Development of a Program for Searching the Optimum Condition using the Design of Experiments

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Abstract: - The Design of Experiments (DOE) is a method that is widely used due to its effectiveness in selecting optimum conditions in the design stage of product development. On the other hand, fast, low-cost, labor-saving, and energy-saving innovative development is also required in the industry. In previous research, a program for quickly searching the optimum condition using the design of experiments is developed and evaluated. Relationships between each parameter and the final property are firstly cleared for an algebraic formula by using the design of experiments. Then the optimum conditions for each parameter were decided by using these formulas in the program. However, when each parameter has several errors in the data, the search accuracy becomes very low. In this research, the improvement for the searching accuracy using the law of error propagation was developed and evaluated. Relationships between each parameter error and the final property are firstly cleared for high-accuracy searching by using the law of error propagation and the previous results in the previous research. And each parameter influence for the final property was cleared. It was found that the large parameter effects could be improved for high-precision search by using high-precision instruments, increasing the number of trials N, and taking measurements in an optimal environment. Relationships between each parameter error and the final property were investigated and evaluated for the proposed method by using a mathematical model. It is concluded from the result that (1) the proposed method is effective for clearing the relationships between each parameter error and the final property, and (2) the proposed method is effective for searching the optimum condition.

Key-Words: - Design of experiments, Optimum condition, Taylar’s law of error propagation, Searching accuracy, High-precision search, Design stage.

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1 Introduction

Design of experiments is often used in industry to efficiently determine the optimal combination of level values of control factors, [1], [2]. In addition, quality engineering (static property) is a highly robust design method that incorporates the concept of error factors based on the design of experiments and has been the subject of much research, [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. However, most of these studies are limited to obtaining the factor effect diagram as an effective case study and using it to perform a two-stage design. Further practical research and development are desired in the industrial world.

In contrast, as a new method, the authors used the design of experiments to clarify the functional relationship between the final property value and each control factor by curve-fitting work and additivity and developed a method to obtain the optimal final property value using the functional relationship. However, when errors were included within the level value of each control factor, the accuracy of the functional relationship equation was reduced and the optimization of the final property value could not be guaranteed.

In this study, we apply Taylor's law of error propagation to the functional relationship between the final property value and each control factor obtained in the authors' previous study, [13] to formulate the functional relationship of the effect of the error of each control factor on the final property value. Then, we develop and evaluate the technology to control the final property value with high accuracy and to improve the accuracy of the optimum final property value. In this paper, orthogonal tables are also used, and if there are interaction or synergy effects between control factors, the proposed method will be affected, [12]. In this research, it is assumed that there are no interaction or synergy effects between the control factors.

This research proposes a method that allows qualitative and quantitative consideration of the influence of control factors to improve the accuracy of the final property values, something that conventional experimental design methods, [1], [2] and quality engineering, [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] have failed to do. By introducing the proposed method through WSEAS, many researchers, engineers, scientists, postgraduate students industrial engineers, and managers will be able to contribute to rapid and accurate development research.

2 Explanation of the Program to Search the Optimal Condition using the Design of Experiments

2.1 The Design of Experiments

In this section, the optimum conditions identification program used for the proposed method is explained.

The Design of Experiments (DOE) is commonly used, based on a small number of experiments or CAE analyses, to estimate an optimum parameter combination for the generation of new designs. The control factors (A to D) and their levels (A_1 to A_3, B_1 to B_3, C_1 to C_3, and D_1 to D_3,) are shown in Table 1 (Appendix). The orthogonal table is used to set up the control factors and their levels in Table 1 (Appendix), as shown in Table 2 (Appendix). The experiments are then carried out according to the numbers in the orthogonal table. The results are also given in Table 2 (Appendix) as final properties P. From the principle of orthogonal tables, the relationship between the influence E (=E_{Ax}, E_{By}, E_{Cz}, E_{Dw}) of each control factor and the final property values $P_{Ax \cdot By \cdot Cz \cdot Dw}$ is shown in Equation (1).

$$\left.\begin{aligned}
E_{A1} &= (P_{A1 \cdot B1 \cdot C1 \cdot D1}+P_{A1 \cdot B2 \cdot C2 \cdot D2}+P_{A1 \cdot B3 \cdot C3 \cdot D3}) / 3\\
E_{A2} &= (P_{A2 \cdot B1 \cdot C2 \cdot D3}+P_{A2 \cdot B2 \cdot C3 \cdot D1}+P_{A2 \cdot B3 \cdot C1 \cdot D2}) / 3\\
E_{A3} &= (P_{A3 \cdot B1 \cdot C3 \cdot D2}+P_{A3 \cdot B2 \cdot C1 \cdot D3}+P_{A3 \cdot B3 \cdot C2 \cdot D1}) / 3\\
E_{B1} &= (P_{A1 \cdot B1 \cdot C1 \cdot D1}+P_{A2 \cdot B1 \cdot C2 \cdot D3}+P_{A3 \cdot B1 \cdot C3 \cdot D2}) / 3\\
E_{B2} &= (P_{A1 \cdot B2 \cdot C2 \cdot D2}+P_{A2 \cdot B2 \cdot C3 \cdot D1}+P_{A3 \cdot B2 \cdot C1 \cdot D3}) / 3\\
E_{B3} &= (P_{A1 \cdot B3 \cdot C3 \cdot D3}+P_{A2 \cdot B3 \cdot C1 \cdot D2}+P_{A3 \cdot B3 \cdot C2 \cdot D1}) / 3\\
E_{C1} &= (P_{A1 \cdot B1 \cdot C1 \cdot D1}+P_{A2 \cdot B3 \cdot C1 \cdot D2}+P_{A3 \cdot B2 \cdot C1 \cdot D3}) / 3\\
E_{C2} &= (P_{A1 \cdot B2 \cdot C2 \cdot D2}+P_{A2 \cdot B1 \cdot C2 \cdot D3}+P_{A3 \cdot B3 \cdot C2 \cdot D1}) / 3\\
E_{C3} &= (P_{A1 \cdot B3 \cdot C3 \cdot D3}+P_{A2 \cdot B2 \cdot C3 \cdot D1}+P_{A3 \cdot B1 \cdot C3 \cdot D2}) / 3\\
E_{D1} &= (P_{A1 \cdot B1 \cdot C1 \cdot D1}+P_{A2 \cdot B2 \cdot C3 \cdot D1}+P_{A3 \cdot B3 \cdot C2 \cdot D1}) / 3\\
E_{D2} &= (P_{A1 \cdot B2 \cdot C2 \cdot D2}+P_{A2 \cdot B3 \cdot C1 \cdot D2}+P_{A3 \cdot B1 \cdot C3 \cdot D2}) / 3\\
E_{D3} &= (P_{A1 \cdot B3 \cdot C3 \cdot D3}+P_{A2 \cdot B1 \cdot C2 \cdot D3}+P_{A3 \cdot B2 \cdot C1 \cdot D3}) / 3
\end{aligned}\right\} \quad (1)$$

