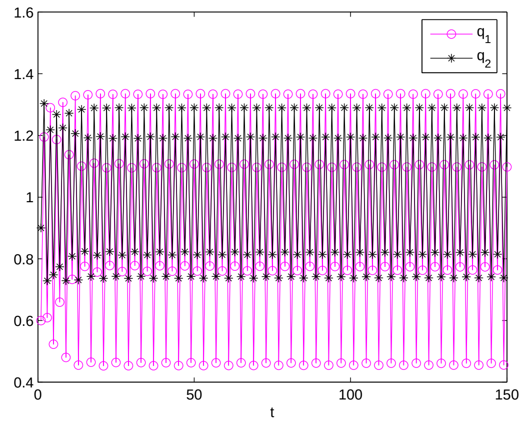

Figure 6: Phase portrait of system (5) for ($\alpha_1 = 0.956, \alpha_2 = 0.67$)

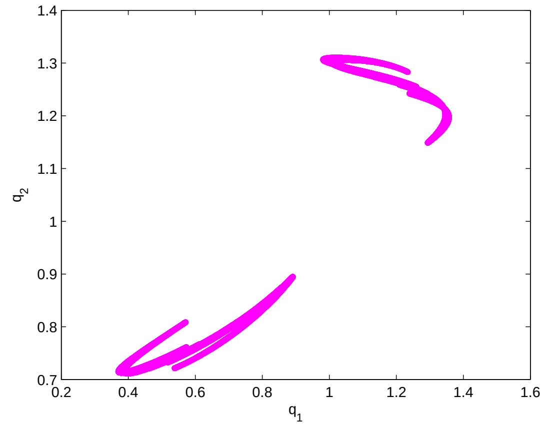

Figure 7: Phase portrait of system (5) for ($\alpha_1 = 0.987, \alpha_2 = 0.67$)

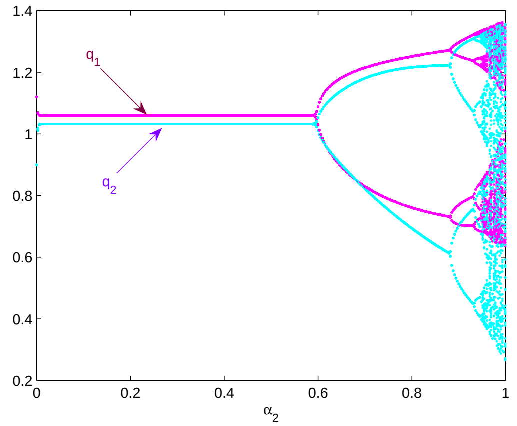

Figure 8: Bifurcation diagram of system (5) with $\alpha_2 \in [0, 1]$ when $\alpha_1 = 0.72$

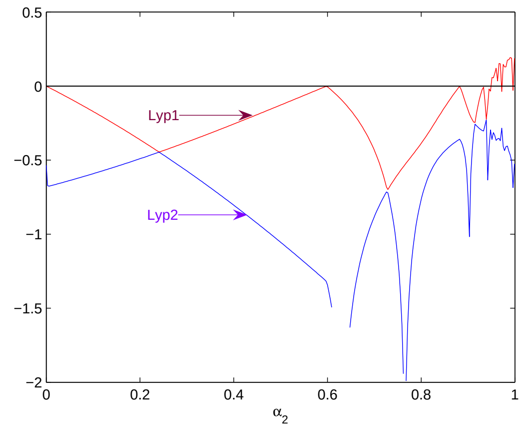

Figure 9: The Lyapunov exponents of system (5) with $\alpha_2 \in [0, 1]$ when $\alpha_1 = 0.72$

