The Effect of a Supersonic Gas Flow on Resonance Regimes of Drill String Nonlinear Vibrations

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Abstract: - This article focuses on the nonlinear effect of the excessive pressure of a supersonic gas flow along with the influence of centrifugal and axial compressive forces on resonance regimes of drill string lateral vibrations. The nonlinear mathematical model of the drill string vibrations is based on Novozhilov's nonlinear theory of elasticity. The initial curvature of the drill string is also included in the model. Two types of nonlinearities: geometric nonlinearity induced by the drill string finite deformations and the nonlinearity from the gas flow are considered. The Bubnov-Galerkin method is used to derive the equation of motion in generalized parameters. Applying the Fourier series expansion with undetermined coefficients and the harmonic balance method, the amplitude-frequency characteristics for the fundamental frequency and higher harmonics resonance are obtained. A comprehensive numerical analysis of the influence of the drilling system and gas flow parameters on the resonance curves is carried out with the use of the Wolfram Mathematica software package. Visualization of the research results is also provided.

Key-Words: drill string, vibration, resonance, nonlinearity, gas flow, harmonic balance.

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1 Introduction

Currently, the petroleum industry is the leading sector of the fuel and energy complex of Kazakhstan and continues to develop actively. This is facilitated by the growth of funding from the national sector and foreign investors, and the use of new production technologies. However, more than 50 percent of all hydrocarbons come from three large fields: Tengiz, Kashagan, and Karachaganak. According to preliminary estimates, the existing reserves in the developed fields will only last for the next several decades [1]. In view of this, there is a need to develop new oil and gas fields, increase the current pace of geological exploration, and prevent accidents that occur during well drilling, which often stop the development of entire fields. This requires extensive research related to the vibrations and stability of drill strings during well drilling to ensure a reliable and highly efficient drilling process.

To perform drilling operations with compressed air or gas acting as circulation fluids, special drilling techniques and equipment are needed. Most deep drilling operations are conducted with the use of direct circulation methods, whereas shallow depth boreholes of larger diameter are commonly drilled by rotating drill strings with reverse circulation of air or gas. The technique of reverse circulation is less studied and limited in scope but continues to be developed for broader utilization in oil and gas drilling operations, [2].

One of the first works on the application of air in drilling was [3], where the mechanics of air drilling, foam drilling, equipment requirements and various downhole problems were discussed. Experimental and numerical study of drill string nonlinear vibrations in gas drilling was performed in [4]. In this work, the authors considered lateral vibrations of a horizontal drill string taking into account the frictional interaction with the borehole wall and analyzed the effect of weight on bit and rotation speed on the drill string motion. Lateral deflection of a vertical drill string consisting of a drill pipe, drill collar and heavy weight drill pipe, its critical length, the influence of drilling parameters and stabilizers on the drill string buckling were studied in [5]. To obtain numerical results, the mass-spring model and the finite element method were utilized. The results showed that gravity, the drill-string components, and the rotary speed had a great impact on the drill string buckling behavior. Lateral vibrations of drill strings applied to air drilling were also considered in [6], where a new finite element model was developed to account for the effect of contact interaction, static-kinetic friction, mass eccentricity, and Rayleigh damping. It was obtained that the use of air as a circulating medium in highly deviated wells resulted in larger drill string lateral displacements compared to those when mud drilling was performed.

Amongst the most recent works related to studying drill string dynamics, it is worth noting the papers [7] and [8]. In [7], the internal resonance of drill string axial, torsional and lateral vibration modes with fluid-structure interaction was studied using the ANSYS software system. The numerical analysis showed that the natural frequencies of all three considered vibration modes decreased when the fluid velocity increased. At the same time, the influence of fluid pressure on natural frequencies of vibrations was negligible. Similar results were also obtained in [9], where drill string nonlinear spatial lateral vibrations under the effect of drilling fluid flow and gravitational energy of the system were investigated. Moreover, the research results demonstrated that accounting for gravitational energy led to a significant rise in the vibration amplitude in both planes considered. Returning to [8], the authors of this work constructed various schemes of a composite drill string consisting of titanium and steel materials to analyze its static and dynamic characteristics when drilling ultra-deep wells. It was shown that the use of titanium drill pipes could noticeably reduce the velocity of whirling, dynamic stress, and lateral vibration acceleration of the composite drill string.

The attention of researchers dealing with drilling problems is also paid to studying wellbore stability as in [10]. In this work, a finite element model of drill string dynamics was constructed and the wellbore stability at different parameters of the drilling system and various drilling tool combinations was studied. It was revealed that using a single-centralized drilling tool when one centralizer is located close to the drill bit increases the stability of the system compared to using a double-centralized tool.

