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Application of Digital Technology for Oil and Gas Fields

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Abstract: The paper investigates the problem of isothermal filtration using an approximate method for solving the theory of partial differential equations. The mathematical model under consideration is nonlinear and does not lend itself to analytical methods of solution. The results obtained indicate the need for wide application in the development of oil and gas fields in the Republic of Kazakhstan. In particular, the results of the study make it possible to solve the problems of adapting mathematical models and evaluating changes in technological indicators, which are necessary attributes in the digital technology "Information System for the Analysis of oil and Gas Field Development" (ISAR). Many problems and mathematical problems of filtration theory arose while working on specific oil and gas fields in the western region of the Republic of Kazakhstan. The above approximate solution methods have found applications not only in filtration theory, but also in other problems (geophysics, ecology, etc.)

Key-Words: absolute permeability tensor, relative phase permeability, capillary pressure, viscosity, ISAR, Darcy's laws, porous medium, technological data, saturation.

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1 Introduction

Currently, the use of digital technology for oil and gas fields is developing very intensively ("Eclipse" and "Black Oil by Schlumberger", "Tempest by Roxar VIP by Landmark" and "TimeZY by Standard Oil" and "Trust"). All the listed systems consist of basic blocks: block - technological data, block - engineering models and block - mathematical models. For predictive calculations, there are mainly block mathematical models. Approximate methods for solving nonlinear partial differential equations are mainly used to solve problems of filtration theory and numerical implementation. The structure of the study consists of the derivation of the equations of the filtration theory, the application of the variational method and the obtained scientific result formulated in the form of theorems. Similar mathematical results can be found in [1-5].

2 Problem Formulation

The theory of filtration of two immiscible liquids in a porous medium is based on the following analogues of Darcy's laws and continuity equations for each of the phases:

$$\vec{v}_i = -K_i(\nabla P_i + \rho_i \cdot \vec{g}), \frac{\partial(m \cdot \rho_i \cdot s_i)}{\partial t} + div(\rho_i \cdot \vec{v}_i) = 0 \quad i = 1,2, \tag{1}$$

where $\vec{v}_i, P_i, \rho_i$ and s_i are, respectively, the phase volume flow rates (filtration rates), pressure, density and saturation, and $s_1 + s_2 = 1$; $\vec{g}$ - acceleration vector of gravity; $m(x)$ – porosity of the medium; $K_i = k_0 \cdot k_i = k_0 \cdot f_i/\mu_i$- phase permeability tensors; $k_0(x)$ – absolute permeability tensor; $f_i = k_{0_i} \cdot \mu_i$ – relative phase permeability; μ_i– phase viscosity coefficients. In the case of non-compressibility of the liquid, it is assumed $\rho_i = const$ that the pressures differ by the amount of capillary pressure:

$$P_2 - P_1 = P_k(x, s_1). \tag{2}$$

Equations (1), (2) are a closed system of relatively unknown functions. From equations (1), (2) by introducing the following notation (see, for example, [1]):

$$a = \left|\frac{\partial P_k}{\partial s}\right| \cdot \frac{k_{0_1} \cdot k_{0_2}}{k_{0_1} + k_{0_2}} = \left|\frac{\partial P_k}{\partial s}\right| \cdot \frac{f_1 \cdot f_2}{f_2 \cdot \mu_1 + f_1 \cdot \mu_2}; \quad b = \frac{k_{0_1}}{k_{0_1} + k_{0_2}} = \frac{\mu_2 \cdot f_1}{f_2 \cdot \mu_1 + f_1 \cdot \mu_2},$$

$$\vec{f} = K \cdot \int_s^1 \nabla\left(\frac{\partial P_k}{\partial s}\right) \cdot \frac{\mu_1 \cdot f_2}{f_2 \cdot \mu_1 + f_1 \cdot \mu_2} d\xi + k_0 \cdot \frac{f_2}{\mu_2} \cdot (\nabla P_2 + (\rho_2 - \rho_1) \cdot \vec{g}) \equiv$$