Then, the final property values can be estimated based on the additivity of the orthogonal sequences, which is the most important feature of the design of experiments. Therefore, the relationship between the influence E of each control factor and all final property values $P_{Ax \cdot By \cdot Cz \cdot Dw}$ can be estimated by Equation (2).

$$P_{Ax \cdot By \cdot Cz \cdot Dw} = E_{Ax}+E_{By}+E_{Cz}+E_{Cw} - (4-1)\, P_{ave} \quad (2)$$

Where, P_{ave} is the average of the final property values (the average of the final property values shown in Table 2, Appendix). In the design of experiments, the additivity of the orthogonal sequences can be used to estimate all combinations results (81 different results in this case) from a small number of experimental results. Therefore, the best combination search can be performed among all combination results, however it is not the optimal condition.

2.2 The Program to Search the Optimal Condition using the Design of Experiments

The additivity (equation (2)) of the orthogonal sequences was used for the optimal condition identification program. In the previous explanations, all control factors had three levels. By increasing the number of these levels, the accuracy of the causal relationship increases, however, it requires a long working time and a large cost. The relationship between the influence E (=E_{Ax}, E_{By}, E_{Cz}, E_{Dw}) of each control factor and each level (A_x, B_y, C_z and D_w) of each control factor was then displayed as four curves ($f(A_x)$, $g(B_y)$, $h(C_z)$, $i(D_w)$) by curve fitting, [13]. In this way, the influence of an infinite number of level values can be processed quickly Equation. (2) is accordingly rewritten as Equation (3).

$$P_{Ax\cdot By\cdot Cyz\cdot Dw}=f(A_x)+g(B_y)+h(C_z)+i(D_w)-(4-1)P_{ave} \quad (3)$$

This allows us to estimate the final properties for an infinite number of combinations within the range of levels in Table 1 (Appendix). This is the optimal condition identification program used in the proposed method. This is the author's original technology, which allows us to search for optimal conditions.

3 Explanation of the Algorithm for Improving the Accuracy of the Searching for Optimal Condition

3.1 Understanding the Relationship between Control Factors Errors and Final Property Values using Taylar's Law of Error Propagation

In this chapter, based on Equation (3) for the relationship between the influence E of each control factor and final property values P in the previous section, Taylor's law of error propagation is used to clarify the effect of the level value error of each control factor on the final property value error. If the error in the final property value $P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ is $\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}$, and the error in the level values A_x, B_y, C_z, and D_w of each control factor is δA_x, δB_y, δC_z and δD_w respectively, then by assuming that the error is sufficiently smaller than the value of the variable of interest, Equation (4) is obtained by dropping the higher-order terms in the Taylor expansion.

$$\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}= (\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial A_x)\delta A_x+(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial B_y)\delta B_y +(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial C_z)\delta C_z+(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial D_w)\delta D_w \quad (4)$$

Where $\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial A_x$ denotes the partial differentiation of $P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ by A_x. Since the error can be positive or negative, Equation (5) is obtained by taking the absolute value of each term.

$$|\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}| \leqq |(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial A_x)\delta A_x|+|(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial B_y)\delta B_y| +|(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial C_z)\delta C_z|+ |(\partial P_{Ax\cdot Bx\cdot Cz\cdot Dw}/\partial D_w)\delta D_w| \quad (5)$$

Equation (6) is obtained by substituting Equation (3) into Equation (5).

$$|\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}|\leqq |(\partial f(A_x)/\partial A_x)\delta A_x|+|(\partial g(B_y)/\partial B_y)\delta B_y| +|(\partial h(C_z)/\partial C_z)\delta C_z |+ (\partial i(D_w)/\partial D_w)\delta D_w| \quad (6)$$

From this Equation (6), it can be seen that the errors δA_x, δB_y, δC_z and δD_w contained in each of the level values A_x, B_y, C_z, and D_w of each control factor propagate to the error $\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ contained in the final property value $P_{Ax\cdot Bx\cdot Cz\cdot Dw}$.