The present work is the continuation of the authors' work, [11], in which resonance regimes and stability of drill string nonlinear lateral vibrations without the gas flow action were studied and analyzed. The obtained results illustrated a significant nonlinear effect and appearance of bifurcation zones on the drill string amplitude-frequency characteristics. In contrast to the studies mentioned above, where the authors considered the horizontal drill string, as in [4], or studied the influence of the drilling fluid pressure [7], [9], the current paper investigates the resonance of the vertical drill string under the effect of a supersonic gas flow.

As it was demonstrated in [12], the torque transmitting rotation to the drill string has less impact on its lateral frequency than the axial load. Therefore, in this paper, the nonlinear effect of the excessive pressure of the supersonic gas flow as well as the influence of centrifugal and axial compressive forces (without accounting for the torque) on the resonance regimes of the drill string lateral vibrations is studied. The lateral vibration mode is under consideration as it can cause severe instabilities of the drill string and even destruction of the well without an appearance at the surface, [13].

2 Nonlinear Mathematical Model of Drill String Vibrations

The problem of stability of the drill string lateral vibrations during shallow drilling of vertical oil and gas wells is investigated. Based on the design features, the drill string is modeled as a rotating isotropic elastic rod of length l (Figure 1). The cross-section of the drill string is assumed to be constant and symmetric.

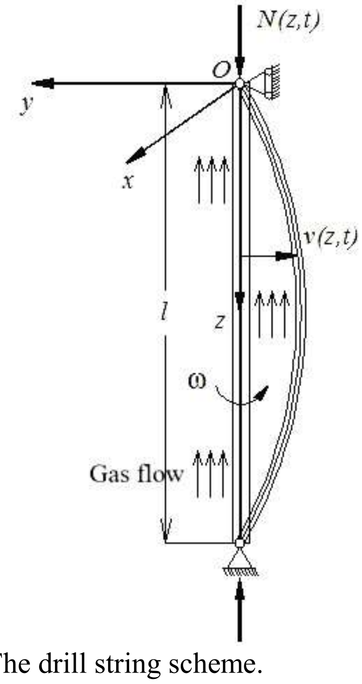

Fig. 1: The drill string scheme.

The global Cartesian coordinate system *Oxyz* is considered. The *z*-axis is directed along the drill string axis. The rotation of the drill string is taken into account by using the additional coordinate system *Ox'y'z'* that moves anticlockwise along with the rod. The *z*- and *z'*-axes have the same direction. For the case of the drill string bending in the *Oyz* plane, the position of any point of the rod is defined as:

$$\begin{cases} x' = x\cos\varphi + \left[y + v(z,t)\right]\sin\varphi, \\ y' = -x\sin\varphi + \left[y + v(z,t)\right]\cos\varphi, \\ z' = z - \dfrac{\partial v(z,t)}{\partial t} y, \end{cases} \qquad (1)$$

where $\varphi = \omega t$ is the angle of rotation, and ω is the angular speed of the rod rotation.

A variable compressive load $N(z,t)$, which is equal to the support reaction of its lower end on the bottomhole, is applied to the upper end of the drill string. The effect of the load is accounted for in the potential of external forces Π:

$$\Pi = \frac{1}{2}\int_0^l N(z,t)\left[\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial^2 v}{\partial z^2}\right)^2 y^2\right]dz \qquad (2)$$

with $v(z,t)$ denoting the displacement of the flexural center of the rod cross-section along the y-axis due to bending.

The expressions for the kinetic and potential energies of the system are given respectively by:

$$T = \frac{1}{2}\rho\int_0^l \left[A\left(\frac{\partial v}{\partial t}\right)^2 + I_x\left(\frac{\partial^2 v}{\partial z \partial t}\right)^2 + \omega^2\left(I_x + Av^2\right)\right]dz \qquad (3)$$

$$U_0 = \frac{G}{1-2\nu}\int_0^l \left[\frac{A}{2}\left(\frac{\partial v}{\partial z}\right)^4 + (1-\nu) I_x \left(\frac{\partial^2 v}{\partial z^2}\right)^2\right]dz, \qquad (4)$$

where ρ is density of the drill string material, A is the cross-section area of the drill string, I_x is the axial moment of inertia, $G = \dfrac{E}{2(1+\nu)}$ is the shear modulus, E is Young's modulus, and ν is Poisson's ratio.

The drill string deformations are assumed to be finite in accordance with Novozhilov's nonlinear theory of elasticity [14], which introduces geometric nonlinearity into the mathematical model of the drill string dynamics. In addition, the drill string axis is not rectilinear; it has an initial curvature, which is caused by technological imperfections of the drill pipe manufacturing and installation. Under the action of the longitudinal compressive load, the initial curvature $v_0(z)$ begins to have a significant effect on the dynamic behavior of the drilling system.