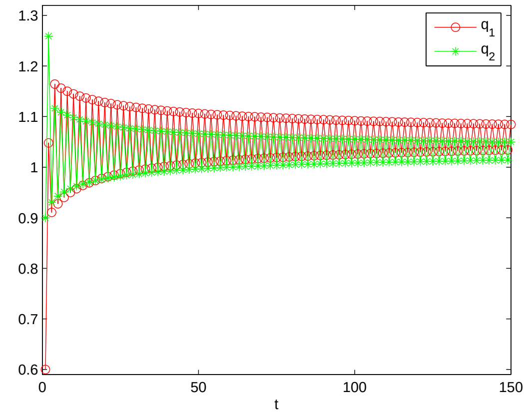

Figure 10: Phase portrait of system (5) for $(\alpha_1 = 0.72, \alpha_2 = 0.596)$

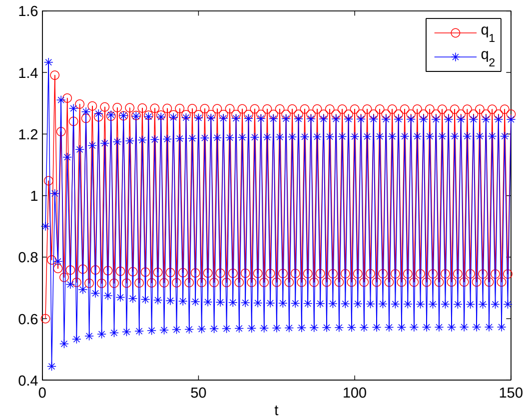

Figure 11: Phase portrait of system (5) for $(\alpha_1 = 0.72, \alpha_2 = 0.885)$

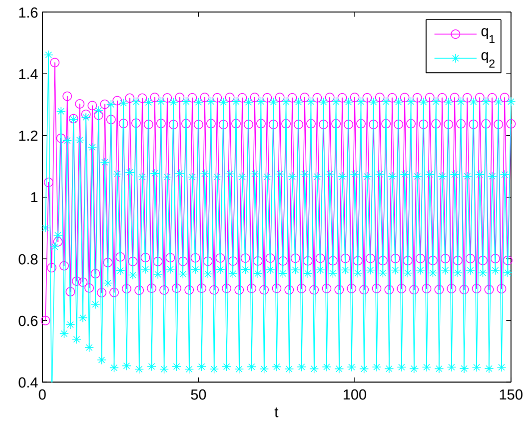

Figure 12: Phase portrait of system (5) for $(\alpha_1 = 0.72, \alpha_2 = 0.932)$

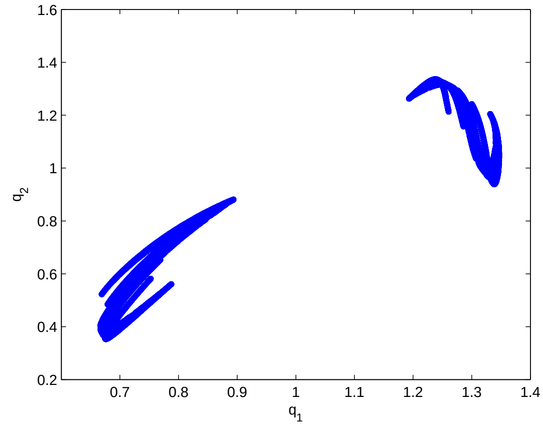

Figure 13: Phase portrait of system (5) for $(\alpha_1 = 0.72, \alpha_2 = 0.97)$

the relationships between output and time to verify whether system (5) depends on initial datum sensitively. Firstly, they are indistinguishable, with the time passing, the difference between them is huge. For $(\alpha_1 = 0.987, \alpha_2 = 0.67)$ and $(\alpha_1 = 0.97, \alpha_2 = 0.72)$, it further proves that system (5) is in a chaotic state. When the system is in a chaotic state, the market will be damaged, and it is difficult for the companies to make long-term plan. Therefore, each action from companies may cause enormous loss.

