Moving Horizon Fault Estimation of a Hybrid System using the Switched ARX-Laguerre Model

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Abstract: Using moving horizon Fault estimation, we propose in this article a fault estimation of a linear hybrid system using a reduced complexity SARX-Laguerre model. The linear hybrid systems are approximated by a SARX-Laguerre model (Switched ARX-Laguerre).This latter is obtained by expanding a discrete time SARX model parameters on Laguerre orthonormal bases. The resulting model ensures an efficient complexity reduction with respect to the classical SARX model. This parametric complexity reduction is still subject to an optimal choice of the Laguerre poles defining Laguerre bases. Therefore, we propose in this paper to identify the parameters of the SARX-Laguerre model by using a recursive algorithm to identify the Fourier coefficients and a metaheuristic algorithm to identify the poles. The proposed model is built from the system input / output observations and is used to develop a fault estimation scheme. The scheme of the fault estimation based on the Moving Horizon fault Estimation (MHE). The proposed fault estimation using moving horizon procedure is tested on numerical simulation.

Key-Words: - Hybrid system, Switching models, Orthogonal bases, SARX-Laguerre model, Moving Horizon, Fault Estimation.

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1 Introduction

Modern technological systems always present an increase on complexity and become more and more susceptible to faults. Therefore, in the last two decades considerable research has been focused on model based fault estimation, [1], [2], [3], [4], [5], [6], [7], [8], [9]. The most important work in fault estimation uses the moving horizon faults estimation (MHE) to optimize faults. The MHE method is based on an estimate over a horizon using the past input and output information and with this data, it is able to estimate the current and past states based on a model of the system. This estimator is formulated to minimize the quadratic estimation error between the system output and the model output over a horizon, [10].

The goal of this work is to synthesize a moving horizon fault estimation (MHE) for linear hybrid systems using a reduced complexity switching model. A hybrid system is a dynamic system that explicitly and simultaneously involves continuous and discrete behaviors. These systems are classically consisting of continuous processes interacting with or supervised by discrete processes. Switched systems form an important class of hybrid systems, [11], [12], [13], [14],i which consist of a finite number of subsystems and a switching rule indicating the active subsystem at each instant, [15]. The MHE is used to estimate the fault actuator for a class of linear hybrid system known as switching system by solving an optimization problem over a past time horizon under certain constraints. The reduced complexity model used in the fault estimation is the SARX-Laguerre obtained after the expansion of a Switched ARX model (SARX) on Laguerre orthonormal bases by filtering the input and the output of every submodel on two independent Laguerre bases, [16], [17].

In last recent years, the use of orthogonal basis in modelling both of linear systems, [18], [19], and nonlinear systems, [16], [20], [21], has experienced a very great development due to its capacity in the reduction of parametric complexity. This modelling is applied by filtering the input of the linear model as FIR model, [22], or ARX model, and nonlinear model as Volterra model by decomposing each kernel on Laguerre basis, [23]. This input filtering guarantees a parametric reduction which can be significant when the considered system is linear with a dominant first order dynamic. However, in the case of scattered poles and an oscillating system the Laguerre model requires a huge number of Laguerre functions and then many numbers of parameters to represent systems with various representative modes. To overcome this drawback, the study, [24], propose an alternative solution to represent any complex dynamic linear systems with a reduced parameter complexity model using the Laguerre bases. This idea is based on the expansion of the linear model on two independent Laguerre bases by filtering the input and the output of the linear model as ARX model. Due to its high efficiency in reducing the parameter complexity, this approach was extended to the case of linear multiple model approach were a ARX-Laguerre multiple model approach is proposed by [16], and to the case of Multiple Input Multiple Output (MIMO) systems, where a ARX-Laguerre MIMO model is proposed by [25]. Thus, to reduce the number of parameters of the SARX-Laguerre model an optimal choice of the poles is necessary. In this paper we propose to use a meta-heuristic algorithm to optimize the SARX-Laguerre poles and using recursive procedure to identify the Fourier coefficients.