Then, applying the Ostrogradsky-Hamilton variation principle, the following nonlinear mathematical model of the rotating drill string lateral vibrations in the Oyz plane, taking into account the influence of external compressive load $N(z,t)$, initial curvature of the drill string $v_0(z)$ and nonlinear effect of the supersonic gas flow, is obtained:

$$\rho A\frac{\partial^2 v}{\partial t^2} + EI_x\frac{\partial^4 v}{\partial z^4} - \rho I_x\frac{\partial^4 v}{\partial z^2 \partial t^2} + \frac{\partial}{\partial z}\left(N(z,t)\frac{\partial (v+v_0)}{\partial z}\right) - \frac{EA}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3 - \rho A\omega^2 v + h\Delta P = 0, \qquad (5)$$

where h is the drill string thickness, and ΔP is the incremental pressure of the supersonic gas flow, the nonlinear expression for which is deduced with the use of postulates of the piston theory, [15]:

$$\Delta P = -P_0\kappa\left(\overline{M}\frac{\partial v}{\partial z} - \frac{\kappa+1}{4}\overline{M}^2\left(\frac{\partial v}{\partial z}\right)^2 + \frac{\kappa+1}{12}\overline{M}^3\left(\frac{\partial v}{\partial z}\right)^3\right). \qquad (6)$$

Here P_0 denotes the unperturbed flow pressure, κ is the polytropic exponent, and $\overline{M} = \dfrac{V_g}{C_0}$ is the Mach number, where V_g characterizes the speed of the unperturbed gas flow and C_0 is the speed of sound in the unperturbed flow (at infinity). Note that the gas is assumed to be incompressible while its supersonic flow is stationary and moves in the direction opposite to the drill string motion. It is worth also emphasizing that the nonlinear expression (6) is applicable only for values $\overline{M} > 2$, [15].

The piston theory is also successfully utilized for analyzing nonlinear dynamics of composite plates, as in [16], and for studying flutter and nonlinear vibrations of circular cylindrical panels [17] and cylindrical shells [18] in a supersonic gas flow.

Boundary conditions, corresponding to the simply supported rod, have the form:

$$v(z,t)=0, \quad EI_x \frac{\partial^2 v(z,t)}{\partial z^2}=0 \qquad (z=0, z=l). \quad (7)$$

3 Modeling of Resonance Regimes in a Gas Flow

To avoid excessive vibrations and damage of the drilling equipment caused by the occurrence of resonance in the drilling system, one needs to thoroughly analyze the dynamic response of drill strings and operate outside the resonance frequency range. Resonance in the system is triggered when the frequency of external excitations matches the natural vibration frequencies of the drill string under axial compression and nonlinear environmental influences. This results in a significant amplification of forced vibrations at the resonant frequency. Also, the initial curvature of the drill string substantially affects the system frequency response and can be approximated by a series of smooth functions.

The longitudinal compressive load $N(z,t)$ is conservative and changes by the periodic law with the external frequency Ω:

$$N = N_0 + N_t \cos\Omega t, \quad (8)$$

where N_0 and N_t are constant and time-varying components of the load, respectively.

To represent the overall behavior of the studied system, the nonlinear mathematical model (5) reduces to the equation in generalized parameters. According to the Bubnov-Galerkin method, the transverse displacement $v(z,t)$ in the *Oyz* plane is approximated by a finite series of periodic functions, which are in full compliance with boundary conditions (7):

$$v(z,t)=\sum_{i=1}^{n} f_i(t)\sin\left(\frac{i\pi z}{l}\right). \quad (9)$$

At the initial time, the drill string is assumed to have a half-sine wave profile. Hence, the initial curvature $v_0(z)$ is given by:

$$v_0(z)= f_0 \sin\left(\frac{\pi z}{l}\right). \quad (10)$$

Denoting by Ω_0 the natural vibration frequency of the drill string, one introduces the dimensionless time $\tau = \Omega_0 t$. Considering the one-mode approximation in (9) and substituting it along with expression (10) into Eq. (5) with subsequent application of the Bubnov-Galerkin technique, a nonlinear ordinary differential equation (ODE) in terms of the generalized time function $f(\tau)$ is obtained:

$$\frac{d^2 f}{d\tau^2}+\left(\gamma-2\beta\cos\Omega\tau\right) f+\alpha f^3 = F_0 + F_1\cos\Omega\tau, \quad (11)$$

where β is the excitation parameter, $\alpha=\alpha_2$ is the coefficient of the nonlinear component, and