3.2 High Accuracy of Final Property Values using Error Management of Control Factors

In this section, as shown in Table 3 (Appendix), the relationship between the impacts ($\partial f(A_x)$ /∂A_x, $\partial g(B_y)$ /∂B_y, $\partial h(C_z)/\partial C_z$, $\partial i(D_w)$ /∂D_w) of the control factor errors and the final property value error $\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ are firstly calculated using Equation (6). Then, referring to the impact, to reduce the error $\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ of the final property value $P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ as much as possible, measures are taken to reduce the error δA_x, δB_y, δC_z and δD_w of the level value of each control factor. The countermeasures include (1) changing the equipment to increase the accuracy of the level values of the control factors, and (2) increasing the number of experiments (N values) to increase the accuracy of the level values. Measures to improve the accuracy of the final property values can be determined using the error $\delta P_{Ax\cdot Bx\cdot Cz\cdot Dw}$ of the final property values in Table 3 (Appendix) as a guide, taking into account cost-effectiveness, time gain and effort required. The proposed method can contribute to the development. Studies of many researchers, engineers, scientists, and industrial engineers.

4 Explanation of the Algorithm Evaluation of the Proposed Method using a Mathematical Model

4.1 Effect of Control Factor Errors on Final Property Values

Two structural equations, Equations (7) and (8), are used to evaluate the influence of the error of the control factor on the final property value. These two equations are prepared to evaluate individually the influence of the error of the control factor on the final property value for two problems of a completely different nature, respectively, and there is no interrelationship between Equations (7) and (8).

$$P_{A\cdot B\cdot C} = A^2 + 9A - B^2 - 3B + 5C - 46 \quad (7)$$

$$P_{A\cdot B\cdot C} = 6\ e^{0.1A} + 2B^2 + 5B + 6C \quad (8)$$

The procedure of searching the functional relationship equation between the final property value and each control factor is explained using the authors' previous study, [13]. The previous Equation (7) or Equation (8) is a functional relationship between the final property value and each control factor, respectively, which is used here for calculation when carrying out Taylor's law of error propagation. The control factors and their level values in Table 4 (Appendix) were used. Table 5 (Appendix) shows the combinations of the level values of each control factor according to the L9 orthogonal table. The final property values $P_{(7)}$ and $P_{(8)}$, calculated using the structure Equation (7) and Equation (8) respectively, are also shown. Figures 1 and Figures 2 in Appendix show the relationship between the final property values $P_{(7)}$ and $P_{(8)}$ in Table 5 (Appendix) and the control factor influence E. The equations obtained by an automatic curve-fitting operation, [13] are also shown in the figures. Also shown in the titles of Figures 1 and Figures 2 in Appendix are the functional relationships between the final property value $P_{Ax,By,Cz}$ and the control factors A_x, B_y, C_z in each figure, and the mean value of the final property value $P_{(7)ave'}$ and $P_{(8)ave'}$ are also shown. As Equation (8) contains an exponential function, it was difficult to separate the influence of each control factor contained in the exponential function on the mean value of Eave, so the coefficients of the exponential part are different between Equation (8) and the calculated functional equation, however, it is believed that this does not have a significant effect. So far, this is the procedure to search for the functional relationship between the final property value and each control factor in the author's previous work, [13]. Then, based on Equations (7) and (8) of the structural equation, errors $\pm\Delta A_S$, $\pm\Delta B_T$ and $\pm\Delta C_M$ were added to the level values of each control factor as shown in Equations (9) and (10).

$$F_L = (A_S\pm\Delta A_S)^2+9\times(A_S\pm\Delta A_S)-(B_T\pm\Delta B_T)^2 -3\times(B_T\pm\Delta B_T)+5\times(C_M\pm\Delta C_M)-46 \quad (9)$$

(Where L = 1 ~ 9; S, T and M = 1, 2 or 3)

$$F_L = 6\,e^{0.1(AS\pm\Delta AS)} + 2\times(B_T\pm\Delta B_T)^2 + 5\times(B_T\pm\Delta B_T) + 6\times(C_M\pm\Delta C_M) \quad (10)$$

(Where L = 1 ~ 9; S, T and M = 1, 2 or 3)

Equations (9) and (10) respectively. The relation of the two final property values P' was calculated and compared with the final property value P of the structure Equation (7) and Equation (8) without the error term, respectively, and the error of the calculation was determined. The accuracy was evaluated as the mean value and standard deviation of 10 calculations.

Figure 3 (Appendix) shows the relationship between the error in the final property value and the error in the level value of each control factor. In the structural equation of Figure 3(a) in Appendix, when the error included in the level value of the control factor is less than ±3 %, the estimation can be done with good accuracy, however, when the error is ±5 %, the mean value of the calculation error is 5.1 % with a standard deviation of 2.2 %. In the structural equation in Figure 3(b) in Appendix, even when the error contained in the level value of the control factor is ± 3 %, the mean value of the calculation error is 7.5 % with a standard deviation of 2.8 %, which is large. This is a major problem in the authors' previous study, [13]. This is also a major problem in the industry, where the design of experiments and quality engineering are used in research and development. Therefore, as a countermeasure, the number of trials is increased (N-value is increased) without any reason, and the measurement equipment and facilities are renewed as much as possible with high accuracy without any clear reason. In this research, it is provides the technology to carry out effective and efficient countermeasures in the right place by clarifying the basis and the reason for the countermeasures.