This paper is organized as follows: in section 2, we present the recursive representation of the SARX-Laguerre model obtained after the expansion of every submodel on two Laguerre bases. In section 3, firstly we focus on the parametric identification of the SARX-Laguerre model where we propose a recursive algorithm to estimate the Fourier coefficients so that the regularized square error will be minimized. Secondly, we use the genetic algorithm to optimize the poles. In section 4, we present the horizon fault estimation scheme of a linear hybrid system using SARX-Laguerre model. Finally, the proposed MHE using SARX-Laguerre model scheme is validated by a numerical example.

2 SARX-Laguerre Model

After the expansion of the strictly causal discrete time linear hybrid systems known as Switched Auto Regressive eXogenous (SARX) models on Laguerre orthonormal independent bases in the goal to reduce the parameters complexity, we obtain the flowing recursive representation of the new model entitled SARX-Laguerre model.

$$\begin{aligned} X(k+1) &= A^{\lambda_k}X(k) + b_y^{\lambda_k}y(k) + b_u^{\lambda_k}u(k) \\ y(k) &= (C^{\lambda_k})^T X(k) \end{aligned} \quad (1)$$

with, the state vector $X(k)$ and the parameters vector C^{λ_k} are a $\left(N_a^{\lambda_k} + N_b^{\lambda_k}\right)$ dimensional vectors, where $N_a^{\lambda_k}$ and $N_b^{\lambda_k}$ are the truncated orders, defined as:

$$X(k) = \begin{bmatrix} x_{0,y}(k) \\ \vdots \\ x_{N_a^{\lambda_k}-1,y}(k) \\ x_{0,u}(k) \\ \vdots \\ x_{N_b^{\lambda_k}-1,u}(k) \end{bmatrix}, C^{\lambda_k} = \begin{bmatrix} g_{0,a}^{\lambda_k} \\ \vdots \\ g_{N_a^{\lambda_k}-1,a}^{\lambda_k} \\ g_{0,b}^{\lambda_k} \\ \vdots \\ g_{N_b^{\lambda_k}-1,b}^{\lambda_k} \end{bmatrix} \quad (2)$$

and A^{λ_k} is a square matrix of dimension $N_a^{\lambda_k} + N_b^{\lambda_k}$ defined as:

$$A^{\lambda_k} = \begin{bmatrix} A_y^{\lambda_k} & \underline{0}_{N_a^{\lambda_k},N_b^{\lambda_k}} \\ \underline{0}_{N_b^{\lambda_k},N_a^{\lambda_k}} & A_u^{\lambda_k} \end{bmatrix} \quad (3)$$

where $\underline{0}_{N_a^{\lambda_k},N_b^{\lambda_k}}$ and $\underline{0}_{N_b^{\lambda_k},N_a^{\lambda_k}}$ are two null matrices of dimensions $(N_a^{\lambda_k} \times N_b^{\lambda_k})$ and $(N_b^{\lambda_k} \times N_a^{\lambda_k})$ respectively, and $A_y^{\lambda_k}$ and $A_u^{\lambda_k}$ are two square matrices of dimension $N_a^{\lambda_k}$ and $N_b^{\lambda_k}$ respectively. The matrix $A_y^{\lambda_k}$ is given, for $r = t = 1, \ldots, N_a$, by the following relation:

$$A_y^{\lambda_k}(r,t) = \begin{cases} \xi_a^{\lambda_k} & if\ r = t \\ (-\xi_a^{\lambda_k})^{(r-t-1)}(1-(\xi_a^{\lambda_k})^2) & if\ r > t \\ 0 & if\ r < t \end{cases}, \quad (4)$$

and the matrix $A_u^{\lambda_k}$ is given, for $r = t = 1, \ldots, N_b$, as:

$$A_u^{\lambda_k}(r,t) = \begin{cases} \xi_b^{\lambda_k} & if\ r = t \\ (-\xi_b^{\lambda_k})^{(r-t-1)}(1-(\xi_b^{\lambda_k})^2) & if\ r > t \\ 0 & if\ r < t \end{cases} \quad (5)$$

