Approximation Method by Variational Splines of Moroccan's Covid Data

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Abstract: This paper is devoted to presenting a study of Covid-19 data in Morocco and analyzing the impact on the Moroccan economy. In this work, we approximate the Covid-19 data using a variational method approach, specifically through smoothing variational splines. We propose an application of the pandemic's effects on the Moroccan economy. Finally, we demonstrate the effectiveness of our method by presenting graphical results of the Moroccan data.

Key-Words: Spline approximation, variational methods, Covid data, Application to Economic.

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1 Introduction

The Covid-19 pandemic has affected nearly every country in the world within a span of just four months. By mid-April 2020, the epidemic had overwhelmed, and in many cases saturated, the hospital capacities of numerous nations, resulting in over 150,000 deaths worldwide and severely impacting the health of at least half a million people. The virus's transmission time proved to be significantly shorter than the response times of human institutions, whether they are medical, scientific, or political. The duration of the latency period, the incubation time, the length of contagiousness, the duration of hospitalization, and the delays between infection and death are all crucial parameters that must be estimated and integrated into models to understand the dynamics of Covid-19.

In addition, one of the challenges in studying this pandemic arises from the number of infected individuals, as some people contract the virus without showing symptoms while still being contagious.

The paper in [13] deals with two Canadian banks daily stock market price changes are examined by ten data mining algorithms to see which algorithm or algorithms classify the financial data well. The paper [9] explores the visualization of LI (L and I) from the perspective of big data analysis and fusion.

The author in [6] solves the second-order linear hyperbolic equations by using a new three level method based on nonpolynomial spline in the space direction and Taylor expansion in the time direction. In [11] the author proposed a method for constructing a fast-computable mathematical model of a family of energy characteristics based on its comparison with a family of integral curves in the class of linear differential equations.

In [2], we present a new method to address the statistical difficulties encountered when splicing statistical series. We explored various splicing methods in the literature and proposed an approximation method for statistical splicing of economic data using smoothing quadratic splines. We demonstrated the effectiveness of our method by presenting a comprehensive dataset of Venezuela's Gross Domestic Product (GDP) by productive economic activity from 1950 to 2005, expressed in base-year 1997 prices. Additionally, we showed results for Moroccan data across various economic activities, including GDP, agriculture, trade, and electricity generation from petroleum sources, covering the period from 1971 to 2015.

The primary goal of this investigation is to approximate Covid-19 data using the variational method approach, specifically through smoothing variational splines. For more details, reference [8] can be consulted, and for related research, reference [4] may also be useful. As a real application, like what we did in [2], we present a study of Covid-19 data in Morocco and analyze the impact on the Moroccan economy.

The information was obtained from official sources, specifically from databases published by: https://ourworldindata.org/coronavirus/country/morocco.

2 Problem Formulation

Let $p, n, m \in \mathbb{N}^*$ and $N, \mu \in \mathbb{N}^*$. We denote by $\langle\cdot\rangle_{\mathbb{R}^n}$ and $\langle\cdot,\ \cdot\rangle_{\mathbb{R}^n}$, respectively, the Euclidean norm and inner product in $\mathbb{R}^n$, such that

$$m > \frac{p}{2} + \mu \qquad (1)$$

Likewise, for any non-empty open set Ω in $\mathbb{R}^p$, we denote by $H^m(\Omega, \mathbb{R}^n)$, the usual Sobolev space of (classes of) functions u which belong to $L^2(\Omega; \mathbb{R}^n)$, together with all their partial derivatives $\partial^\beta u$, in the distribution sense, of order $|\beta| \leq m$, where $\beta = (\beta_1, \beta_2, \dots, \beta_p) \in \mathbb{N}^p$ and $|\beta| = \beta_1 + \beta_2 + \cdots + \beta_p$. This space is equipped with the norm

$$\|u\|_{m,\Omega,\mathbb{R}^n} = \left(\sum_{|\beta|\leq m} \int_\Omega \left\langle \partial^\beta u(x)\right\rangle_{\mathbb{R}^n}^2 \, dx\right)^{\frac{1}{2}},$$

the semi–norms

$$|u|_{l,\Omega,\mathbb{R}^n} = \left(\sum_{|\beta|=l} \int_\Omega \left\langle \partial^\beta u(x)\right\rangle_{\mathbb{R}^n}^2 \, dx\right)^{\frac{1}{2}},$$

with $0 \leq l \leq m$, and the corresponding inner semi–products

$$(u, v)_{l,\Omega,\mathbb{R}^n} = \sum_{|\beta|=l} \int_\Omega \left\langle \partial^\beta u(x), \partial^\beta v(x)\right\rangle_{\mathbb{R}^n} \, dx,$$

with $0 \leq l \leq m$.