4.2 Evaluation of the Proposed Method

Two structural equations of the previous section are used to evaluate the proposed technique. Substituting Equations (7) and (8) into Equation (5) respectively, we obtain Equations (11) and (12), respectively.

$$|\delta P_{Ax\cdot Bx\cdot Cz}| \leqq |(\partial f(A_x)/\partial A_x)\delta A_x|+|(\partial g(B_y)/\partial B_y)\delta B_y| +|(\partial h(C_z)/\partial C_z)\delta C_z| = |(2A_x+9)\delta A_x|+|(-2B_y-3)\delta B_y|+|5\delta C_z| \quad (11)$$

$$|\delta P_{Ax\cdot Bx\cdot Cz}| \leqq |(\partial f(A_x)/\partial A_x)\delta A_x|+|(\partial g(B_y)/\partial B_y)\delta B_y| +|(\partial h(Cz)\partial Cz)\delta Cz| = |(0.6\,e^{0.1}A_x)\delta A_x|+|(4By+5)\delta B_y|+|6\delta C_z| \quad (12)$$

From these equations, by examining the coefficients of the three terms on the right-hand side of each equation, it is possible to understand how the errors δA_x, δB_y, and δC_z of the control factors A, B, and C, respectively, propagate concerning the error in the final property value $|\delta P_{Ax\cdot Bx\cdot Cz}|$.

Table 6 in Appendix shows, based on Table 4 (Appendix), the control factors, their level values, and the error of each level value. In Table 7 (Appendix) each level value including the error is set according to the L9 orthogonal table. The errors are the values +3 %, +6 % and +10 % of each level value, which is considered to have the greatest influence within the error range. Table 8 and Table 9 in Appendix show the results of the evaluation of the proposed a technique using the structural Equations (7) and (8) respectively. The final property values $P_{(7)}$ and $P_{(8)}$ when the error is 0 % are taken from the results in Table 5 (Appendix). The final property values $P_{(7)'}$ and $P_{(8)'}$ when the error is +3 %, +6 %, and +10 % are calculated using the mathematical model Equations (9) and (10) and Table 6 and Table 7 (Appendix). After calculating the influence of the error $|P_{(7)}-P_{(7)'}|$ and $|P_{(8)}- P_{(8)'}|$ from the difference between the values with and without the error, the error of the final property value errors $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$ from Equations (11) and (12), and compared them. There is a good correspondence between $|P_{(7)}-P_{(7)'}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ in Table 8 (Appendix) and $|P_{(8)}-P_{(8)'}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$ in Table 9 (Appendix). The proposed law of error propagation can be effectively used to consider the effect of level error in the control factors. $|P_{(7)}-P_{(7)'}|$ in Table 8 (Appendix) and $|P_{(8)}-P_{(8)'}|$ in Table 9 (Appendix) are random data generated by using computer-generated random numbers, whereas $|\delta P_{Ax \cdot Bx \cdot Cz\ (11)}|$ in Table 8 (Appendix) and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$ in Table 9 (Appendix) use the maximum (fixed) value within each level error range, so that $|P_{(7)}-P_{(7)'}| \geq |\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ and $|P_{(8)}-P_{(8)'}| \geq |\delta P_{Ax \cdot Bx \cdot Cz(12)}|$. The same results as in Table 9 (Appendix) are obtained by replacing 6.0001 $e^{0.1A}$ in the first term of the right-hand side of the calculated (estimated) structural Equation (8)' in Figure 2 (Appendix) with 6 $e^{0.1A}$ in the first term of the right-hand side of the original structural Equation (8).

Figure 4 (Appendix) shows the effect of each term on the final property value errors $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$, focusing on the three terms on the right-hand side of each of Equations (11) and (12) (corresponding to the level values of the control factors A, B, and C) applying the proposed law of error propagation. The errors of the final property values, $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$, are also shown as black lines in the figure (the black line is the sum of the effects of the level values of the control factors A, B, and C). The vertical axis is a logarithmic scale. The parameters are the error of each level value of the control factors +3 %, +6 %, and +10 %. First of all, both Equations (11) and (12) of the proposed law of error propagation show that the effect of the level error of the control factor A is very large, and therefore it is effective in increasing the accuracy of the level of the control factor A to obtain accurate final property values. The control factors A and B in Figure 4(a) in Appendix are both quadratic equations in the structural Equation (7), however, the value of the target level value of control factor A is larger than that of control factor B. This is because of the error $\delta A_x > \delta B_y$ in Equation (11). The control factor B in Figure 4(a) (Appendix) has some influence on the error of the final property values $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$ compared to the control factor B in Figure 4(b) (Appendix). This is because the second term on the right-hand side of Equation (12), $|(4B_y+5)|$, is larger than the second term on the right-hand side of Equation (11), $|(-2B_y-3)|$. In both structural equations, the level value of the control factor C and the level value error have a very small influence on the final property value. These trends can also be seen in Table 8 and Table 9 in Appendix, however, Figure 4 (Appendix) is physically easier to understand. Thus, the proposed Equations (11) and (12) of the law of propagation of error are useful for considering and examining the relationships between the final property values, errors, control factors, and level values in the search for optimal conditions using the design of experiments. Then, when managing the level value error of each control factor, we first select and focus on the control factor that is effective in improving the accuracy of the final property value from Figure 4 (Appendix), and then increase the number of trials (increase the N value) or consider the use of a measurement device with high accuracy to reduce the level value error of that control factor. In this case, the accuracy of the final property value can be improved by a factor of $N^{0.5}$ by increasing the N value, however, this requires a great deal of time and effort. It is also possible to improve the accuracy of the final property value with high-precision equipment and measuring instruments, but this requires a great deal of money and there are limits to the accuracy of the equipment and measuring instruments. Then, in this process, it is possible to decide the most appropriate countermeasure from among many options, considering cost performance and life cycle assessment (LCA) while using Figure 4 (Appendix).

