An Analytical Analysis of Mixed Convective MHD Casson Flow with Ramped Wall Temperature

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Abstract: - This research discusses the behavior of magnetohydrodynamic Casson fluid subjected to a ramped wall temperature along an infinitely inclined plate. Non-Newtonian fluid, specifically the Casson fluid, is employed to characterize fluid behavior in this research. The effects of porous media, chemical reactions, and radiation are considered. In practical applications, non-Newtonian fluids find use in lubrication such as grease, engine oil, etc. The Laplace transform technique has been applied during this study where ordinary differential equations are converted from the governing partial differential equations with suitable boundary conditions. These transformed dimensionless equations are subsequently solved numerically with Mathematica, and we have achieved the exact solutions of momentum, followed by energy and finally concentration. In the results section, we have analyzed the behavior of flow features under varying conditions of key parameters, including the governing parameters, the unsteadiness parameter, and the Casson parameter. All the results are shown and analyzed in graphs.

Key-Words: - Casson fluid, mixed convection, magnetohydrodynamics, chemical reaction, porous medium, Laplace Transform.

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1 Introduction

In the modern era, non-Newtonian fluids have gained significant attention for study due to their wide-ranging scientific and technological applications surpassing those of Newtonian fluids. These fluids find utility in diverse areas including personal care products, biological substances, and chemical food items. The unique behavior of non-Newtonian fluids which deviates from Newton's law of viscosity with variable viscosity dependent on stress, contributes to the growing interest among researchers and becomes popular among researchers, especially Casson fluid. The model of Casson fluid, introduced by Casson in 1959, has been designed to explain the flow characteristics of pigment–oil suspensions, [1]. The relationship between shear stress and strain in Casson fluids imparts them with their non-Newtonian rheological characteristics. This fluid is particularly suitable for applications necessitating shear-thinning behavior attributable to its high shear viscosity and yield stress, [2]. [3], studied the impact of mass and heat transfer on the Casson fluid hydro-magnetic peristaltic flow through an asymmetric channel in a rotating inclined system. [4], analyzed the effects of gravity modulation, viscous dissipation, and Joule heating on the MHD flow of Casson fluid with second-order slip velocity over a vertically moving, plate. [5] examined the flow of Casson fluid across an exponentially permeable stretched surface along a permeable medium, focusing on mass and heat transfer in a magnetohydrodynamic (MHD), involving the outcomes of velocity, temperature, and single slips. [6], initiated the magnetohydrodynamic (MHD) Casson fluid flow along an infinite vertical plate, taking into account the effects of thermophoresis, Brownian motion, and chemical reactions. To solve the governing equations numerically, the sixth-order Runge–Kutta (R–K) algorithm is employed while the Nachtsheim–Swigert (N–S) shooting iteration technique is employed as the primary method for calculations. One of the significant findings of the present study is skin friction rises with increasing values of the thermal convective parameter. However, it decreases when values of the magnetic factor are elevated, together with the Eckert number and Casson parameter. Analytical and numerical investigations were conducted using the Galerkin technique to assess the effect of the Péclet number on the convective motion within a horizontal porous layer of Casson fluid. The flow within the porous medium is described by a modified Casson-adjusted Darcy equation which accounts for the rheological properties of the Casson fluid. The results show that the stability of the system decreases as the Casson parameter increases, whereas the opposite effect is observed with the Péclet number, [7].