$$\equiv \vec{f}_0 + k_0 \cdot \frac{f_2}{\mu_2} \cdot (\nabla P_k + (\rho_2 - \rho_1) \cdot \vec{g}); \quad \vec{F} = \vec{f}_0 - b \cdot \vec{f}$$

and as a new desired function, the reduced pressure:

$$P = P_1 - \int_s^1 \frac{\partial P_k}{\partial s} \cdot \frac{k_{0_2}}{k} d\xi + \rho_1 \cdot g \cdot h = P_1 \cdot (1 - G) + P_2 \cdot G +$$

$$+ \int_s^1 P_k \cdot G' d\xi + \rho_1 \cdot g \cdot h; \quad G = \frac{f_2 \cdot \mu_1}{\mu_2 \cdot f_1 + \mu_1 \cdot f_2}; \quad \vec{g} = g \cdot \nabla h$$

we obtain a system of equations with respect to $s(x,t), P(x,t)$:

$$\frac{\partial(ms)}{\partial t} = div\left(k_0 \cdot a(s) \cdot \nabla s + k_1 \cdot \nabla P + \overrightarrow{f_0}\right) \tag{3}$$

$$div\left(K \cdot \nabla P + \vec{f}\right) = 0 \tag{4}$$

or an equivalent system for $s(x,t), P(x,t), \vec{v}(x,t)$:

$$\frac{\partial(ms)}{\partial t} = div\left(k_0 \cdot a(s) \cdot \nabla s - b \cdot \vec{v} + \vec{F}\right), \quad (5)$$

$$div\left(K \cdot \nabla P + \vec{f}\right) = 0, \ \ \vec{v} = -\left(K \cdot \nabla P + \vec{f}\right) \quad (6)$$

The filtration flow in a given bounded region with a piecewise smooth boundary is considered $\Gamma = \partial\Omega$. Let $\Omega_T = \Omega \times (0,T)$, $\vec{n}$ - external normal to Γ. It is assumed that the boundary $\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2$ of the filtration area may consist of three, and Γ_0 corresponds to surfaces impervious to both liquids, Γ_1 – production well, Γ_2 – injection well. Then the conditions for both liquids on Γ_0 are equivalent:

$$\vec{v}_1 \cdot \vec{n}|_{\Gamma_{0T}} = -\left(k_0 \cdot a \cdot \nabla s + k_1 \cdot \nabla P + \vec{f}_0\right) \cdot \vec{n}\Big|_{\Gamma_{0T}} = 0,$$

$$\vec{v} \cdot \vec{n}|_{\Gamma_{0T}} = (\vec{v}_1 + \vec{v}_2) \cdot \vec{n}|_{\Gamma_{0T}} = -\left(K \cdot \nabla P + \vec{f}\right) \cdot \vec{n}\Big|_{\Gamma_{0T}} = 0.$$

On Γ_2 you can specify the absence of a displaced phase flow:

$$\vec{v}_1 \cdot \vec{n}|_{\Gamma_{2T}} = -\left(k_0 \cdot a \cdot \nabla s + k_1 \cdot \nabla P + \vec{f}_0\right) \cdot \vec{n}\Big|_{\Gamma_{2T}} = 0, \quad (7)$$

and the total consumption:

$$\vec{v} \cdot \vec{n}|_{\Gamma_{2T}} = -\left(K \cdot \nabla P + \vec{f}\right) \cdot \vec{n}\Big|_{\Gamma_{2T}} = V(x,t) \quad (8)$$

or the pressure of the displaced phase

$$P_2|_{\Gamma_{2T}} = P_{2_0}(x,t). \quad (9)$$

At the same time, the most difficult is the formulation of the boundary condition on $\Gamma_{1_T} = \Gamma_1 \times (0,T)$. Based on the results of work [1] on Γ_1 until the moment of breakthrough, it is possible to similarly (7), (8) set the absence of a displacement fluid flow and the total flow rate of the mixture:

$$\vec{v} \cdot \vec{n}|_{\Gamma_{1T}} = -\left(K \cdot \nabla P + \vec{f}\right) \cdot \vec{n}\Big|_{\Gamma_{1T}} = V(x,t),$$