The vectors $b_y^{\lambda_k}$ and $b_u^{\lambda_k}$ are of dimension $N_a^{\lambda_k} + N_b^{\lambda_k}$ and given by:

$$b_y^{\lambda_k} = \sqrt{1-(\xi_a^{\lambda_k})^2} \begin{pmatrix} 1 \\ -\xi_a^{\lambda_k} \\ (-\xi_a^{\lambda_k})^2 \\ \vdots \\ (-\xi_a^{\lambda_k})^{N_a-1} \\ \underline{0}_{N_b,1} \end{pmatrix} \quad (6)$$

$$b_u^{\lambda_k} = \sqrt{1-(\xi_b^{\lambda_k})^2} \begin{pmatrix} \underline{0}_{Na,1} \\ 1 \\ -\xi_b^{\lambda_k} \\ (-\xi_b^{\lambda_k})^2 \\ \vdots \\ i(-\xi_b^{\lambda_k})^{N_b-1} \end{pmatrix} \quad (7)$$

with $\underline{0}_{N_a,1}$ and $\underline{0}_{N_b,1}$ are two null vectors of dimension N_a and N_b respectively.

We note that the SARX-Laguerre model is characterized by $(2\ s)$ poles to be optimized and (s N) Fourier Coefficients to be identified, where $(N = N_a^{\lambda_k} + N_b^{\lambda_k})$. In this paper, we use a recursive procedure to identify the Fourier coefficients and a metaheuristic algorithm to optimize the poles.

3 Identification of SARX-Laguerre Model

3.1 Recursive Identification of Fourier Coefficients

In this section, we propose a recursive method to identify the Fourier coefficients of the SARX-Laguerre model. The proposed method consists in the minimization of the regularized square error $J_{reg}^{\lambda_h}(h)$ given as:

$$J_{reg}^{\lambda_h}(h) = \sum_{k=1}^{h}\left[y_m(k) - (C^{\lambda_k})^T X(k)\right]^2 + \alpha\left(\sum_{n=0}^{N_a^{\lambda_h}-1}\left(g_{n,a}^{\lambda_{h-1}}(h-1) - g_{n,a}^{\lambda_h}(h)\right)^2 + \sum_{n=0}^{N_b^{\lambda_h}-1}\left(g_{n,b}^{\lambda_{h-1}}(h-1) - g_{n,b}^{\lambda_h}(h)\right)^2\right), \tag{8}$$

where $y_m(k)$ is the measured output at time instant k correspond to the output of the λ_kth subsystem, $g_{n,a}^{\lambda_{h-1}}(h-1)$ and $g_{n,b}^{\lambda_{h-1}}(h-1)$ are the Fourier coefficients at time instant $(h-1)$ and $g_{n,a}^{\lambda_h}(h)$ and $g_{n,b}^{\lambda_h}(h)$ are the Fourier coefficients at time instant (h), for $\lambda_h,\ \lambda_{h-1} \in [1,\ \ldots,\ s]$. The parameter $(\alpha > 0)$ acts on the importance given either to the first part or to the second part of the criterion $J_{reg}^{\lambda_h}(h)$. The cost function $J_{reg}^{\lambda_h}(h)$ given by (8) can be written in matrix form as:

$$J_{reg}^{\lambda_h}(h) = \|Y_h^m - Y_h^{\lambda_h}\|^2 + \alpha\|C_{h-1}^{\lambda_{h-1}} - C_h^{\lambda_h}\|^2 \tag{9}$$

where the vectors Y_h^m and $Y_h^{\lambda_h}$ contains, for $(k = 1,\ \ldots,\ h)$, the λ_kth subsystem output and the λ_kth submodel output respectively. $C_{h-1}^{\lambda_{h-1}}$ and $C_h^{\lambda_h}$ are the parameter vectors of the λ_hth ARX-Laguerre submodel at time instants $h-1$ and h , where $C_h^{\lambda_h}$ is defined as:

$$C_h^{\lambda_h} = \left[g_{0,a}^{\lambda_h}(h),\ \ldots,\ g_{N_a^{\lambda_h}-1,a}^{\lambda_h}(h),\ g_{0,b}^{\lambda_h}(h),\ \ldots,\ g_{N_b^{\lambda_h}-1,b}^{\lambda_h}(h)\right]^T \tag{10}$$

The vector $Y_h^{\lambda_h}$ can be written as:

$$Y_h^{\lambda_h} = \underline{X}_h\, C_h^{\lambda_h} \tag{11}$$

where $\underline{X}(h)$ is a matrix regrouping the state vector $X(k)$ given by (2) of the λ_kth ARX-Laguerre submodel for $(k = 1,\ \ldots, h)$ determined recursively as:

$$\underline{X}_h = \begin{bmatrix} \underline{X}_{h-1} \\ X(h)^T \end{bmatrix} \tag{12}$$