MHD phenomena are vital in various fields such as materials manufacturing, energy generation, magnetofluidic blood pumps, cancer therapy, biomedical flow management, separation technologies, bio-micro-fluidic devices incorporating MHD effects, and drug delivery systems utilizing MHD principles. Magnetohydrodynamics (MHD) studies how fluids with high conductivity behave when subjected to magnetization, [3]. Hannes Alfven pioneered the field of MHD, ultimately receiving the Nobel Prize in Physics in 1970 for his groundbreaking contributions. [8], established free convection flows of unsteady magnetohydrodynamics (MHD) within a porous medium passing an infinitely inclined plate, accounting for both MHD and radiation influences. [9], addressed the formulation of the free convection flow of magnetohydrodynamic through an inclined plate within Newtonian fluid such as gasoline, water, alcohol, and oil considering factors such as the inclination angle, magnetic parameter, chemical reactions, and radiation. From the results, it is concluded that velocity declines as the values of the magnetic parameter increase. This is due to the Lorentz force, a resistive type of force generated by the application of a transverse magnetic field. Additionally, it is observed that fluid motion accelerates as the inclination angle increases along the plate, with buoyancy forces coming into play while the plate is inclined from the vertical. The temperature distribution trend shows that temperature increases with higher radiation values, since radiation offers an additional means to diffuse energy, thereby thickening the thermal boundary layer. Moreover, [10], executed an analysis of dissipative heat transfer and peristaltic pumping in the framework of MHD Casson fluid flow within an inclined channel. Their findings indicated that incremental in both magnetization and Casson parameter led to an elevation in the heat transfer rate. Several intriguing articles, including, [11], [12], [13] and [14], focus on the study of MHD in the context of non-Newtonian Casson fluid.

Commonly, chemical species are defined as a chemical identity or chemical substances that have the same molecular energy level at a specified timescale which can be classified as atomic, molecular, ionic, and radical species. [15], explored the formation and impacts of chemical species on microorganisms using pulsed high-voltage discharge in water. [16], investigated the colloidal stability of nanobubbles at low temperatures and in the presence of chemical species of sulfate. [17], addressed the Jeffrey nanofluid in squeezed flow and horizontal channel, considering the effects of viscous dissipation effects involving thermo-diffusion Darcy-Forchheimer and several chemical species.

The required amount of energy in a chemical system with prospective reactants to begin a chemical reaction is defined as activation energy. This energy is calculated using the Arrhenius equation, which illustrates how rate constants vary with temperature. In areas such as chemical engineering, geothermal engineering, mechanochemistry, water and oil emulsions, and material degradation, mass transfer phenomena involving chemical reactions are critical. The relationship between chemical reactions and mass transfer is complex. This relationship can be explored in the context of mass transfer and fluid flow by varying the speeds at which reactant species are produced and consumed, involving both mass transfer and fluid flow processes, [18]. Research has been conducted to investigate double diffusions in the context of Casson fluid flow, with magnetohydrodynamic over a wedge with the occurrence of a chemical reaction and thermal radiation. An actively rising chemical reaction parameter decreases the concentration of Casson fluid, attributable to the momentum of the fluid being hindered by the chemical reaction, [19]. An analysis is carried out to investigate the impacts of chemical reactions on the two-phase viscous-Casson fluids flow between rotating parallel plates. The findings indicate that higher orders of chemical reaction increase the concentration profile, [20]. [21] considered the steady Casson liquid flow and two-dimensional (2D) laminar, along a stretched porous wedge, considering the effects of chemical reactions and buoyancy to comprehend the characteristics of the dynamic wedge. The governing equations and parameters are converted to their nondimensional forms using appropriate transformations. The Bvp4c numerical tool in MATLAB is used to evaluate the solutions. The key findings reveal that as the chemical reaction parameter elevates, the drag force on the wedge declines. The Nusselt number also declines with higher values of chemical reaction parameters. On the contrary, the Sherwood number increases with a higher chemical reaction parameter. Furthermore, while the chemical reaction parameter leads to a gradual increase in temperature, it causes a decrease in concentration.

Recent research intends to explore the characteristics of unsteady Casson fluid flow with ramped wall temperature along an infinite inclined plate. Generally, we are focusing on the Casson fluid which has a behavior that does not obey Newtonian fluid. This study considers the fluid is static when the fluid remains stationary, and the fluid has force and motion when it is subjected to radiative heat and mass transfer. Applying the Laplace transform, the exact solutions are derived for momentum, energy, and concentration equations. The results of velocity, followed by temperature, and finally concentration are shown graphically. Numerical results are exhibited to complement the analysis. These analytical solutions can assist scientists and engineers in verifying the accuracy of complex mathematical models obtained through numerical schemes. Additionally, the proposed mathematical model is expected to serve as a reference for other researchers in academia, engineering, and industry, facilitating further analysis of flow characteristics, heat transfer, and mass transfer performance of such fluids.