$$\vec{v}_2 \cdot \vec{n}|_{\Gamma_{1T}} = 0, \quad (10)$$

moreover, the relation arising from (6) must be fulfilled:

$$\int_\Gamma \vec{v} \cdot \vec{n} d\Gamma = 0.$$

The resulting model is closed by setting the initial condition with respect to saturation:

$$s(x,t)|_{t=0} = s_0(x). \quad (11)$$

Algorithms for finding approximate solutions to the problem are based on the variational principle (5) – (8), (11). The application of duality principles makes it possible to obtain dual functionals, which make it possible to estimate the minimum values of the initial functionals from below.

It is assumed that the movement of an incompressible liquid in a porous medium occurs in a region of arbitrary cross-section under the action of a pressure gradient (according to Darcy's law) in the time interval [0, T]. The equivalence of the formulation of such a problem in terms of differential equations and the variational principle is shown in [2, 3]. For slow unsteady motion of the medium, the variation scheme is as follows.

Let $\{\Delta t_j^p\}$– splitting the segment [0, T]:

$$\sum_{j=1}^{p} \Delta t_j^p = T, \ \Delta t_j^p = t_j^p \text{-} t_{j-1}^p, \ t_0^p = 0, \ t_p^p = T. \quad (12)$$

Let's introduce a sequence with respect to saturation functions:

$$s^0, \ s^1, ..., s^{j-1}, \ s^j, ..., \ s^p, \quad (12')$$

such that s^j minimizes the functionality:

$$\Phi_j^p(s, s^{j-1}) = \int_\Omega \left\{\frac{m}{2} \cdot \frac{(s - s^{j-1})^2}{\Delta t_j^p} + k_0 \cdot a(s) \cdot (\nabla s)^2 + \vec{F} \cdot \nabla s - Q \cdot s\right\} dx, \quad (13)$$

where $Q = \vec{v} \cdot \nabla b$. Denote by

$$\bar{s}(\left\{\Delta t_j^p\right\}, t, x) \quad (14)$$

a piecewise linear t function. If the sequence of functions (14) converges to some function $s(x,t)$ at any partition (12), then $s(x,t)$ is called the solution of a non-stationary problem.

For an approximate solution of the variational problem (13), we construct a system of functions:

$$s_n(x,t) = \omega(x) \cdot \sum_{k=1}^{n} c_k(t) \cdot \phi_k(x) = \sum_{k=1}^{n} c_k(t) \cdot \psi_k(x),$$

$$\psi_k(x) = \omega(x) \cdot \phi_k(x), \ \ \phi_k(x) = x_1^i \cdot x_2^j \ \ (i + j \le n), \ \ c_k(t) \in R^1 \times (0,T), \quad (15)$$

complete in the space $\overset{0}{W_2^1}(\Omega$, on which it is natural to consider the functionals (13), where the function ω(x) is piecewise continuously differentiable and satisfies the conditions:

$$\omega(x) > 0, x \in \Omega; \omega(x) < 0, x \notin \Omega; \lfloor \nabla\omega \rfloor \neq 0, x \in \partial\Omega.$$

It should be noted that in (13) the functionals are undifferentiable. Therefore, based on the regularization method and from (13), we obtain finite-dimensional optimization problems. The application of the latter approach is due to the fact that the regularization method provides positive certainty of the Hessian $H(z_n)$, where

$$z_n \equiv z_n(t_j^p) = (c_1(t_j^p), c_2(t_j^p), \ldots, c_n(t_j^p)).$$

Next, the sequence is considered

$$z^{k+1}{}_n(t_j^p) = z^k{}_n(t_j^p) + \tau_k \cdot \square_k, \quad k = 0,1,2,\ldots, \tag{16}$$

where h_k defines the direction; t_k is the length of the step in the direction h_k. If the sequence (16) minimizes the functionals (13), then it is called relaxation, and the condition for stopping the construction of the relaxation sequence is usually considered as