When Ω is a bounded, open subset of $\mathbb{R}^p$ with Lipschitz-continuous boundary (in the J. Nečas [12] sense), it follows from the Sobolev's imbedding theorem that

$$H^m(\Omega; \mathbb{R}^n), \text{ is a subset of } C^\mu(\bar{\Omega}; \mathbb{R}^n), \qquad (2)$$

with continuous injection,

where $C^\mu(\bar{\Omega}; \mathbb{R}^n)$ stands for the space of functions with values in $\mathbb{R}^n$ which are bounded and uniformly continuous on Ω, together with all their partial derivatives of order $\leq \mu$.

Finally, we denote by P_m and $\mathbb{R}^{N,n}$, respectively, the space of all polynomials of degree $\leq m$ and the space of real matrices with N rows and n columns, with the inner product

$$\langle A, B\rangle_{\mathrm{N,n}} = \sum_{i,j=1}^{N,n} a_{ij} b_{i,j},$$

and the corresponding norm

$$\langle A\rangle_{\mathrm{N,n}} = \left(\langle \mathrm{A}, \mathrm{A}\rangle_{\mathrm{N,n}}\right)^{1/2}.$$

3 Smoothing variational splines

Let $\Upsilon \subset \mathbb{R}^n$ be either a curve or surface defined by a parametrization f belonging to $H^m(\Omega; \mathbb{R}^n)$, where Ω is a bounded, open subset of $\mathbb{R}^P$, $p = 1,2$, with Lipschitz–continuous boundary. Suppose are given two positive integers N_1 and N_2, an ordered subset $A = \{a_1, a_2, \dots, a_{N_1}\}$ of N_1 points distinct of Ω and for $i = 1,2, \dots, N_1$ let $r_i \in \mathbb{N}$ and $R = \sum_{i=1}^{N_1} r_i$, continuous linear applications L_l: $H^m(\Omega; \mathbb{R}^n) \longrightarrow \mathbb{R}^n$, such that, $L_l = \partial^{\gamma_{ij}} v(a_i)$, being

$l = r_1 + r_2 + \dots r_{i-1} + j$, $j = 1, \dots, r_i$,

and $\gamma_{ij} = ({\gamma_{ij}}^1, \dots, {\gamma_{ij}}^P) \in I$, with $I \subset \mathbb{N}^p$ and $|\gamma_{ij}| = {\gamma_{ij}}^1 + {\gamma_{ij}}^2 + \cdots + {\gamma_{ij}}^P \leq \mu$.

Let $L = (L_1, \dots, L_R)$. we suppose that

$$Ker(L) \cap P_{m-1}(\Omega; \mathbb{R}^n) = \{0\} \qquad (3)$$

Likewise, for any $i = 1, \dots, N_2$, α_i is a continuous inner semi-product in $H^m(\Omega; \mathbb{R}^n)$, we denote $\alpha = (\alpha_1, \dots, \alpha_{N_2})$.

Remark 3.1 We have the following situation, for $i = 1$, let $r_1 \in \mathbb{N}$ and we designate by $L_1, \ldots, L_{r_1}$ the linear applications associated to a_i, and for $i = 2, \ldots, N_1$ let $r_2, \ldots, r_i \in \mathbb{N}$, and $L_{r_1+r_2+\cdots r_{i-1}+1}, \ldots,$ $L_{r_1+r_2+\cdots r_{i-1}+r_i}$ the linear applications associated to a_i. From now on, to simplify, L_{a_i} or L_a will designate the family of r_i or r_a continuous linear applications associated to a_i or a, and we denote by

$$\sum_{i=1}^{N_1} \langle L_{a_i} v\rangle_{r_i,n}^2 = \sum_{a\in A} \langle L_a v\rangle_{r_a,n}^2 = \sum_{l=1}^{R} \langle L_l v\rangle_{\mathbb{R}^n}^2 = \sum_{i=1}^{N_1}\sum_{j=1}^{r_i} \langle \partial^{\gamma_{ij}} v(a_i)\rangle_{\mathbb{R}^n}^2.$$