2 Problem Formulation

The Casson fluid flow near an infinite inclined moving plate is considered, embedded with ramped wall temperature, in a porous medium. We assume a first-order chemical reaction effect to take place and apply a magnetic field, B to the plate in this study. After simplification of the dimensionless process, the governing equations of Casson fluid, radiation heat transfer, and concentration field are presented as follows:

$$\frac{\partial u}{\partial t} = v\left(1+\frac{1}{\gamma_1}\right)\frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K} + GrT\cos\phi + GcC\cos\phi \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{(1+R)}{Pe}\frac{\partial^2 T}{\partial y^2} - \frac{Pe}{1+R}\frac{\partial T}{\partial t} \tag{2}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Pe_c}\frac{\partial^2 C}{\partial y^2} - K_r C \tag{3}$$

in which u indicates velocity, T signifies temperature and C represents concentration respectively. The other parameters such as v indicates kinematic viscosity, γ_1 denotes Casson fluid, M refers to the magnetic field, K signifies permeability of the porous medium, Gr represents the thermal Grashof number, Gc stands for the mass Grashof number, R depicts radiation parameter, Pe symbolises Peclet number of heat transfer, Pe_c is known as the Peclet number of mass transfer and Kr indicates chemical reaction parameter.

The approximation dimensionless initial value and boundary conditions (ivbc) are defined as:

$$\begin{aligned}
&u=0, \quad T=0, \quad C=0 \quad \text{for } y\ge 0, \text{ and } t\le 0\\
&u=1, \quad C=0 \quad \text{for } t>0\\
&T=t, \quad y=0 \quad \text{and } 0<t\le 1\\
&T=1, \quad y=0 \quad \text{and } t>1\\
&u, T, C\to 0, \ y\to\infty \ \text{ for } t>0
\end{aligned} \tag{4}$$

3 Problem Solution

To reduce Partial Differential Equations into Ordinary Differential Equations, the use of the Laplace transform is involved to evaluate equations (1) - (3) with its ivbc (4). The transformed equations (1) – (3) are:

$$\bar{u} = \left(\frac{1}{s} + \frac{a_1 - a_1 e^{-s}}{s^2 (s - a_2)} + \frac{a_3}{s(s - a_4)} \right) e^{-y\sqrt{\frac{\lambda + s}{\gamma}}} - \left(\frac{a_1 - a_1 e^{-s}}{s^2 (s - a_2)} \right) e^{-y\sqrt{Hs}} - \frac{a_3}{s(s - a_4)} e^{-y\sqrt{Pe_c (Kr + s)}} \tag{5}$$

$$\bar{T}(y,s) = \frac{1 - e^{-s}}{s^2} e^{-\sqrt{Hs}\, y} \tag{6}$$

$$\bar{C} = \frac{1}{s} e^{-y\sqrt{Pe_c (Kr + s)}} \tag{7}$$

where

$$H = \frac{Pe}{1 + R},\ \gamma = 1 + \frac{1}{\gamma_1},\ \lambda = M + \frac{1}{K},\ a_1 = \frac{Gr\cos\phi}{\gamma H}$$

$$a_2 = \frac{\lambda}{\gamma H},\ a_3 = \frac{Gc\cos\phi}{Pe},\ a_4 = \frac{\lambda - Pe_c Kr}{Pe_c}.$$

To solve equations (5) – (7), the inverse Laplace transform will be applied to get its analytical solutions. These solutions are:

$$C(y,t) = \left(\frac{t}{2} + \frac{y}{4}\sqrt{\frac{Pe_c}{Kr}} \right) e^{y\sqrt{Pe_c Kr}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}} + \sqrt{Krt} \right) + \left(\frac{t}{2} - \frac{y}{4}\sqrt{\frac{Pe_c}{Kr}} \right) e^{-y\sqrt{Pe_c Kr}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}} - \sqrt{Krt} \right) \tag{8}$$

$$T(y,t) = F(y,t) - F(y,t-1) H_v(t-1) \tag{9}$$

$$u(y,t) = u_1(y,t) + u_2(y,t) + u_3(y,t) - u_2(y,t-1)H_v(t-1) - u_5(y,t) - u_4(y,t) + u_4(y,t-1)H_v(t-1) \tag{10}$$