$$\gamma = 1-\frac{\alpha_1^2}{3\alpha_2}, \quad \alpha=\alpha_2, \quad F_0 = F_0 + \frac{\alpha_1}{3\alpha_2}-\frac{2\alpha_1^3}{27\alpha_2^2},$$

$$F_1 = F_1 - \frac{2\alpha_1\beta}{3\alpha_2}, \quad \beta=\frac{\beta_2}{\beta_1}, \quad \Omega=\frac{\Omega}{\Omega_0},$$

$$\alpha_i = \frac{\tilde{\alpha}_i}{\Omega_0^2}, \quad F_j = \frac{\tilde{F}_j}{\Omega_0^2}, \quad i=1,2;\ j=0,1,$$

$$\beta_1 = \frac{l}{2}\left(EI_x\left(\frac{\pi}{l}\right)^4 - N_0\left(\frac{\pi}{l}\right)^2-\rho A\omega^2\right),$$

$$\beta_2 = \frac{N_t\pi^2}{4l}, \quad \Omega_0=\sqrt{\frac{\beta_1}{\delta_1}}, \quad \delta_1=\frac{\rho l}{2}\left(A+I_x\left(\frac{\pi}{l}\right)^2\right),$$

$$\tilde{\alpha}_1 = \frac{\overline{M}^2 P_0\kappa(\kappa+1)\pi h}{6l\delta_1}, \quad \tilde{\alpha}_2=\frac{3EA\pi^4}{8(1-\nu)l^3\delta_1},$$

$$\tilde{F}_0 = f_0\frac{N_0\pi^2}{2l\delta_1}, \quad \tilde{F}_1 = f_0\frac{N_t\pi^2}{2l\delta_1}.$$

Equation (11) features a symmetric nonlinear response of the system due to eliminating the quadratic term $\alpha_1 f^2$ by making the substitution $f = f + \frac{\alpha_1}{3\alpha_2}$. Hereinafter, the subscript “1” of the function $f(\tau)$ is omitted for convenience. Note that the effect of the supersonic gas flow is taken into account in the parameters α, γ, F_0 and F_1.

In ODE (11), in addition to vibrations at the same frequency as that of the excitation force, higher harmonics and subharmonics can also arise, [19]. Potential resonance conditions affecting the amplitude-frequency characteristics of the system in the supersonic gas flow can be identified by the Fourier series expansion with undetermined coefficients:

$$f(\tau) = r_0 + r_k \cos(k\Omega\tau - \varphi_k), \qquad k = 1, 2, \dots, \quad (12)$$

where r_0, r_k are the unknown vibration amplitudes, and φ_k are phase angles.

Consider the resonance of Eq. (11) at the fundamental frequency, i.e. retain only the constant component r_0 and the first harmonic in series (12). By substituting (12) into (11) and employing the harmonic balance method, that is matching the $\cos 0\Omega\tau$, $\sin\Omega\tau$, and $\cos\Omega\tau$ terms, we arrive at the following system of equations for the Fourier coefficients:

$$\begin{aligned}
&\gamma r_0 - \beta r_1 \cos\varphi_1 + \alpha r_0 \left(r_0^2 + \frac{3}{2} r_1^2 \right) = F_0, \\
&r_1 \left[-\Omega^2 + \gamma + 3\alpha \left(r_0^2 + \frac{r_1^2}{4} \right) \right] \cos\varphi_1 \\
&\quad - 2\beta r_0 = F_1, \\
&r_1 \left[-\Omega^2 + \gamma + 3\alpha \left(r_0^2 + \frac{r_1^2}{4} \right) \right] \sin\varphi_1 = 0.
\end{aligned} \quad (13)$$

By eliminating the phase angle φ_1 from system (13) through a series of trigonometric transformations, the following amplitude-frequency relations for the unknown amplitudes r_0, r_1 and the excitation frequency Ω are obtained:

$$\begin{aligned}
&r_1^2 \left(\lambda(r_0, r_1) - \Omega^2 \right)^2 = \left(F_1 + 2\beta r_0 \right)^2, \\
&r_0 \left(\lambda(r_0, r_1) - \alpha \left(2r_0^2 - \frac{3r_1^2}{4} \right) \right) \\
&\quad - \frac{\beta r_1^2}{F_1 + 2\beta r_0} \left(\lambda(r_0, r_1) - \Omega^2 \right) = F_0,
\end{aligned} \quad (14)$$

where

$$\lambda(r_0, r_1) = \gamma + 3\alpha \left(r_0^2 + \frac{r_1^2}{4} \right).$$

Since the system studied is nonlinear, then its response to the harmonic excitation has to contain not only the first harmonic, but also the higher harmonics in the Fourier expansion (12). Approximating the periodic solution of Eq. (11) in the form:

$$f(\tau) = r_0 + r_1 \cos(\Omega\tau - \varphi_1) + r_3 \cos(3\Omega\tau - \varphi_3), \quad (15)$$

and applying the harmonic balance method, i.e. matching the $\cos 0\Omega\tau$, $\sin\Omega\tau$, $\cos\Omega\tau$, $\sin 3\Omega\tau$, and $\cos 3\Omega\tau$ terms, one can provide valuable insights into the system frequency response by analyzing the following set of the derived equations relative to the unknown amplitudes r_0, r_1, r_3 and frequency Ω:

$$\begin{aligned}
&r_1^2 \left[A_1 \left(A_1 - 3A_3 \frac{r_3^2}{r_1^2} \right) + \frac{3\alpha^2}{16} r_1^2 \left(3r_3^2 - \frac{A_1}{A_3} r_1^2 \right) \right] \\
&\quad = \left(F_1 + 2\beta r_0 \right)^2, \\
&A_3^2 r_3^2 = \frac{\alpha^2}{16} r_1^6, \\
&A_1 r_1^2 - \frac{3\alpha}{32} \left(\frac{\alpha^2}{A_3} r_1^6 + 16 A_3 r_3^2 \right) \\
&\quad = \frac{1}{\beta} \left(A_0 r_0 - F_0 \right) \left(F_1 + 2\beta r_0 \right),
\end{aligned} \quad (16)$$

with

$$A_0 = \gamma + \alpha r_0^2 + \frac{3\alpha}{2} \left(r_1^2 + r_3^2 \right),$$

$$A_1 = -\Omega^2 + \gamma + 3\alpha r_0^2 + \frac{3\alpha}{4} \left(r_1^2 + 2r_3^2 \right),$$

$$A_3 = -9\Omega^2 + \gamma + 3\alpha r_0^2 + \frac{3\alpha}{4} \left(2r_1^2 + r_3^2 \right).$$

Equations (16) represent the amplitude-frequency characteristics of the resonance at higher frequencies.

The amplitude-frequency dependencies (14) and (16) are influenced by the geometric characteristics and material properties of the drill string. This enables a comprehensive analysis of how these parameters affect the resonance regimes of the drill string lateral vibrations in the supersonic gas flow.

4 Numerical Results and Discussions

To conduct numerical analysis of the primary resonance and the resonance at higher frequencies for the considered nonlinear system (11), we use the amplitude-frequency characteristics (14) and (16), which are then calculated in the Wolfram Mathematica software package. The drill string made of steel material is under consideration. The effect of the drill string parameters and the parameters of the supersonic gas flow on the resonance curves is studied.

System parameters used for conducting numerical analysis are the following: $D = 0.2\,\text{m}$ (outer diameter of the drill string), $d = 0.12\,\text{m}$ (inner diameter), $E = 2.1\times10^5\,\text{MPa}$, $\rho = 7800\,\text{kg/m}^3$, $\nu = 0.28$. The gas flow parameters are $\overline{M} = 3$, $\kappa = 1.4$, $P_0 = 1.013\times10^6\,\text{Pa}$.

Figure 2, Figure 3, Figure 4, Figure 5, Figure 6 and Figure 7 demonstrate the resonance curves for drill strings ranging from 100 m to 500 m in length and rotating at angular speed $\omega = 10\,\text{rad/s}$. The axial load is taken as $N(z,t) = 1.7 + 0.5\cos\Omega\tau\,\text{kN}$. The amplitudes r_0, r_1 and r_3 are measured in meters, whereas the frequency Ω is considered as a dimensionless quantity in the graphs below.

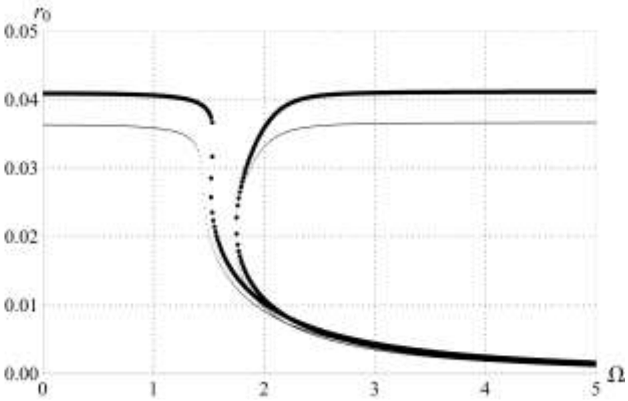

Fig. 2: Amplitude-frequency characteristics $r_0(\Omega)$ of the primary resonance with (bold line) and without (thin line) a gas flow, $l = 100\,\text{m}$, $f_0 = 0.3\,\text{m}$