Now we consider the following minimization problem: find $\tilde{\sigma}_{\varepsilon\tau}$ such that

$$\begin{cases} \tilde{\sigma}_{\varepsilon\tau} \in H^m(\Omega;\mathbb{R}^n) \\ \forall\, v \in H^m(\Omega;\mathbb{R}^n), \quad \tilde{J}_{\varepsilon\tau}(\tilde{\sigma}_{\varepsilon\tau}) \leq \tilde{J}_{\varepsilon\tau}(v) \end{cases} \tag{4}$$

Definition 3.1. The solution of the problem (4), if it exists, will be called the smoothing variational spline relative to A, L, α, Lf, τ and ε.

Theorem 3.1. The problem (4) has a unique solution, which is also the unique solution of the following variational problem: find $\tilde{\sigma}_{\varepsilon\tau}$ such that

$$\begin{cases} \tilde{\sigma}_{\varepsilon\tau} \in H^m(\Omega;\mathbb{R}^n) \quad \mathrm{B} \\ \forall\, v \in H^m(\Omega;\mathbb{R}^n),\ \langle L\,\tilde{\sigma}_{\varepsilon\tau}, Lv\rangle_{R,n} + \langle \tau, \alpha(\tilde{\sigma}_{\varepsilon\tau}, v\rangle_{\mathbb{R}^{N_2}} \\ \qquad + \varepsilon(\tilde{\sigma}_{\varepsilon\tau}, v)_{m,\Omega,\mathbb{R}^n} = \langle Lf, Lv\rangle_{R,n} \end{cases}$$

Proof. Taking into account (1) and that the following norm

$$v \to [[v]] = \left(\sum_{a\in A} \langle L_a v\rangle_{r_a,n}^2 + \langle \tau, \alpha(v,v\rangle_{\mathbb{R}^{N_2}} + \varepsilon |v|_{m,\Omega,\mathbb{R}^n}^2\right)^{1/2},$$

is equivalent in $H^m(\Omega;\mathbb{R}^n)$ to the norm $\|\,.\,\|_{m,\Omega,\mathbb{R}^n}$, one easily checks that the symmetric bilinear $\tilde{a}: H^m(\Omega;\mathbb{R}^n) \times H^m(\Omega;\mathbb{R}^n) \to \mathbb{R}^n$, given by

$$\tilde{a}(u,v) = \sum_{a\in A} \langle L_a u, L_a v\rangle_{r_a,n}^2 + \langle \tau, \alpha(u,v\rangle_{\mathbb{R}^{N_2}} + \varepsilon(u,v)_{m,\Omega,\mathbb{R}^n},$$

is a continuous and $H^m(\Omega;\mathbb{R}^n)$-elliptic. Likewise, the following linear form φ given by

$$v \in H^m(\Omega;\mathbb{R}^n) \to \varphi(v) = \sum_{a\in A} \langle L_a f, L_a v\rangle_{r_a,n},$$

is continuous. The result is a consequence of the Lax-Milgram Lemma (see [5]).

4 Convergence

In order to establish the convergence, we need to introduce some additional hypotheses. Let $\mathcal{D}$ be a subset of real positive numbers which 0 is an accumulation point. Suppose that, for any $d \in \mathcal{D}$, we are given A^d, $L^d, \alpha^d, \varepsilon = \varepsilon(d)$, $\tau = \tau(d)$ and $\tilde{J}_{\varepsilon\tau}^d$, satisfying the same properties as, respectively, $A, L, \alpha, \varepsilon, \tau$ and $\tilde{J}_{\varepsilon\tau}$, in the previous Section. Let $\tilde{\sigma}_{\varepsilon\tau}^d$ be the smoothing variational spline in Ω relative to A^d, $L^d, \alpha^d, L^d f, \tau$ and ε, which minimize the functional $\tilde{J}_{\varepsilon\tau}^d$ in $H^m(\Omega;\mathbb{R}^n)$.