where

$$F(y,t) = \left[\begin{array}{l} \left(t + \frac{Hy^2}{2} \right) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{H}{t}} \right) \\ - y\sqrt{\frac{Ht}{\pi}} e^{-\frac{Hy}{4t}} \end{array} \right]$$

$$u_1(y,t) = \frac{1}{2} \left[\begin{array}{l} e^{y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\lambda}{\gamma t}} + \sqrt{\lambda t} \right) \\ + e^{-y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\lambda}{\gamma t}} - \sqrt{\lambda t} \right) \end{array} \right]$$

$$u_2(y,t) = \frac{a_1 e^{a_2 t}}{2a_2^2} \left[\begin{array}{l} e^{y\sqrt{\frac{\lambda + a_2}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} + \sqrt{(\lambda + a_2)t} \right) \\ + e^{-y\sqrt{\frac{\lambda + a_2}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} - \sqrt{(\lambda + a_2)t} \right) \end{array} \right] + \frac{a_1}{2a_2} \left[\begin{array}{l} e^{y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} + \sqrt{\lambda t} \right) \\ + e^{-y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} - \sqrt{\lambda t} \right) \end{array} \right] + \frac{a_1}{a_2} \left[\begin{array}{l} \left(\frac{t}{2} + \frac{y}{4\sqrt{\lambda\gamma}} \right) e^{y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} + \sqrt{\lambda t} \right) \\ + \left(\frac{t}{2} - \frac{y}{4\sqrt{\lambda\gamma}} \right) e^{-y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} - \sqrt{\lambda t} \right) \end{array} \right]$$

$$u_3(y,t) = \frac{a_3 e^{a_4 t}}{2} \left[\begin{array}{l} e^{y\sqrt{\frac{\lambda + a_4}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} + \sqrt{(\lambda + a_4)t} \right) \\ + e^{-y\sqrt{\frac{\lambda + a_4}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} - \sqrt{(\lambda + a_4)t} \right) \end{array} \right] + \frac{a_3}{2a_4} \left[\begin{array}{l} e^{y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} + \sqrt{\lambda t} \right) \\ + e^{-y\sqrt{\frac{\lambda}{\gamma}}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{1}{\gamma t}} - \sqrt{\lambda t} \right) \end{array} \right]$$

$$u_4(y,t) = \frac{a_1 e^{a_2 t}}{2a_2^2} \left[\begin{array}{l} e^{y\sqrt{Ha_2}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{H}{t}} + \sqrt{a_2 t} \right) \\ + e^{-y\sqrt{Ha_2}} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{H}{t}} - \sqrt{a_2 t} \right) \end{array} \right] + \frac{a_1}{a_2^2} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{H}{t}} \right) + \frac{a_1}{a_2} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{H}{t}} \right)$$

$$u_5(y,t)=\frac{a_3e^{a_4t}}{2}\left[\begin{array}{l} e^{y\sqrt{Pe_c(Kr+a_4)}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}}+\sqrt{(Kr+a_4)t}\right) \\ +e^{-y\sqrt{Pe_c(Kr+a_4)}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}}-\sqrt{(Kr+a_4)t}\right)\end{array}\right]$$
$$+\frac{a_3}{2a_4}\left[\begin{array}{l} e^{y\sqrt{Pe_cKr}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}}+\sqrt{Krt}\right) \\ +e^{-y\sqrt{Pe_cKr}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{Pe_c}{t}}-\sqrt{Krt}\right)\end{array}\right]$$

4 Results

In this part, we present the findings from the previous section to facilitate a clear insight into both the graphical and physical interpretations. We apply $\gamma = 0.5$, $R = 0.3$, $Gc = 2.0$, $Gr = 2.0$, $\phi = \pi/4$ and $M = 0.5$. All the values stated above are fixed across all graphs.

Figure 1 elucidates the velocity profile with the Casson parameter. The graph explains that a rise in the Casson parameter causes a reduction in velocity. This happened due to the low resistance to the yield stresses, which causes the flow field to narrow when we elevate the value of the Casson parameter. Hence, the velocity profile declines.