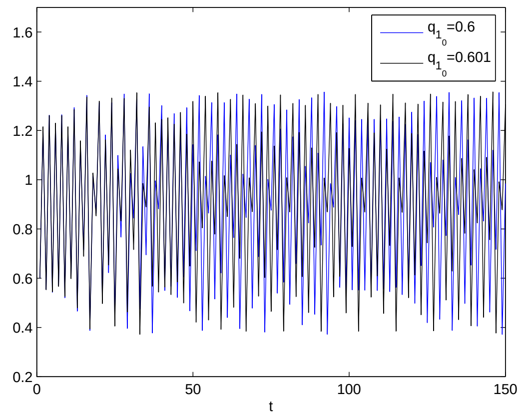

Figure 14: The two orbits of q_1-coordinates for initial points are $(0.6, 0.9)$ and $(0.601, 0.9)$

Fractal dimension is taken as another criterion to judge whether the system is in a chaotic state. There are many ways to define the fractal dimension, but none of them can be treated as the universal one. According to [32], the following definition of fractal di-

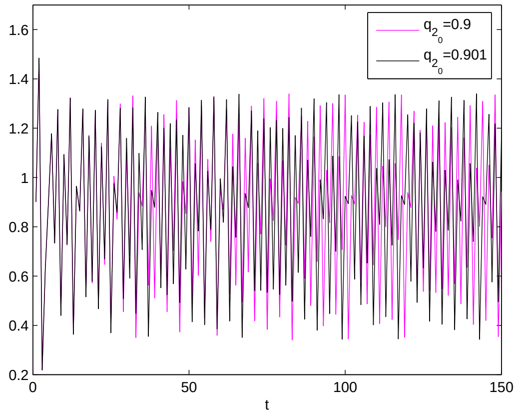

Figure 15: The two orbits of q_2-coordinates for initial points are $(0.6, 0.9)$ and $(0.6, 0.901)$

mension is adopted.

$$d = j - \frac{\sum_1^j \lambda_i}{\lambda_{j+1}}, \tag{11}$$

where $\lambda_1 > \lambda_2 >, ..., \lambda_n$ are the Lyapunov exponents, and j is the maximum integer for which satisfies $\sum_1^j \lambda_i > 0$ and $\sum_1^{j+1} \lambda_i < 0$. If $\lambda_i \geq 0, i = 1, 2, ..., n$, the $d = n$. If $\lambda_i < 0, i = 1, 2, ..., n$, then $d = 0$.

The Lyapunov exponents of system (5) are $\lambda_1 = 0.144942, \lambda_2 = -0.378793$ for $(\alpha_1 = 0.97, \alpha_2 = 0.72)$. As the largest Lyapunov exponent λ_1 is positive, it indicates system (5) is in a chaotic state. Fractal dimension demonstrates that the chaotic motion has self-similar structure. The fractal dimension of system (5) is $d = 1 - \frac{\lambda_1}{\lambda_2} = 1.3826$. The fractal dimension manifests the space density of the strange attractor [33]. The larger the dimension of the chaotic attractor is, the bigger the occupied space is. The fractal dimension of the 2D system (5) is more than 1, so the occupied space is big and the structure is tight, which can be seen in Fig. 7.

4 Control of the system (5)

From above section, we can see that system (5) will become unstable and eventually fall into chaos with the output adjustment speed increasing. When chaos occurs, all the players will be harmed and the market will be damaged. Thus, nobody will be able to make good strategies and decide reasonable output. To avert the risk, it is a good ideal for duopoly to maintain at Nash equilibrium output. There are many methods can be used to control or anticontrol bifurcations and chaos. For the details, one can see relevant Refs. [34, 35, 36, 37, 38, 39, 40].

In this section, the chaos can be controlled by the means of the straight-line stabilization method, parameters adjustment method and time-delayed feedback method, respectively.

4.1 Straight-line stabilization method

Recently, Ling Yang et al. [34] and Haibo Xu et al. [35] proposed a new chaos control method which is called the straight-line stabilization method.

Denote

$$\begin{aligned} \delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = (\mu I - J_1) \begin{pmatrix} q_1(t) - q_1^* \\ q_2(t) - q_2^* \end{pmatrix} \\ = \begin{pmatrix} [\mu - (1 + j_{11})](q_1(t) - q_1^*) - j_{12}(q_2(t) - q_2^*) \\ -j_{21}(q_1(t) - q_1^*) + [\mu - (1 + j_{22})](q_2(t) - q_2^*) \end{pmatrix}. \end{aligned} \tag{12}$$

Where $|\nu| < 1$ is the feedback control parameter and other parameters are the same as above.