Traditionally, the accuracy of the final property values has been improved by trial and error over a long period and at great expense and effort. However, the proposed method can easily provide qualitative and quantitative measures to improve the accuracy of the final property values.

As a practical application of the proposed method, we are planning to improve its forced cooling characteristics (heat transfer coefficient) to a high degree of accuracy by using the previous study on "forced cooling of strong alkaline water mist", [13], which will be reported in the next paper.

5 Conclusion

By managing the error of each control factor, the property value can be managed accurately, and the technology to increase the accuracy of the optimum final property value was developed and evaluated. The following conclusions were obtained. (1) Using Taylor's law of propagation of error, the effect of the error of each control factor on the final property value is formulated as a functional relationship, and by efficiently managing each control factor, it is possible to achieve high accuracy of the optimum property value. (2) About the calculation accuracy using two structural equations, without using the proposed technique, the errors of the respective final property values within the range of ±1 %, ±3 %, ±5 %, and ±10 % for the errors ΔAS, ΔB T and ΔCM contained in the control factors A, B, and C are ±0.9 to 2.0 %, ±3.4 to 7.5 %, ±5. 1 to 11.0 % and ±8.1 to 18.34 %, respectively. By using the proposed technique, the error of the final property value can be improved preferentially and efficiently. (3) By using the proposed law of error propagation, the qualitative and quantitative effects of the level error of the control factors on the final property value can be understood in advance, which can be effectively used for the error management and error control of the final property value.

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Conflict of Interest

The authors have no conflicts of interest to declare.

APPENDIX

Table 1. Control factors and these levels

Control factors				
Name	A	B	C	D
Levels	A_1	B_1	C_1	D_1
	A_2	B_2	C_2	D_2
	A_3	B_3	C_3	D_3

Table 2. Orthogonal array and final properties

	Control factors				Final properties P
No.	A	B	C	D	
1	A_1	B_1	C_1	D_1	$P_{A1 \cdot B1 \cdot C1 \cdot D1}$
2	A_1	B_2	C_2	D_2	$P_{A1 \cdot B2 \cdot C2 \cdot D2}$
3	A_1	B_3	C_3	D_3	$P_{A1 \cdot B3 \cdot C3 \cdot D3}$
4	A_2	B_1	C_2	D_3	$P_{A2 \cdot B1 \cdot C2 \cdot D3}$
5	A_2	B_2	C_3	D_1	$P_{A2 \cdot B2 \cdot C3 \cdot D1}$
6	A_2	B_3	C_1	D_2	$P_{A2 \cdot B3 \cdot C1 \cdot D2}$
7	A_3	B_1	C_3	D_2	$P_{A3 \cdot B1 \cdot C3 \cdot D2}$
8	A_3	B_2	C_1	D_3	$P_{A3 \cdot B2 \cdot C1 \cdot D3}$
9	A_3	B_3	C_2	D_1	$P_{A3 \cdot B3 \cdot C2 \cdot D1}$

Table 3. Control of all control factor errors for searching the optimum conditions with high accuracy using the Equation (6) with the law of propagation of error

Relationship between each control factor level A_x , B_y, C_z, D_w and the final property $P_{Ax \cdot Bx \cdot Cz \cdot Dw}$:

$$P_{Ax \cdot Bx \cdot Cz \cdot Dw} = f(A_x) + g(B_y) + h(C_z) + i(D_w) - (4-1)\,P_{ave} \qquad (3)$$

Relationship between each control factor's level error δA_x, δB_y, δC_z, δD_w and the final property error $\delta P_{Ax \cdot Bx \cdot Cz \cdot}$:

$$|\delta P_{Ax \cdot Bx \cdot Cz \cdot Dw}| \leqq |(\partial f(A_x)/\partial A_x)\,\delta A_x| + |(\partial g(B_y)/\partial B_y)\,\delta B_y| + |(\partial h(C_z)/\partial C_z)\,\delta C_z| + |(\partial i(D_w)/\partial D_w)\,\delta D_w| \qquad (6)$$

Control factors	Each levels	Each control factor's level error	Influence and impact of each control factor's level error for the final property error δ $P_{Ax \cdot Bx \cdot Cz \cdot Dw}$	Control of all control factor errors for searching the optimum conditions with high accuracy
A	A_x	δA_x	$\partial f(A_x)/\partial A_x$	Influences and impacts of each control factor's level error were referred. Then effectively control factor's level errors for the small influences were improved.
B	B_y	δB_y	$\partial g(B_y)/\partial B_y$	
C	C_z	δC_z	$\partial h(C_z)/\partial C_z$	
D	D_w	δD_w	$\partial i(D_w)/\partial D_w)$	

Table 4. Control factors and these levels

Control factors			
Name	A	B	C
Level 1	40	8	5
Level 2	50	12	5.5
Level 3	60	16	6

Table 5. Orthogonal table and final properties ($P_{(7)ave}$=2807.5, $P_{(8)ave}$=1615.214)

L9	A	B	C	$P_{(7)}$	$P_{(8)}$
1	40	8	5	1851	526
2	40	12	5.5	1762	709
3	40	16	6	1640	956
4	50	8	5.5	2844	1091
5	50	12	6	2754	1274
6	50	16	5	2625	1512
7	60	8	6	4036	2625
8	60	12	5	3939	2799
9	60	16	5.5	3818	3046

Table 6. Control factors with level errors.