For any $d \in \mathcal{D}$, we suppose that

$$\sup_{x\in\Omega} \max_{a\in A_L^d} \langle x - a\rangle_{\mathbb{R}^p} = d \tag{5}$$

Where $A_L^d = \{a_i \in A^d \mid |\gamma_{ij}| = 0,\ for\ some\ j = 1, \ldots, r_i,\}$ is the subset of A^d, whose points are associated to the degrees of freedom of the Lagrangian data.

Proposition 4. 1. Let $B_0 = \{b_{01}, b_{02}, \ldots, b_{0\Delta}\}$ be a P_{m-1}-unisolvent subset of points of $\overline{\Omega}$. Then, it exists $\eta > 0$ such that, if B_n designates the set of a $B = \{b_1, b_2, \ldots, b_\Delta\}$ Δ-tuples of points in $\bar{\Omega}$, verifying the following condition

$$\forall j = 1, \ldots, \Delta,\ \langle b_j - b_{0j}\rangle_{\mathbb{R}^n} < \eta,$$

The application $[[\,.\,]]_m^B$ defined for all $B \in B_\eta$ by

$$[[v]]_m^B = \left(\sum_{j=1}^{\Delta} \langle v(b_j)\rangle_{\mathbb{R}^n}^2 + |v|_{m,\Omega,\mathbb{R}^n}^2\right)^{1/2},$$

for any $v \in H^m(\Omega;\mathbb{R}^n)$, is a norm in $H^m(\Omega;\mathbb{R}^n)$, uniformly equivalent in B_η to the usual Sobolev's norm $\|\,.\,\|_{m,\Omega,\mathbb{R}^n}$.

Proof. It is analogous to Proposition 2.1 of [10].

Corollary 4.2. Suppose that (1) and $m > \frac{p}{2}$ hold. Then, it exists $\eta > 0$ and a subset A_0^d of A^d such that, for all $d \in \mathcal{D}$, $d \leq \eta$, the following application

$$[[v]]_d^0 = \left(\sum_{a\in A_0^d} \langle v(a)\rangle_{\mathbb{R}^n}^2 + |v|_{m,\Omega,\mathbb{R}^n}^2\right)^{1/2},$$

$\forall v \in H^m(\Omega,\mathbb{R}^n)$, is a norm in $H^m(\Omega;\mathbb{R}^n)$ uniformly equivalent, with respect to d, to the norm $\|\,.\,\|_{m,\Omega,\mathbb{R}^n}$.

Theorem 4.3. Suppose that the hypotheses (1) and (5) hold, and that

$$\langle \tau \rangle_{\mathbb{R}^{N_2}} = o(\varepsilon),\ d \to 0, \qquad (6)$$

and

$$\varepsilon = o(d^{-p}),\ d \to 0 \qquad (7)$$

Then

$$\lim_{d\to 0}\left\|\tilde{\sigma}^{d}_{\varepsilon\tau} - f\right\|_{m,\Omega,\mathbb{R}^n} = 0.$$

Proof. **1**) In the first step we want to prove that the approximating function is bounded. To this end, it follows

$$\left|\tilde{\sigma}^{dh}_{\varepsilon\tau}\right|^2_{m,\Omega,\mathbb{R}^n} \le |f|^2_{m,\Omega,\mathbb{R}^n} + o(1), d \to 0, \quad (8)$$

and that

$$\left\langle L^d\left(\tilde{\sigma}^{dh}_{\varepsilon\tau} - f\right)\right\rangle^2_{\mathrm{N,n}} = O(\varepsilon), d \to 0. \qquad (9)$$

2) For the second step, let $B_0 = \{b_{01}, b_{02}, \dots, b_{0\Delta}\}$, and $P_{m-1}(\Omega;\mathbb{R}^n)$–unisolvent subset of points of $\bar{\Omega}$ and let η the constant in the proposition 4.1. As Ω is an open set, there exists $\eta' \in (0,\eta]$ such that

$$\forall j = 1,\dots,\Delta,\ \bar{B}\left(b_{0j},\eta'\right) \subset \bar{\Omega}.$$

Then, from (1), one has $\forall d \in \mathcal{D}, d < \eta'$

$$\forall j = 1,\dots,\Delta,\ \bar{B}(b_{0j},\eta' - d) \subset \bar{B}(a,d).$$

With $a \in A^d_L \cap \bar{B}\left(b_{0j},\eta'\right)$.