Added the external control signal (12) to the system (5), the controlled system is as follows

$$\begin{cases} q_1(t+1) = & q_1(t) + \alpha_1 q_1(t)[a - bQ_T(t) \\ & -bq_1(t) - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}] + \delta_1, \\ q_2(t+1) = & q_2(t) + \alpha_2 q_2(t)[a - bQ_T(t) \\ & -bq_2(t) - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)}] + \delta_2 \end{cases} \tag{13}$$

It can be seen from Fig. 16, for $\alpha_1 = 0.987, \alpha_2 = 0.67$, controlled system (13) stabilized at Nash equilibrium point when $-1 < \mu < 0.04895$. It reveals that the chaos control of the model can be realized while the perturbation is very small.

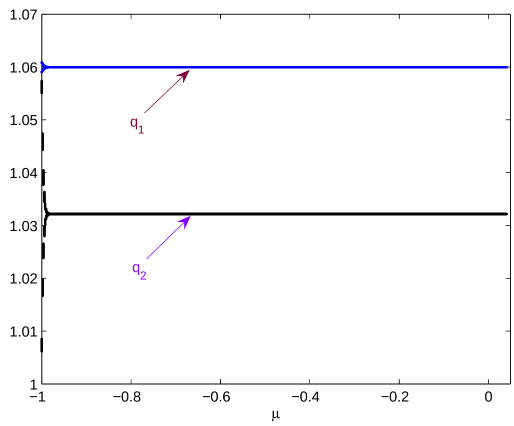

Figure 16: Bifurcation diagram with $\mu \in [-1, 0.04895]$, and $\alpha_1 = 0.987, \alpha_2 = 0.67$

4.2 Parameters adjustment method

Parameters adjustment method is also used to control the effect of parameter $\alpha_i (i = 1, 2)$ on system (5).

The controlled system can be expressed as follows:

$$\begin{cases} q_1(t+1) & = (1-k)[q_1(t) + \alpha_1 q_1(t)(a - bQ_T(t) \\ & \quad -bq_1(t) - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)})] + kq_1(t), \\ q_2(t+1) & = (1-k)[q_2(t) + \alpha_2 q_2(t)(a - bQ_T(t) \\ & \quad -bq_2(t) - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)})] + kq_2(t) \end{cases} \tag{14}$$

where k is an adjustment parameter and other parameters are the same as above. From Fig. 17, for $\alpha_1 = 0.72, k = 0.1$, we can see that with control parameter increasing, that the system is gradually controlled at 8-cycle, 4-cycle, 2-cycle and at fixed point.

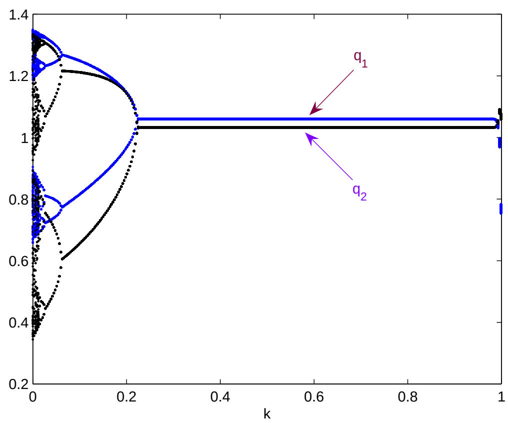

Figure 17: Bifurcation diagram with $k \in [0, 1]$, and $\alpha_1 = 0.72, \alpha_2 = 0.97$