Control factors				
Name		A	B	C
Level 1		A_1=40	B_1=8	C_1=5
Level 2		A_2=50	B_2=12	C_2=5.5
Level 3		A_3=60	B_3=16	C_3=6
Control factor level error	+3%	δA_1=1.2	δB_1=0.24	δC_1=0.150
		δA_2=1.5	δB_2=0.36	δC_2=0.165
		δA_3=1.8	δB_3=0.48	δC_3=0.180
	+6%	δA_1=2.4	δB_1=0.48	δC_1=0.30
		δA_2=3.0	δB_2=0.72	δC_2=0.33
		δA_3=3.6	δB_3=0.96	δC_3=0.36
	+10%	δA_1=4.0	δB_1=0.8	δC_1=0.50
		δA_2=5.0	δB_2=1.2	δC_2=0.55
		δA_3=6.0	δB_3=1.6	δC_3=0.60

Table 7. Orthogonal table using the control factor with level errors

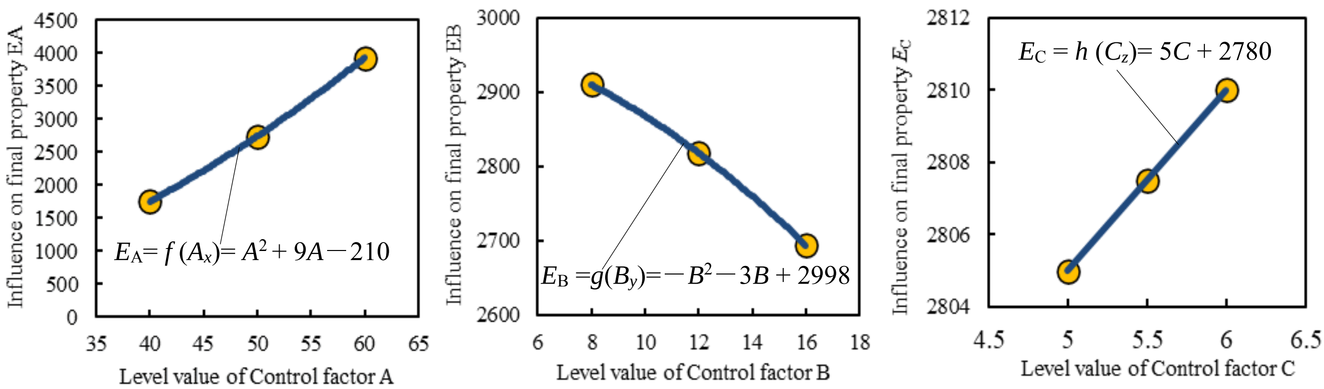
L9	A	B	C
1	$A_1+\delta A_1$	$B_1+\delta B_1$	$C_1+\delta C_1$
2	$A_1+\delta A_1$	$B_2+\delta B_2$	$C_2+\delta C_2$
3	$A_1+\delta A_1$	$B_3+\delta B_3$	$C_3+\delta C_3$
4	$A_2+\delta A_2$	$B_1+\delta B_1$	$C_2+\delta C_2$
5	$A_2+\delta A_2$	$B_2+\delta B_2$	$C_3+\delta C_3$
6	$A_2+\delta A_2$	$B_3+\delta B_3$	$C_1+\delta C_1$
7	$A_3+\delta A_3$	$B_1+\delta B_1$	$C_3+\delta C_3$
8	$A_3+\delta A_3$	$B_2+\delta B_2$	$C_1+\delta C_1$
9	$A_3+\delta A_3$	$B_3+\delta B_3$	$C_2+\delta C_2$

Table 8. The calculated the final property values $P_{(7)}$' using Table 6, Table 7 and Equation (7) . The final property values $P_{(7)}$ is in Table 5. $|P_{(7)}-P_{(7)}'|$ is the level error influence in the final property $P_{(7)}$'. The final property error $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ is calculated using Equation (11) by the proposed law of error propagation. The proposed law of error propagation is very useful for grasping of the level error influence in the final property values $P_{(7)}$'; $|P_{(7)}-P_{(7)}'| \fallingdotseq |\delta P_{Ax \cdot Bx \cdot Cz(11)}|$

Lever error	Equations (7) & (11)									
	0 %	3 %			6 %			10 %		
L9	$P_{(7)}$	$P_{(7)}$'	$\|P_{(7)}-P_{(7)}'\|$	$\|\delta P_{Ax \cdot Bx \cdot Cz(11)}\|$	$P_{(7)}$'	$\|P_{(7)}-P_{(7)}'\|$	$\|\delta P_{Ax \cdot Bx \cdot Cz(11)}\|$	$P_{(7)}$'	$\|P_{(7)}-P_{(7)}'\|$	$\|\delta P_{Ax \cdot Bx \cdot Cz(11)}\|$
1	1851	19558	104	112	2063	212	224	2210	359	374
2	1762	1861	99	117	1963	201	235	2102	341	391
3	1640	1732	92	125	1827	187	249	1956	316	415
4	2844	3005	161	169	3172	328	338	3400	557	563
5	2754	2911	156	174	3072	318	348	3293	539	580
6	2625	2774	149	181	2928	303	362	3139	514	604
7	4036	4268	232	238	4506	470	475	4833	797	792
8	3939	4165	226	243	4398	459	485	4718	779	809
9	3818	4037	219	250	4262	444	500	4572	754	833