If $\mathcal{N}_j = card\left(A^d_L \cap \bar{B}\left(b_{0j},\eta'\right)\right)$, it follows that,

$$\forall d \in \mathcal{D}, d < \eta', \forall j = 1,\dots,\Delta;$$
$$(\eta' - d)^p \le \mathcal{N}_j d^p,$$

this implies, for each $d_0 \in (0,\eta')$, that

$$\forall d \in \mathcal{D}, d \le d_0, \forall j = 1,\dots,\Delta; \qquad (10)$$
$$\mathcal{N}_j \ge (\eta' - d_0)^p d^{-p}$$

Now well, from (9) we obtain for all $j = 1,\dots,\Delta$,

$$\sum_{a\in A^d_L\cap\bar{B}(b_{0j},\eta')} \left\langle\left(\tilde{\sigma}^{dh}_{\varepsilon\tau} - f\right)(a)\right\rangle^2_{\mathbb{R}^n} = o(d^{-p}), \quad (11)$$

when $d \longrightarrow 0$.

If a^d_j is the point of $A^d_L \cap \bar{B}\left(b_{0j},\eta'\right)$ such that

$$\left\langle\left(\tilde{\sigma}^{dh}_{\varepsilon\tau} - f\right)\left(a^d_j\right)\right\rangle_{\mathbb{R}^n} = \min_{a\in A^d\cap\bar{B}(b_{0j},\eta')}\left\langle\left(\tilde{\sigma}^{dh}_{\varepsilon\tau} - f\right)(a)\right\rangle_{\mathbb{R}^n}$$

then, it's deduced from (10) and (11), that for every $j = 1,\dots,\Delta$,

$$\left\langle\left(\tilde{\sigma}^{dh}_{\varepsilon\tau} - f\right)\left(a^d_j\right)\right\rangle_{\mathbb{R}^n} = o(1), d \to 0. \quad (12)$$

Denote by B^d the subset $\{a^d_1,\dots,a^d_\Delta\}$. By applying again, the proposition 4.1, with $B = B^d$, for d sufficiently closed to 0, it results from (8) and (12) that, $\exists C > 0, \exists t > 0$,

$$\forall d \in \mathcal{D},\ d \le t,\ \left\|\tilde{\sigma}^{dh}_{\varepsilon\tau}\right\|_{m,\Omega,\mathbb{R}^n} \le C,$$

this means, the family $\left(\tilde{\sigma}^{dh}_{\varepsilon\tau}\right)$ with $d \in \mathcal{D}, d \le t$ is bounded in $H^m(\Omega;\mathbb{R}^n)$.

Hence, there exist a subsequence $\left(\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}\right)_{l\in\mathbb{N}}$, extracted from the such family, with $\varepsilon_l = \varepsilon(d_l)$, $\tau_l = \tau(d_l)$, $\lim_{l\to+\infty} d_l = 0$, $\lim_{l\to+\infty} h_l = 0$, and an element $f^* \in H^m(\Omega;\mathbb{R}^n)$ such that

$$f^* = \lim_{l\to+\infty} weakly\ \tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}\ in\ H^m(\Omega;\mathbb{R}^n) \quad (13)$$

3) In the third step, by reasoning by reduction to absurd let prove that $f^* = f$. In fact, suppose that $f^* \neq f$. From the continuous injection of $H^m(\Omega;\mathbb{R}^n)$ into $C^0(\bar{\Omega};\mathbb{R}^n)$ there exist $\theta > 0$ and an open set $w \subset \Omega$ such that

$$\forall x \in w,\ \langle f^*(x) - f(x)\rangle_{\mathbb{R}^n} > \theta.$$

Such injection is also compact, from (13) it follows that $\exists l_0 \in \mathbb{N}, \forall l \ge l_0, \forall x \in w$,

$$\left\langle\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}(x) - f^*(x)\right\rangle_{\mathbb{R}^n} \le \frac{\theta}{2}.$$

Then, for all $l \ge l_0$ and $x \in w$, it obtains that

$$\begin{aligned}\langle\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}(x) - f(x)\rangle_{\mathbb{R}^n} &\ge \langle f^*(x) - f(x)\rangle_{\mathbb{R}^n} \qquad (14)\\ &\quad - \langle\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}(x) - f^*(x)\rangle_{\mathbb{R}^n}\\ &> \frac{\theta}{2}.\end{aligned}$$

Now well, for l sufficiently great, there exists a point $b^{d_l} \in A^d \cap w$ such that

$$\left\langle\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}\left(b^{d_l}\right) - f\left(b^{d_l}\right)\right\rangle_{\mathbb{R}^n} = o(1), l \to +\infty,$$

this contradicts (14). For consequently, $f^* = f$.