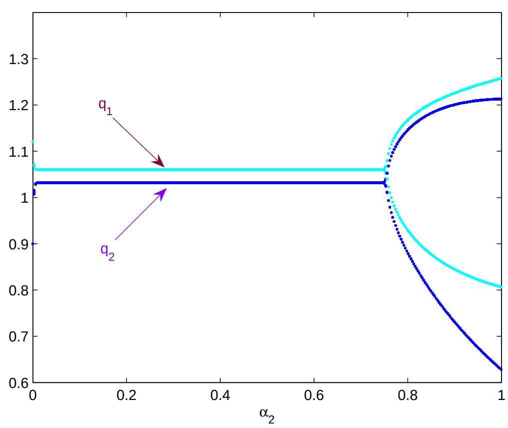

Figure 18: Bifurcation diagram with $\alpha_2 \in [0, 1]$, and $\alpha_1 = 0.72, k = 0.1$

If $k > 0.2168$, controlled system (14) stabilized at the Nash equilibrium point.

When $k = 0.1$, the stable region of α_2 expands from the original 2-cycle bifurcation point 0.5944 (in Fig. 8) to 0.7483 (in Fig. 18), which indicates that once system is controlled, chaos can be delayed or completely eliminated.

4.3 Time-delayed feedback method

Added time-delay feedback ($T = 1$) to the first equation of system (5), the controlled system is as follows:

$$\begin{cases} q_1(t+1) = & q_1(t) + \alpha_1 q_1(t)[a - bQ_T(t) \\ & -bq_1(t) - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}] \\ & +\nu_1(q_1(t) - q_1(t-1)), \\ q_2(t+1) = & q_2(t) + \alpha_2 q_2(t)[a - bQ_T(t) \\ & -bq_2(t) - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)}] \end{cases} \tag{15}$$

where ν_1 is the control coefficient .

For the purpose of studying the stability of system (15), we rewrite system (15) as a third dimensional system in the form

$$\begin{cases} x(t+1) = q_1(t), \\ q_1(t+1) = & q_1(t) + \alpha_1 q_1(t)[a - bQ_T(t) \\ & -bq_1(t) - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}] \\ & +\nu_1(q_1(t) - x(t)), \\ q_2(t+1) = & q_2(t) + \alpha_2 q_2(t)[a - bQ_T(t) \\ & -bq_2(t) - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)}] \end{cases} \tag{16}$$

The Jacobian matrix of system (16) at the Nash equilibrium point p^* is The Jacobian matrix at Nash equilibrium point can be represented by the following form:

$$J_2 = \begin{pmatrix} 0 & 1 & 0 \\ -\nu_1 & 1 + j_{11} + \nu_1 & j_{12} \\ 0 & j_{21} & 1 + j_{22} \end{pmatrix}, \tag{17}$$

Moreover, the characteristic polynomial of system (16) is:

$$\lambda^3 + B_2\lambda^2 + B_1\lambda + B_0 = 0 \tag{18}$$

where

$$\begin{aligned} B_2 &= -(2 + j_{11} + j_{22} + \nu_1), \\ B_1 &= (1 + j_{11} + \nu_1)(1 + j_{22}) - j_{12}j_{21} + \nu_1, \\ B_0 &= -\nu_1(1 + j_{22}) \end{aligned} \tag{19}$$

The necessary and sufficient conditions for the local stability of Nash equilibrium can be gained by Jury test [31] as follows:

$$\begin{cases} 1+B_2+B_1+B_0>0, \\ 1-B_2+B_1-B_0>0, \\ B_0^2-1<0, \\ |B_0^2-1|>|B_1-B_2B_0| \end{cases} \tag{20}$$