Then, the cost function $J_{reg}^{\lambda_h}(h)$ given by (9) can be rewritten as:

$$J_{reg}^{\lambda_h}(h) = \left(Y_h^m - \underline{X}_h\, C_h^{\lambda_h}\right)^T\left(Y_h^m - \underline{X}_h\, C_h^{\lambda_h}\right) + \left(C_{h-1}^{\lambda_{h-1}} - C_h^{\lambda_h}\right)^T \alpha\left(C_{h-1}^{\lambda_{h-1}} - C_h^{\lambda_h}\right) \tag{13}$$

Since the criterion (13) is quadratic in $C_h^{\lambda_h}$, its minimization with respect to this vector gives a global minimum such as:

$$\frac{\partial J_{reg}^{\lambda_h}(h)}{\partial C_h^{\lambda_h}} = -2\left[\underline{X}_h^T\, Y_h^m - \underline{X}_h^T\, \underline{X}_h C_h^{\lambda_h}\right]^T - 2\alpha\left(C_{h-1}^{\lambda_{h-1}} - C_h^{\lambda_h}\right) \tag{14}$$

The vector $\widehat{C}_h^{\lambda_h}$ grouping the estimated parameters of the SARX-Laguerre model, for $\lambda_k \in \{1,\ \ldots,\ s\}$, is calculated by the following relation:

$$\widehat{C}_h^{\lambda_h} = \left(\underline{X}_h^T\, \underline{X}_h + \alpha I\right)^{-1}\left[\underline{X}_h^T\, Y_h^m + \alpha\, \widehat{C}_{h-1}^{\lambda_{h-1}}\right] \tag{15}$$

where I is a $(N_a^{\lambda_k} + N_b^{\lambda_k})$ identity matrix.

3.2 SARX-Laguerre Poles Optimization using Genetic Algorithms

Genetic Algorithms are considered as function optimizers inspired by the evolution theory, and which are applied to a broad range of optimization problems. The implementation of a genetic algorithm requires an initial population randomly generated. The optimality of each solution from the population is quantified by computing a criterion named "fitness". Ranking the fitness values assigned to the solutions allows Genetic Algorithms to eliminate the bad ones and make copies of the best ones, so that the population size remains unchanged. This operator is named "selection". From two good solutions "Parents", the next operator named "Cross Over" gives rise to two "Children" which are closer than "Parents" to the optimal solution. Changes then these new individuals during the "Mutation" operator preventing the algorithm to converge to a local optimum.

In this paper, we use the Genetic Algorithm to optimize the poles of the SARX-Laguerre model. Where, in the case of modeling linear hybrid system decomposed to (s) subsystem, we are led to optimize $(2\ s)$ poles and to identify $(N\ S)$ Fourier coefficients, where $(N = N_a^{\lambda_k} + N_b^{\lambda_k})$. To optimize the poles we propose to use the genetic algorithm and to identify the poles we use the recursive algorithm given in the previous subsection. The proposed identification procedure of the SARX-Laguerre mode is illustrated by Figure 1. The optimization begins by generating m initial populations, each population is a set of $(2\ s)$ poles. The best poles are used to compute the outputs of the SARX-Laguerre model used in term to evaluate the fitness in the next step. The aim of the optimization is to select the best poles that minimize the Normalized Mean Square Error "$NMSE$" between the system outputs and the SARX-Laguerre model output. The fitness function associated with the output is given by:I

$$NMSE = \frac{\sum_{k=1}^{M}[y_m(k) - y(k)]^2}{\sum_{k=1}^{M}[y_m(k)]^2} \tag{16}$$

where $y_m(k)$ is the measured output at time instant k and $y(k)$ is the output model at time instant k given by relation (1).