4) From (13), given that $f^* = f$ and as $H^m(\Omega;\mathbb{R}^n)$ it's compactly in $H^{m-1}(\Omega;\mathbb{R}^n)$ it follows that

$$f = \lim_{l\to+\infty}\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}\ \text{ in }\ H^{m-1}(\Omega;\mathbb{R}^n).$$

Then,

$$\lim_{l\to+\infty}\left(\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}, f\right)_{m-1,\Omega,\mathbb{R}^n} = \|f\|^2_{m-1,\Omega,\mathbb{R}^n}, \quad (15)$$

from what, using again (13) and that $f^* = f$, it obtains that

$$\begin{aligned}\lim_{l\to+\infty}\left(\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}, f\right)_{m,\Omega,\mathbb{R}^n} &\qquad (16)\\ = \lim_{l\to+\infty}\Big(\left(\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}, f\right)_{m,\Omega,\mathbb{R}^n} &- \left(\tilde{\sigma}^{d_l h_l}_{\varepsilon_l\tau_l}, f\right)_{m-1,\Omega,\mathbb{R}^n}\Big)\\ = |f|^2_{m,\Omega,\mathbb{R}^n}&\end{aligned}$$

Also, given that, for all $l \in \mathbb{N}$, one has

$$\left|\tilde{\sigma}_{\varepsilon_l \tau_l}^{d_l h_l} - f\right|_{m,\Omega,\mathbb{R}^n}^2 = \left|\tilde{\sigma}_{\varepsilon_l \tau_l}^{d_l h_l}\right|_{m,\Omega,\mathbb{R}^n}^2 + |f|_{m,\Omega,\mathbb{R}^n}^2 - 2\left(\tilde{\sigma}_{\varepsilon_l \tau_l}^{d_l h_l}, f\right)_{m,\Omega,\mathbb{R}^n},$$

it deduces from (8) together with (16), that

$$\lim_{l \to +\infty} \left|\tilde{\sigma}_{\varepsilon_l \tau_l}^{d_l h_l} - f\right|_{m,\Omega,\mathbb{R}^n} = 0.$$

and using (15), implies that

$$\lim_{l \to +\infty} \left\|\tilde{\sigma}_{\varepsilon_l \tau_l}^{d_l h_l} - f\right\|_{m,\Omega,\mathbb{R}^n} = 0.$$

5) In the last step, finally, it is concluded by reasoning by reduction to absurd, that fulfils the result.

5. The impact of the pandemic on the Moroccan economy

The Covid-19 pandemic has drastically altered human life since its emergence in China at the end of 2019. Economically, it has triggered the most severe crisis since at least the Second World War. The pandemic is a global phenomenon of immense scale. Its rapid spread, significant danger, and ease of transmission have disrupted even the most effective healthcare systems in developed countries worldwide. The COVID-19 pandemic has had harmful health and economic consequences, causing unprecedented upheavals in global economies and international trade. Beyond its devastating health impacts, COVID-19 has severely affected the economies of entire countries. It has led to a sharp contraction in global economic activity and disrupted international trade and financial exchanges.

5.1. The negative effects of confinement on the digital economy.

At the macroeconomic level, the containment measures implemented to manage the spread of the pandemic, combined with the effects of drought, led to a significant economic downturn in 2020. The national economy contracted by 6.3%, in stark contrast to the 2.6% growth recorded in 2019. This demonstrates that the pandemic's impact on health is not the only factor influencing the economy; government policies and other determinants also play a crucial role. The containment measures, while necessary for public health, have rapidly produced adverse effects on the economy.

5.1.1. Tourism.

The tourism sector, which contributes around 8% to the country's GDP, has been one of the most severely affected by the health crisis. All related activities, including hotels, restaurants, travel agencies, and car rental services, have experienced significant distress. The sector's total turnover loss is estimated at 34 billion dirhams in 2020 across all branches. The near-total lockdown imposed in response to the pandemic led to a dramatic 90% decrease in international tourist arrivals compared to the previous year.