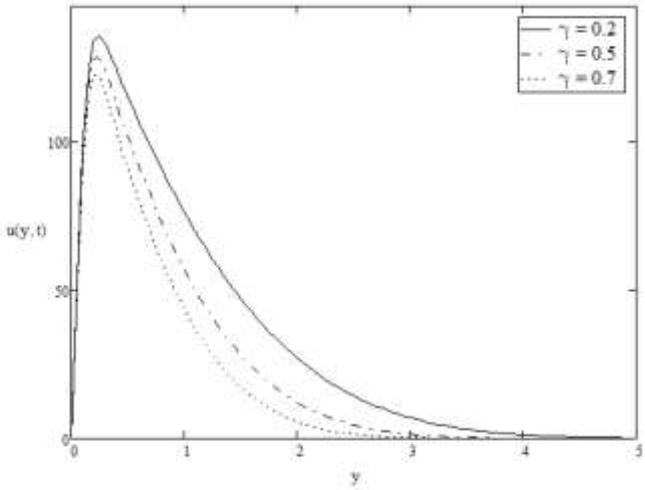

Fig. 1: Velocity field with varying values of Casson parameters, γ

Figure 2 presents the velocity profile relative to the chemical reaction. As depicted in the graph, the velocity rises with an increase in the chemical reaction. As we increase the volume of the chemical reaction, the boundary layer is affected and becomes thin due to the combined effects of concentration and hydrodynamic along the path.

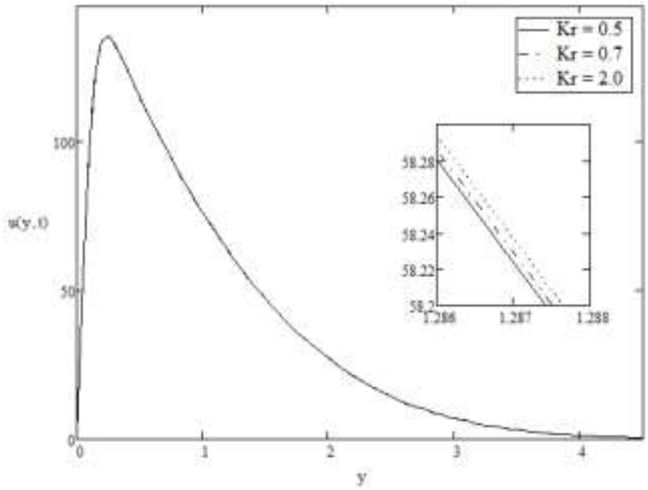

Fig. 2: Velocity field with varying values of chemical species parameters, *Kr*

Figure 3 illustrates the velocity profile with the Peclet number of heat transfer. The graph demonstrates that as the Peclet number increases, the velocity decreases. This happened due to the small amount of diffusion when the Peclet number is increased, hence the diffusive processes are delayed, and the velocity becomes dropped.

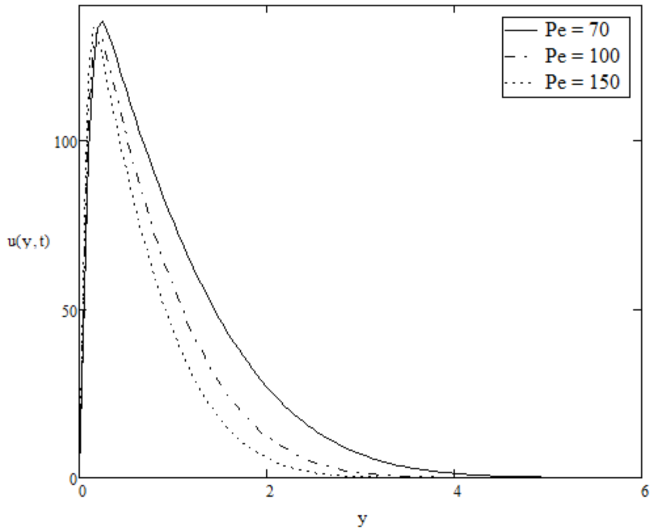

Fig. 3: Velocity field with varying values of heat Peclet number, *Pe*

Figure 4 shows the velocity profile with different values of porosity. As shown in the graph, the velocity increases with the elevation of the porosity parameter. A higher value of the porous medium creates more resistance to flow, leading to an increase in flow velocity. The frictional force opposing the flow decreases, resulting in a higher velocity.