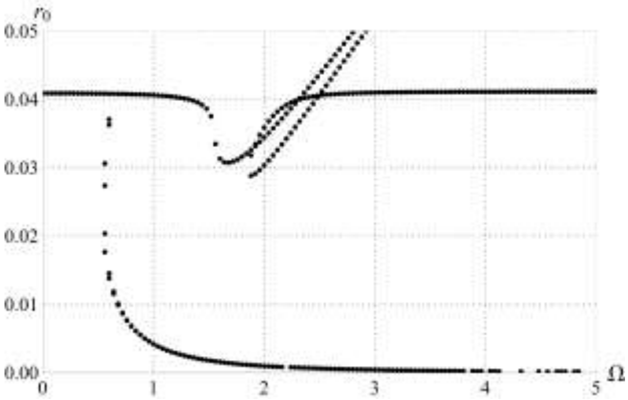

Fig. 3: Amplitude-frequency characteristics $r_0(\Omega)$ of the resonance at higher frequencies with the gas flow, $l = 100\,\text{m}$, $f_0 = 0.3\,\text{m}$

It follows from Figure 2 and Figure 3 that taking into account the third harmonic leads to the appearance of an additional curve $r_0(\Omega)$ in the region of change Ω from 0.55 to higher frequency values, and also gives the intersection of the curve branches at dimensionless frequency $\Omega = 2.3 \div 2.5$. Concurrently, the gas flow shifts the resonance curve upward along the axis representing the change in amplitude.

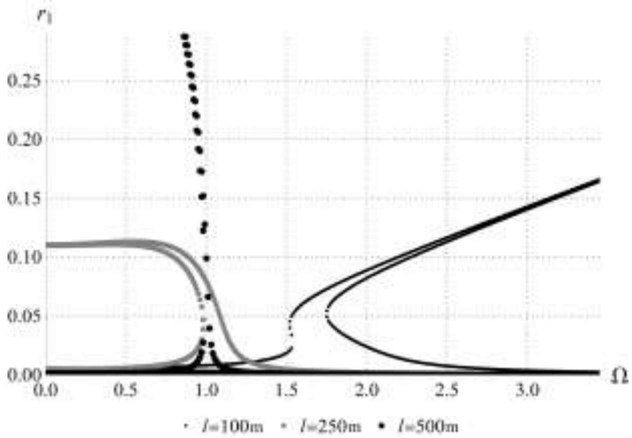

Fig. 4: Resonance curves for different lengths of the drill string with the gas flow at the fundamental frequency, $f_0 = 0.3\,\text{m}$

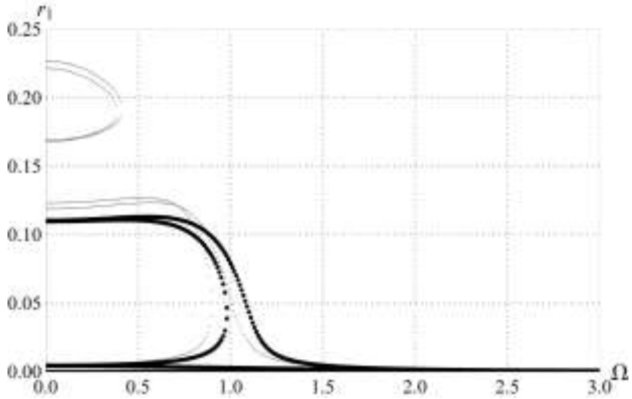

Fig. 5: Resonance curves of the drill string vibrations at the fundamental frequency with (bold line) and without (thin line) the gas flow, $l = 250\,\text{m}$, $f_0 = 0.3\,\text{m}$

As depicted in Figure 4, the resonance curves of the drill string of length $l = 100\,\text{m}$ extend towards higher frequencies. This rightward shift can be attributed to the effect of geometric nonlinearity. We observe a similar trend characterized by the appearance of resonance at higher frequencies and an enlargement of the instability region when reducing Young's modulus, e.g., selecting the duralumin material. Additionally, the initial curvature of the drill string causes an upward shift in the resonance curves, leading to higher resonance frequencies.

Notably, as the length of the drill string increases, the resonance curves shift leftward. This phenomenon can potentially destabilize the system in the low-frequency range. Furthermore, when the influence of supersonic gas flow is neglected, anomalous loop-shaped behavior is observed in the amplitude range of 0.17 to 0.23 meters at the fundamental frequency, as illustrated in Figure 5.

Figure 6, Figure 7, Figure 8, Figure 9 and Figure 10 account for the impact of supersonic gas flow on the system's behavior.