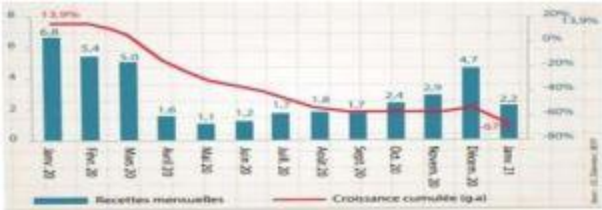

Figure 1. Evolution of tourism receipts.

5.1.2. Transportation.

Receipts for the entire land, marine, rail, and air transport sectors have declined for both passenger and freight services. Air transport is suffering due to the implemented precautionary measures and a drop in demand. According to the International Air Transport Association, the pandemic could cause significant losses in Morocco and a sharp reduction in passenger numbers. On March 15, 2020, the government suspended all international flights without announcing an expected date for their resumption. Road and rail transport are also expected to be affected by the crisis as a result of the applied measures.

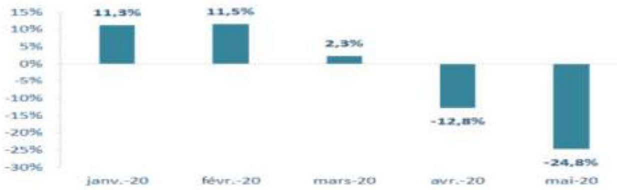

Figure 2. Evolution of cumulative tourist receipts in Morocco.

5.1.3. Manufacturing industries.

The effect of the coronavirus is evident in two main areas: the supply of raw materials and inputs, which are becoming increasingly scarce, and the decline in foreign demand. As a result, certain industries are at a standstill, such as the automotive sector, which is the country's leading export industry. Any reduction in its activity will have a significant impact on the trade balance. Other sectors, such as textiles, are struggling to find markets, while some risk being blocked due to the lack of raw materials and intermediate products, like the agri-food sector, which represents 25% of Morocco's industrial GDP and provides more than 110,000 jobs.

5.1.4. Service sector.

Business services and trade are among the sectors most affected by the health crisis. Among those still operating, 43% of companies have reduced their production to adapt to the conditions imposed by the situation. As a result, 27% of companies have temporarily or permanently reduced their workforce. Industrial companies have reduced their workforce by 22%, resulting in the elimination of 195,000 jobs.

5.1.5. Trade.

The crisis is indeed impacting Morocco's foreign trade, which represents 32% of GDP. In terms of the trade balance, a slowdown in exports is expected due to the disruption of supply chains,

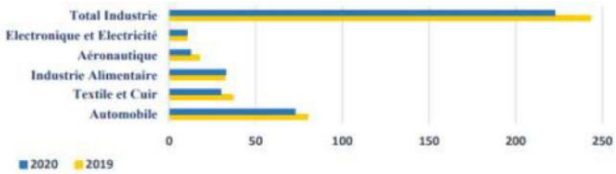

Figure 3. Evolution of exports in the industrial sector.

longer processing times, and the decline in foreign demand for Moroccan goods.

5.2. The positive effects of confinement on the digital economy.

The effects of the Covid-19 pandemic have driven an increase in household equipment among Moroccans, particularly in computer and telephone devices in 2020, according to a survey published by the National Telecommunications Regulatory Agency (ANRT). The survey also indicates that access to the internet and the use of e-commerce in Morocco have risen significantly. Moroccans were heavily equipped with PCs and tablets in 2020, the year of confinement. While the household equipment rate stagnated at 60% in 2019, it saw a substantial increase in 2020, reaching 64%.

Additionally, the pandemic's impact on teleworks, distance education, and e-commerce led to an expansion of the internet community by 3.5 million more people than in 2019. Within this community, social network usage was widespread, reaching 98% in 2020. Except for individuals aged over 75, all age groups had a social network usage rate exceeding 96%. The survey also reveals that one in four individuals made an online purchase in 2020. Among them, 78% made two to five purchases, 7% made more than 10, and 10% made just one purchase.