$$\left|\Phi_j^p(z_n^N) - \Phi_j^p(z_n^{N-1})\right| < \varepsilon \|z_n^N - z_n^{N-1}\|, \tag{17}$$

which defines the function

$$s_n^N(t_j^p, x) = \sum_{k=1}^n c_k^N(t_j^p) \cdot \psi_k(x). \tag{18}$$

Minimizing sequences of the form are constructed:

$$\bar{s}_n^N(t_j^p, x) = \frac{1}{2}(s_n^N(t_j^p, x) + \square_*^j - \left|s_n^N(t_j^p, x) - \square_*^j\right|), \quad \square_*^j \in \Delta_m^j,$$

$$\Delta_m^j: 0 < \square_1^j \leq \square_2^j \leq \square_3^j \leq \ldots \leq \square_k^j \leq \ldots \leq \square_m^j = \max_{\Omega} s_n^N(t_j^p, x),$$

$$\Phi_j^p(\square_*^j) = \min[\Phi_j^p(\bar{s}_n^N(t_j^p, x), \bar{s}_n^N(t_{j-1}^p, x)) : \square^j \in \Delta_m^j]. \tag{19}$$

The functions $\bar{s}_n^N(t_j^p, x)$ are obtained from the functions $s_n^N(t_j^p, x)$ by cutting off local maxima with horizontal planes along h.

3 Main results

By virtue of (15) – (19), as well as the assumed smoothness for the given problem (5) – (8), (11) fair

Theorem 1. If there is a limit to the sequence $\bar{s}(\{\Delta t_j^p\}, t, x)$ for p→∞, independent of the method of dividing the segment [0, T], then this limit is a solution to a non-stationary problem and the estimate is valid:

$$\left|\bar{s}(\{\Delta t_j^p\}, t, x) - s(t, x)\right| < M \cdot \max \sqrt{\Delta t_j^p}.$$

Finally, according to the scheme for obtaining dual functionals for stationary problems and by introducing the condition for defrosting differential connections, the functionals (13) can be represented as an upper bound for arbitrary smooth functions μ and θ:

$$\Phi_j^p(s, s^{j-1}) = \sup_{\mu,\theta}\{\Phi_j^p(s, s^{j-1}) + G_1(\mu, \theta) - G_2(\mu, \theta)\},$$

$$\inf_s \Phi_j^p(s, s^{j-1}) = \inf_s \sup_{\mu,\theta} L_j^p(s, s^{j-1}, \mu, \theta),$$

$$L_j^p(s, s^{j-1}, \mu, \theta) = \Phi_j^p(s, s^{j-1}) + G_1(\mu, \theta) - G_2(\mu, \theta). \tag{20}$$

The given relations in (20) make it possible to investigate the dual problem and obtain a final statement regarding the original problem (5) – (8), (11).

Theorem 2. If the statement of Theorem 1 is true and the gradients of the functional $L_j^p(s, s^{j-1}, \mu, \theta)$ with respect to μ and θ are nonzero, then the following equality holds:

$$\inf_s \Phi_j^p(s, s^{j-1}) = \sup_{\mu,\theta} L_j^p(s, s^{j-1}, \mu, \theta). \tag{21}$$

The proof is based on the construction of approximate solutions in finite-dimensional spaces of functions generated by systems of linear independent elements, then minimizing sequences are constructed using the Newton process.

Equality (21) allows us to obtain lower estimates of the minimum values of the functionals $\Phi_j^p(s, s^{j-1})$. In addition, the statement of theorem 2 is a convenient tool in the study of mathematical models for anomalous liquids.

4 Conclusion

Further application of the minimum of functionals (13) is carried out as follows. Until the moment of the breakthrough, pressure and velocity are determined according to Darcy's law, then saturation is specified using the above method. This is how the task data is adjusted. This method is also convenient to use in the cyclic processing of wells. Numerical experiments with specific technological indicators and forecast calculations for real oil and gas fields in the western region of the Republic of Kazakhstan have been carried out. Due to limitations, the full results are not given, but they are presented in the scientific project.

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The author has no conflict of interest to declare that is relevant to the content of this article.