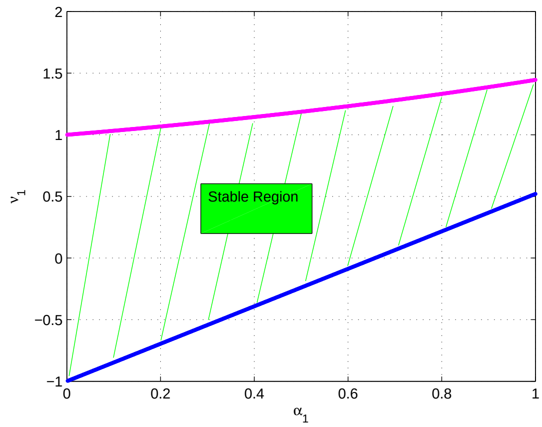

Figure 19: The stable region of system (16) at Nash equilibrium point

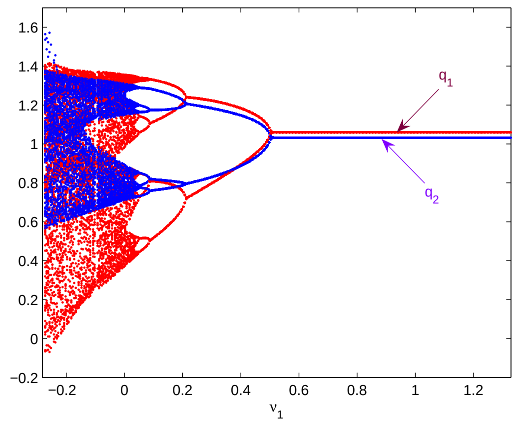

Figure 20: Bifurcation diagram with $\nu_1 \in [-0.2825, 1.3287]$, and $\alpha_1 = 0.987, \alpha_2 = 0.67$

Through computing the above equations, local stable region of Nash equilibrium point can be obtained. It is bounded hyperbolic plane region with positive $(\alpha_1, \alpha_2, \nu_1)$. If α_2 held fixed, the stable region in the phase plane of (α_1, ν_1) can be obtained, such as the stable region of (α_1, ν_1) is shown in Fig. 19 when $\alpha_2 = 0.67$. The Nash equilibrium is stable for the values of (α_1, ν_1) inside the stable region. Likewise, the stable regions in the phase plane

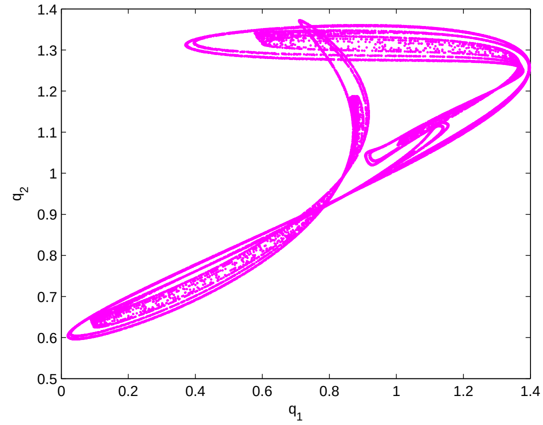

Figure 21: Chaos attractor of system (16) for $(\alpha_1 = 0.987, \alpha_2 = 0.67, \nu_1 = 0.22)$

of (α_2, ν_1) and (α_1, α_2) with α_1 and ν_1 respectively hold fixed can also be obtained, which are omitted here. Fig. 20 shows the bifurcation diagram with $\nu \in [-0.2825, 1.3287]$ and $\alpha_1 = 0.987, \alpha_2 = 0.67$ to show the stable region of (α_1, ν_1). Fig. 21 shows chaos attractor of system (16) for $(\alpha_1 = 0.987, \alpha_2 = 0.67, \nu_1 = 0.22)$.

5 Conclusion

In this paper, a dynamical nonlinear duopoly game in exploitation of a renewable resource is established. We have investigated the local stability of equilibria, bifurcation and chaotic behaviors of the duopoly game. We have found that bifurcation, chaos and other complex phenomena occur with the output adjustment speed parameter increasing. The oligopoly market will became unstable and fall into chaos if output adjustment speed parameter out of the stable region. The duopoly output stable at the Nash equilibrium point can be respectively realized by the straight-line stabilization method, parameters adjustment method and time-delayed feedback method when the system under a chaotic state. For the traditional non-renewable energy is being depleted, the development of renewable resource is an inevitable choice. It has a very theoretical and practical significance to research the complexity of new nonlinear dynamical system. The obtained results give a light for companies in exploitation of the renewable resource to make strategies of output and exploit renewable resources, and are helpful for the government to formulate relevant policies to the adjustment of economic structure.

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