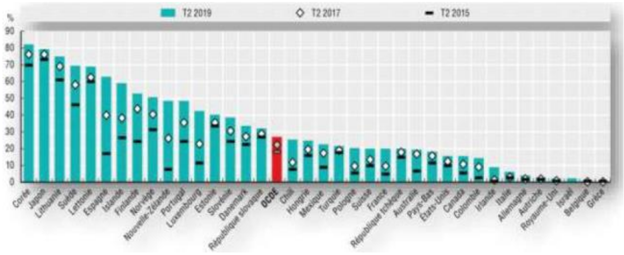

Figure 4. Total number of fixed broadband subscriptions.

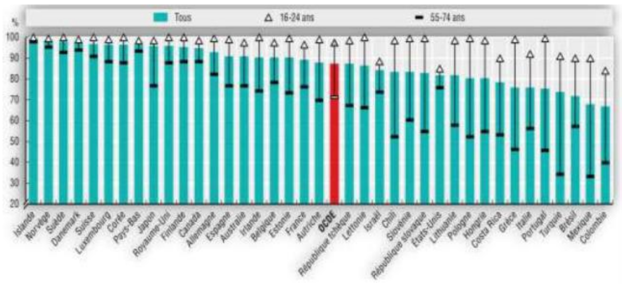

Figure 5. Internet users by age.

5.3. Case studies.

The purpose of this investigation is to present an approximation method for COVID-19 data in Morocco. We demonstrate the effectiveness of our method by presenting graphical results of Moroccan data from 2020 to 2022.

Figure 6 shows the data and its approximating curve defined by a smoothing quadratic spline for the year 2020.

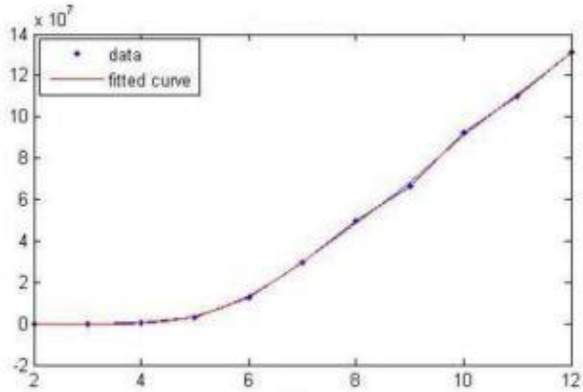

Figure 6. Moroccan covid data in 2020 and its approximating curve defined by a smoothing variational spline.

Figure 7 presents the data and its approximating curve defined by a smoothing quadratic spline for the year 2021.

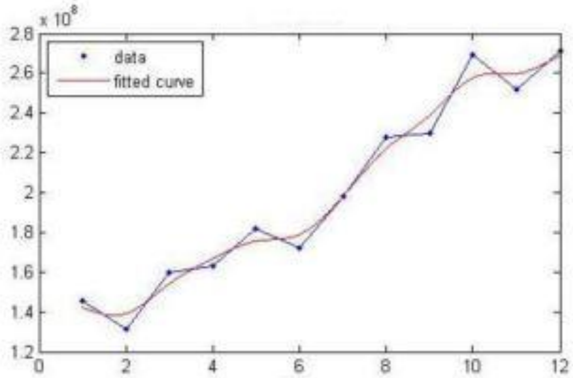

Figure 7. Moroccan covid data in 2021 and its approximating curve defined by a smoothing variational spline.

Figure 8 shows the data and its approximating curve defined by a smoothing quadratic spline for the year 2022.

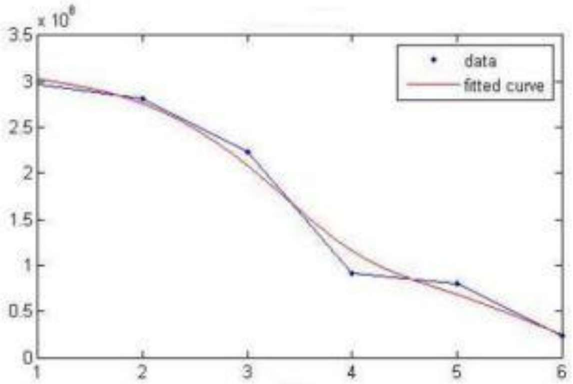

Figure 8. Moroccan covid data in 2022 and its approximating curve defined by a smoothing variational spline.

Remark: To demonstrate the generality of our spline functions, this approximation study can also be conducted using classical methods, such as fitting with quadratic splines. We used MATLAB and Mathematica programs to compute the functions.

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