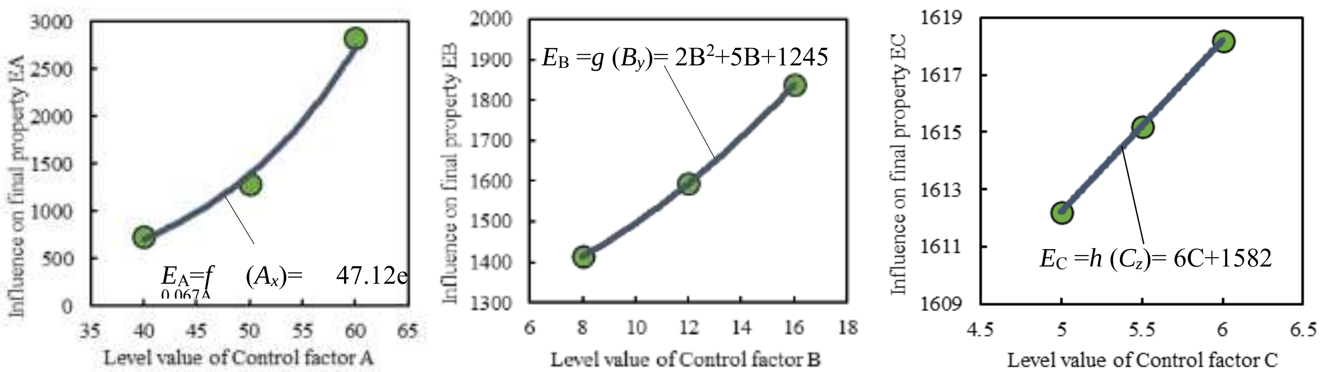
Table 9. The calculated the final property $P_{(8)}$' using Table 6, Table 7 and Equation (8) . The final property $P_{(8)}$ is in Table 5. $|P_{(8)}-P_{(8)}'|$ is the level error influence in the final property $P_{(8)}$'. The final property influence $|\delta P_{Ax\cdot Bx\cdot Cz(12)}|$ is calculated using Equation (12) by the proposed law of error propagation. The proposed law of error propagation is very useful for grasping of the level error influence in the final property $P_{(8)}$'; $|P_{(8)}-P_{(8)}'| \fallingdotseq |\delta P_{Ax\cdot Bx\cdot Cz(12)}|$.

Lever error	Equations (8) & (12)									
	0 %	3 %			6 %			10 %		
L9	$P_{(8)}$	$P_{(8)}$'	$\|P_{(8)}-P_{(8)}'\|$	$\|\delta P_{Ax\cdot Bx\cdot Cz(12)}\|$	$P_{(8)}$'	$\|P_{(8)}-P_{(8)}'\|$	$\|\delta P_{Ax\cdot Bx\cdot Cz(12)}\|$	$P_{(8)}$'	$\|P_{(8)}-P_{(8)}'\|$	$\|\delta P_{Ax\cdot Bx\cdot Cz(12)}\|$
1	526	577	52	54	634	109	112	721	195	199
2	709	771	62	64	839	130	132	939	231	233
3	956	1032	76	79	1115	159	161	1236	280	281
4	1091	1246	154	165	1423	332	346	1703	612	634
5	1274	1439	165	175	1627	353	367	1922	648	668
6	1512	1691	179	189	1894	381	394	2209	696	714
7	2625	3112	487	532	3694	1069	1128	4649	2024	2115
8	2799	3296	498	542	3888	1090	1148	4858	2059	2148
9	3046	3558	512	556	4165	1119	1176	5154	2109	2195



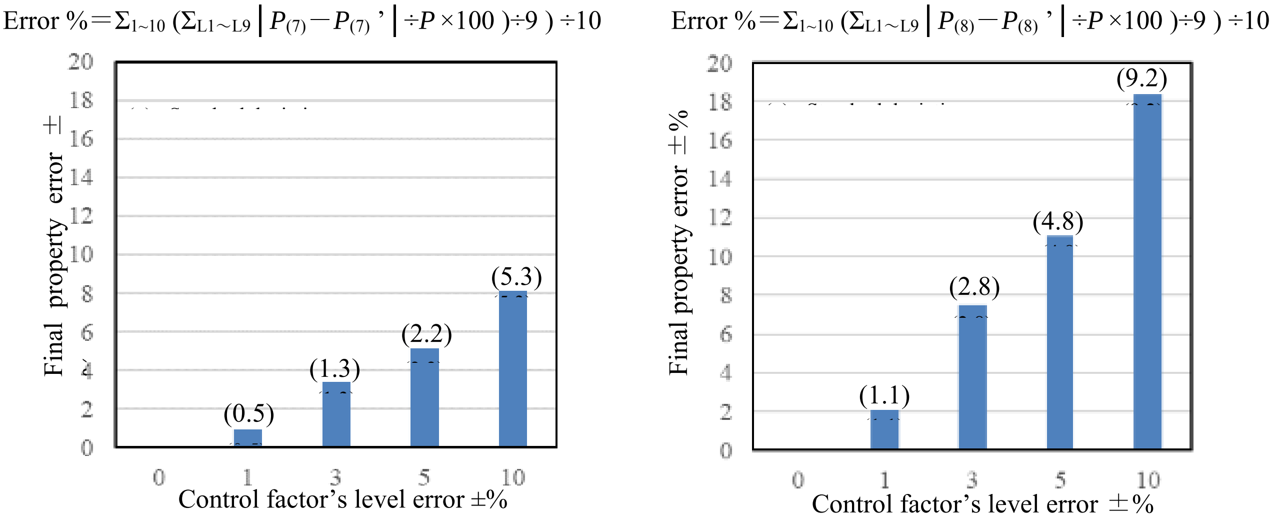
(a) Control factor A (b) Control factor B (c) Control factor C