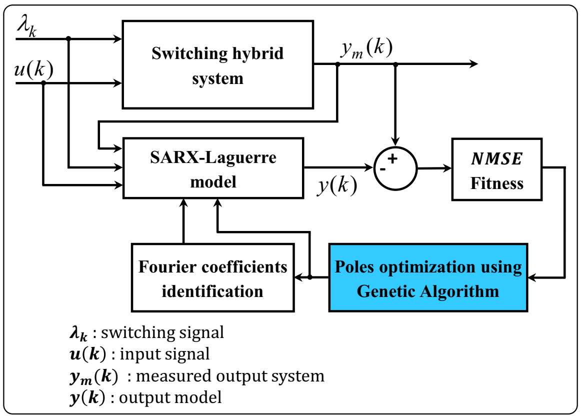

Fig. 1: Identification procedure of SARX-Laguerre model

4 Moving Horizon Fault Estimation based on SARX-Laguerre Model

In the following, we propose to use the moving horizon fault estimation (MHE) based on the SARX-Laguerre model to solve the fault estimation of the linear hybrid systems. The MHE is used to estimate the actuator faults from the error between the estimated output $\widehat{y}(k)$ and the system output $\overline{y}(k)$. The SARX-Laguerre model taking account of actuator faults can be written from (1) as:

$$\begin{aligned} \widehat{X}(k+1) &= A^{\lambda_k} X(k) + b_y^{\lambda_k} y(k) + \\ & \quad b_u^{\lambda_k}(u(k) + f(k)), \\ \widehat{y}(k) &= (C^{\lambda_k})^T \widehat{X}(k), \end{aligned} \tag{17}$$

where $\widehat{X}(k)$ and $\widehat{y}(k)$ are the estimated state vector and the estimated output model respectively, when $f(k)$ is the actuator faults. The estimate actuators faults $\widehat{f}(k)$ is obtained by an online minimization at every sample time of the quadratic cost function $J_f(k)$ defined as follows:

$$J_f(k) = \sum_{j=k-N_f+1}^{k} (\widehat{y}(j) - y_m(j))^2 \tag{18}$$

with, N_f is the fault estimation horizon, $\widehat{y}(j)$ is the estimated model output when the actuator fault is considered as given by (17) and $y_m(j)$ is the measured system output. The quadratic criterion $J_f(k)$ given by (18) can be rewritten as:

$$\begin{aligned} J_f(k) = \sum_{j=k-N_f+1}^{k} \Big[(C^{\lambda_j})^T \Big(A^{\lambda_j} \widehat{X}(j-1) + \\ b_y^{\lambda_j} \widehat{y}(j-1) + b_u^{\lambda_j}(u(j-1) + \widehat{f}(j-1)) \Big) - y_m(j) \Big]^2, \end{aligned} \tag{19}$$

where $\widehat{f}$ is the fault actuator to be optimized using the MHE method. The MHE method is formulated by minimizing the criterion $J_f(k)$ given by (19) at every sample time. The criterion $J_f(k)$ can be written in matrix form as:

$$J_f(k) = \left\| \widehat{Y}_{N_f}(k) - Y_{m,N_f}(k) \right\|^2 \tag{20}$$

where the vector $Y_{m,N_f}(k)$ is determined from the measured output system as follows:

$$Y_{m,N_f}(k) = [y_m(k - N_f + 1), \ldots,\ y_m(k)]^T \tag{21}$$

and the vector $\widehat{Y}_{N_f}(k)$ is determined from the model output $\widehat{y}(k)$ given by (17) as follows:

$$\widehat{Y}_{N_f}(k) = \left[\widehat{y}(k - N_f + 1),\ \ldots,\ \widehat{y}(k)\right]^T \tag{22}$$

From relation (17), the vector $\widehat{Y}_{N_f}(k)$ can be written in matrix form as:

$$\begin{aligned} \widehat{Y}_{N_f}(k) = \ & \mathbf{CA}X(k) + \mathbf{CB_y} Y_{N_f}(k) + \\ & \mathbf{CB_u} U_{N_f}(k) + \mathbf{CB_u} F_{N_f}(k) \end{aligned} \tag{23}$$

with :