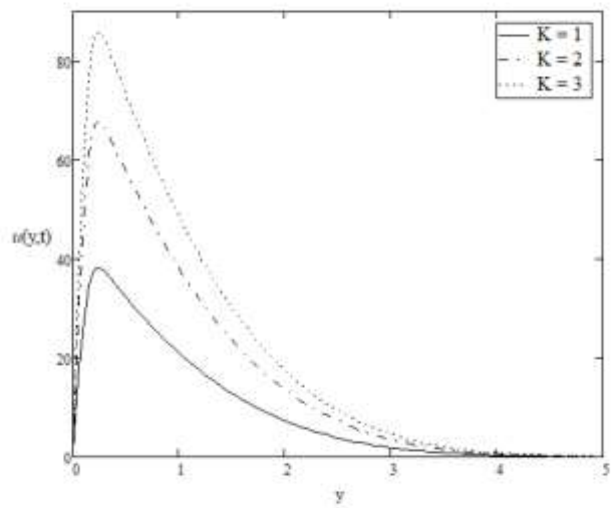

Fig. 4: velocity field with varying values of porosity, *K*

Figure 5 displays the velocity profile with various values of the magnetic field. The rising value of the magnetic field makes the velocity profiles decrease. This occurred attributable to the application of a transverse magnetic field that generates Lorentz force that retards the flow.

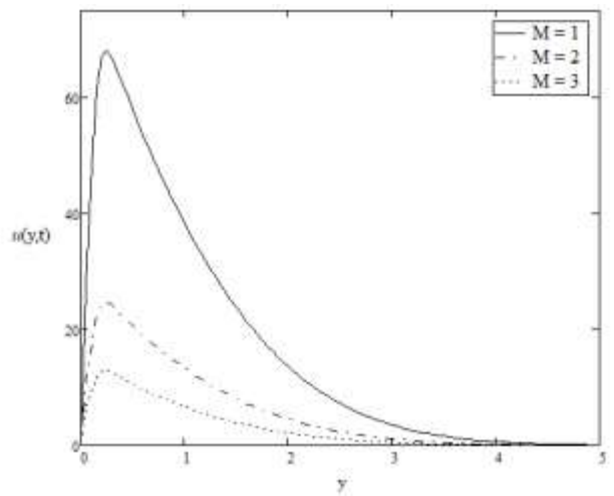

Fig. 5: Velocity field with varying values of the magnetic field, *M*

5 Conclusion

The unsteady flow of Casson fluid flow with ramped wall temperature near an infinite inclined plate is investigated. The findings illustrate the interactions among the Casson fluid, chemical reaction, and Peclet number throughout the study. Laplace transform method is employed to acquire results about velocity, temperature, and concentration. The decisive findings are summarised below:

- When the Casson parameter rises, the velocity declines. This is due to the low resistance to the yield stresses, resulting in a reduction in the flow field as the Casson parameter is elevated.
- The velocity increases as the chemical reaction intensifies. The boundary layer is affected and becomes thin due to the combined effects of concentration and hydrodynamic along the path.
- As the Peclet's number increases, the velocity decreases. This happened due to the small amount of diffusion when the Peclet number is increased, hence the diffusive processes are delayed, and the velocity becomes dropped.
- The velocity elevates with the increasing porosity parameter. The resistance was developed to flow hence the velocity increases. The frictional force opposing the flow decreases, which leads to a rise in velocity.
- The rising value of the magnetic field makes the velocity profiles decrease. It is attributable to the application of a transverse magnetic field that acts as Lorentz's force, retarding the flow.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process
During the preparation of this work, the authors used ChatGPT.com in order to improve and rephrase the sentence of the article. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Conflict of Interest

The authors have no conflicts of interest to declare.