Fig. 1: Relationship between the control factors A, B, C and the final property P using the experimental design



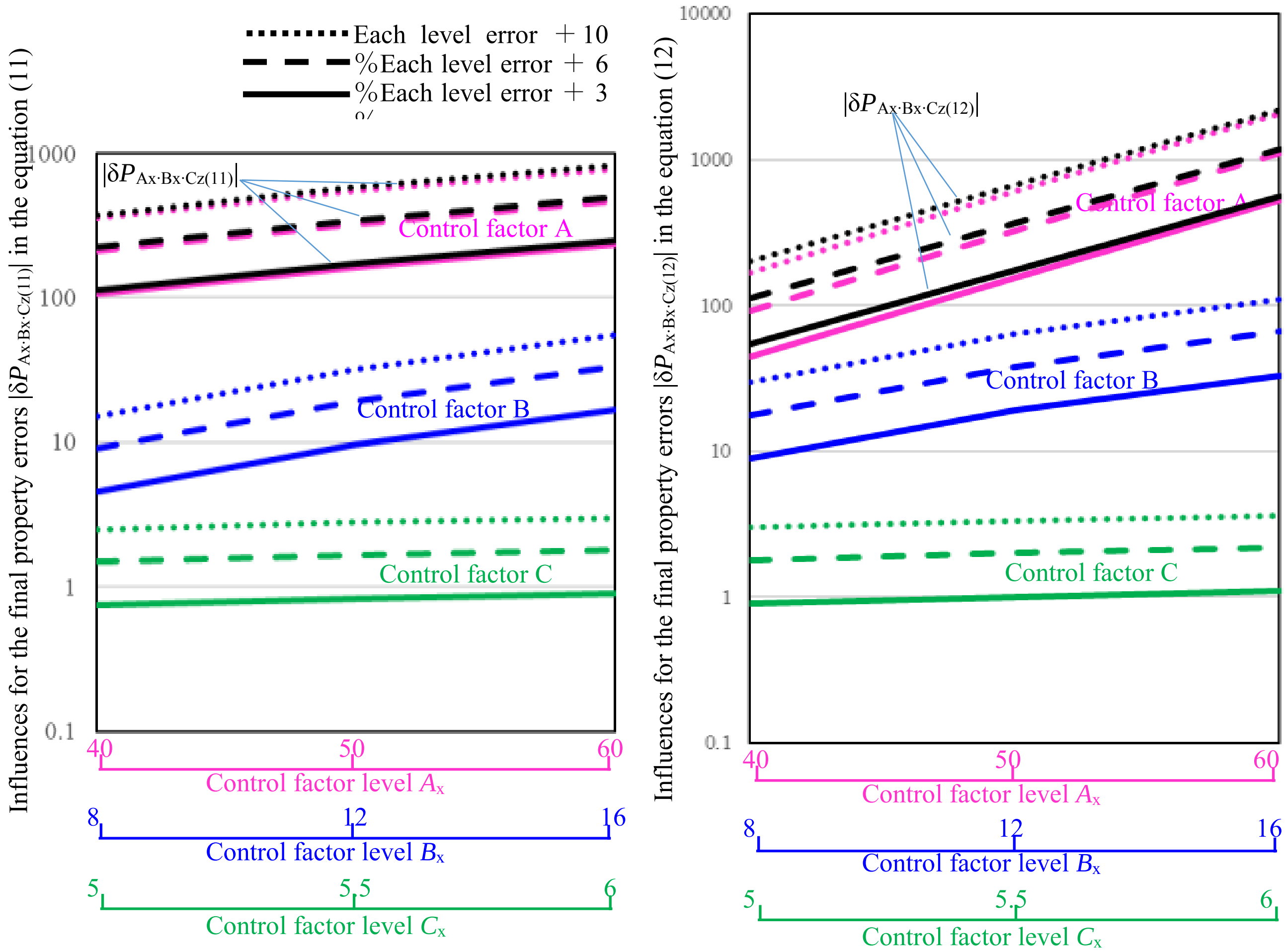
(a) Control factor A (b) Control factor B (c) Control factor C

Fig. 2: Relationship between the control factors A, B, C and the final property P using the experimental design Structural equation (8): $P_{A\cdot B\cdot C} = 6\,e^{0.1A} + 2B^2 + 5B + 6C$, and $P_{(8)ave}$=16158.214
Calculated equation (8)': $P_{A\cdot B\cdot C} = 6.0001\,e^{0.1A} + 2B^2 + 5B + 6C - 0.0001$, and $P_{(8)ave'}$= 16058.234



(a) $P_{A \cdot B \cdot C} = A^2 + 9A - B^2 - 3B + 5C - 46$ (b) $P_{A \cdot B \cdot C} = 6\, e^{0.1A} + 2B^2 + 5B + 6C$

Fig. 3: Relationship between the control factor's level error and the final property error. When the control factor's level error became gradually large, the final property error also became large. Therefore, the control of all control factor errors for searching the optimum conditions with high accuracy was required.



(a) Equations (11) (b) Equations (12)

Fig. 4: The final property errors $|\delta P_{Ax \cdot Bx \cdot Cz(11)}|$ and $|\delta P_{Ax \cdot Bx \cdot Cz(12)}|$ were influenced by 3 paragraphs the right side of the Equations (11) and (12). The influences of 3 paragraphs were clearly shown in this figure.