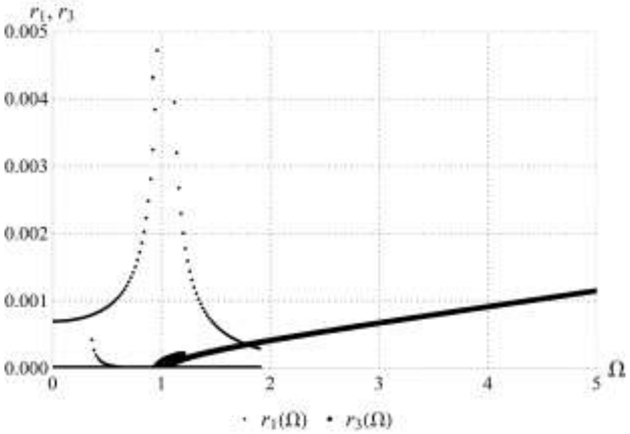

Fig. 6: The first (dotted) and third (bold dotted) harmonics of the drill string vibrations, $l = 100\,\text{m}$, $f_0 = 0.01\,\text{m}$

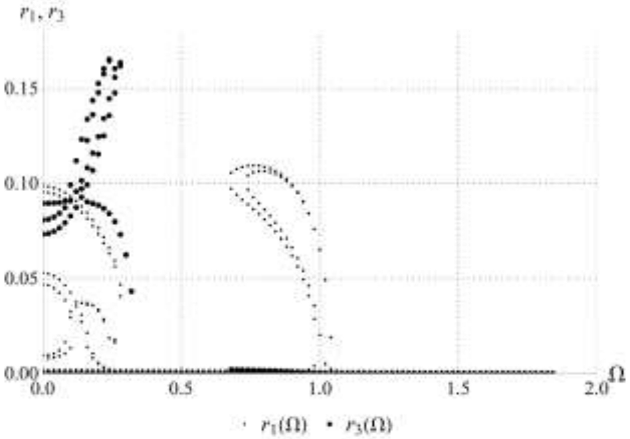

Fig. 7: Resonance curves of the first (dotted) and third (bold dotted) harmonic vibrations of the drill string, $l = 250\,\text{m}$, $f_0 = 0.01\,\text{m}$

When the third harmonic is incorporated into the solution (15), the resonance curves are shifted towards higher frequencies as the external frequency increases. This shift is accompanied by a substantial reduction in vibration amplitudes compared to those observed in the resonance curves depicted in Figure 6. This result is in good agreement with the results obtained in [10], where the gas flow was not taken into account.

As can be seen in Figure 7, a decrease in the amplitude of the primary resonance leads to the emergence of vibrations at the third harmonic frequency. Moreover, at the bifurcation points of the primary resonance, where multiple stable solutions coexist, the amplitude of the third harmonic resonance experiences a significant increase.

Figure 8 and Figure 9 demonstrate the effect of the drill string's angular speed of rotation and its wall thickness on the resonance curves at the fundamental frequency.

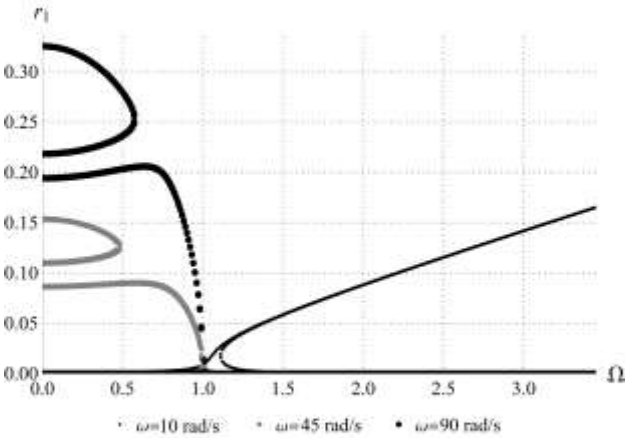

Fig. 8: Resonance curves for different values of the drill string angular speed of rotation, $l = 100\,\text{m}$, $f_0 = 0.02\,\text{m}$

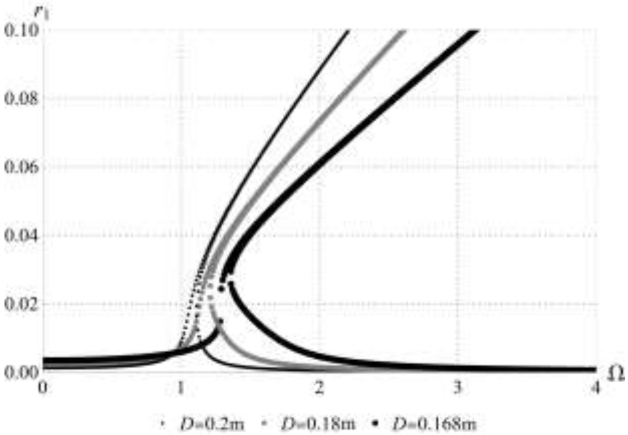

Fig. 9: Resonance curves for different values of the drill string wall thickness, $l = 100\,\text{m}$, $\omega = 10\,\text{rad/s}$, $f_0 = 0.02\,\text{m}$

Analysis of the results indicates that the system resonance occurs at lower external excitation frequencies as the drill string angular speed of rotation increases from 10 to 90 rad/s (Figure 8). Figure 9 also reveals that decreasing the drill string outer diameter shifts the resonance curves towards higher frequencies. Consequently, thinner drill string walls amplify the impact of geometric nonlinearity on the drill string vibration resonance curves.

The comparison of resonance curves for the nonlinear mathematical model (5) and its linear analog, which does not consider the finite deformations of the drill string and nonlinear influence of the gas flow, is presented in Figure 10. The results confirm the validity of the nonlinear model used for conducting the numerical simulations. Furthermore, our previous research showed that the use of the linear model could result in unstable solutions, while the consideration of geometric nonlinearity led to their stabilization, [9]. Growing discrepancy between nonlinear and linear cases when conducting the stability analysis of drill strings for longer wellbores was also demonstrated in [20].

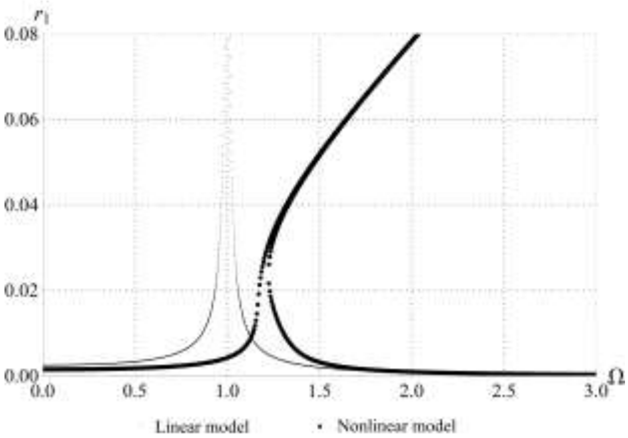

Fig. 10: Influence of nonlinearity on resonance curves, $l = 100\,\text{m}$, $f_0 = 0.02\,\text{m}$, $N_0 = 3.5\,\text{kN}$ (linear model – dotted, nonlinear model – bold dotted line)

5 Conclusion

This research explored the nonlinear impact of supersonic gas flow on the resonance behavior of drill string lateral vibrations. The lateral vibration mode was considered as it could cause severe instabilities of the drill string and even destruction of the borewell without an appearance at the surface. Utilizing the Bubnov-Galerkin technique, the nonlinear ODE in terms of the generalized time function was obtained. Potential resonance conditions affecting the amplitude-frequency characteristics of the drilling system in the gas flow were identified using the Fourier series expansion with undetermined coefficients and the harmonic balance method. The analysis of the constructed amplitude-frequency dependencies provided valuable insights into the system's nonlinear frequency response.

The numerical analysis showed that the consideration of the supersonic gas flow caused the rise of the resonance curves up along the axis of the vibration amplitude change. It was revealed that the resonance curves stretched to the left as the drill string length increased resulting in system instability in the low-frequency region. Further analysis indicated the occurrence of the system resonance at lower external excitation frequencies as the drill string's angular speed of rotation increased. It was also shown that thinner drill string walls amplified the impact of geometric nonlinearity on the drill string resonance curves. Moreover, at the bifurcation points of the primary resonance, where multiple stable solutions coexist, the amplitude of the third harmonic resonance experienced a significant increase. The comparative analysis of the nonlinear and linear models of the drill string lateral motions confirmed the validity of the numerical results obtained.

The presented work has some limitations. One of them is that the borehole interaction was not taken into account in the model. It could restrict the maximum values of the vibration amplitudes, at which the drilling process remains stable without affecting the wall integrity. Therefore, to further our research, we will delve into the nonlinear dynamics of drill strings, focusing on the influence of drilling fluid and contact interactions with borehole walls on the resonance regimes and stability.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Askat K. Kudaibergenov created the nonlinear mathematical model of the drill string lateral vibrations and determined the amplitude-frequency characteristics.
- Askar K. Kudaibergenov participated in developing the mathematical model and conducting the numerical analysis of the drill string resonance regimes.
- L. A. Khajiyeva was responsible for the conceptual framework and methodological approach of the problem studied.

All the authors took part in writing and preparation of the article.

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Conflict of Interest

The authors have no conflicts of interest to declare.