- The matrices $\mathbf{C}$, $\mathbf{A}$, $\mathbf{B_y}$ and $\mathbf{B_u}$ are defined as:

$$\mathbf{C} = \begin{bmatrix} (C^{\lambda_{(k-N_f+1)}})^T & 0_{1,N} & \cdots & \underline{0}_{1,N} \\ \underline{0}_{1,N} & (C^{\lambda_{(k-N_f+2)}})^T & \cdots & \underline{0}_{1,N} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{0}_{1,N} & \underline{0}_{1,N} & \cdots & (C^{\lambda_k})^T \end{bmatrix} \tag{24}$$

$$\mathbf{A} = \begin{bmatrix} A^{\lambda_{(k-N_f+1)}} & 0_{1,N} & \cdots & \underline{0}_{1,N} \\ \underline{0}_{N,N} & A^{\lambda_{(k-N_f+2)}} & \cdots & \underline{0}_{N,N} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{0}_{N,N} & \underline{0}_{N,N} & \cdots & A^{\lambda_k} \end{bmatrix} \tag{25}$$

$$\mathbf{B_y} = \begin{bmatrix} b_y^{\lambda_{(k-N_f+1)}} & \underline{0}_{(N,1)} & \cdots & \underline{0}_{(N,1)} \\ \underline{0}_{(N,1)} & b_y^{\lambda_{(k-N_f+2)}} & \cdots & \underline{0}_{(N,1)} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{0}_{(N,1)} & \underline{0}_{(N,1)} & \cdots & b_y^{\lambda_k} \end{bmatrix} \tag{26}$$

$$\mathbf{B_u} = \begin{bmatrix} b_u^{\lambda_{(k-N_f+1)}} & \underline{0}_{(N,1)} & \cdots & \underline{0}_{(N,1)} \\ \underline{0}_{(N,1)} & b_u^{\lambda_{(k-N_f+2)}} & \cdots & \underline{0}_{(N,1)} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{0}_{(N,1)} & \underline{0}_{(N,1)} & \cdots & b_u^{\lambda_k} \end{bmatrix} \tag{27}$$

- The vectors $Y_{N_f}(k)$ and $U_{N_f}(k)$ are respectively the vector of the output model without actuator fault and the vector of the input system for $(j = k-N_f,\ k-N_f+1,\ \ldots,\ k)$ defined as follows:

$$Y_{N_f}(k) = \begin{bmatrix} y(k-N_f) \\ y(k-N_f+1) \\ \vdots \\ y(k-1) \end{bmatrix}, \quad U_{N_f}(k) = \begin{bmatrix} u(k-N_f) \\ u(k-N_f+1) \\ \vdots \\ u(k-1) \end{bmatrix} \tag{28}$$

- The vector $F_{N_f}(k)$ is regroup the actuator fault for $(j = k-N_f,\ k-N_f+1,\ \ldots,\ k)$ to be optimized by minimizing the criterion $J_f(k)$, defined as:

$$F_{N_f}(k) = \begin{bmatrix} f(k-N_f) \\ f_{N_f}(k-N_f+1) \\ \vdots \\ f_{N_f}(k-1) \end{bmatrix} \tag{29}$$

- The vector $\mathbf{X}(k)$ regroup the state vector $X(j)$ for $(j = k-N_f+1,\ k-N_f+2,\ \ldots,\ k)$

$$\mathbf{X}(k) = \begin{bmatrix} X(k-N_f) \\ X(k-N_f+1) \\ \vdots \\ X(k-1) \end{bmatrix} \tag{30}$$

Then, the criterion $J_f(k)$ given by (20) can be written as:

$$J_f(k) = \Big\| \mathbf{CAX}(k) + \mathbf{CB_y} Y_{N_f}(k) + \mathbf{CB_u} U_{N_f}(k) + \mathbf{CB_u} \widehat{F}_{N_f}(k) - Y_{m,N_f}(k) \Big\|^2 \tag{31}$$

The criterion (31) can be rewritten as:

$$J_f(k) = \left[\mathbf{CAX}(k) + \mathbf{CB_y} Y_{N_f}(k) + \mathbf{CB_u} U_{N_f}(k) - Y_{m,N_f}(k)\right]^2 + \left[\mathbf{CB_u} \widehat{F}_{N_f}(k)\right]^2 + 2\left[\mathbf{CAX}(k) + \mathbf{CB_y} Y_{N_f}(k) + \mathbf{CB_u} U_{N_f}(k) - Y_{m,N_f}(k)\right]\left[\mathbf{CB_u} \widehat{F}_{N_f}(k)\right]. \tag{32}$$

The criterion $J_f(k)$ given by (32) is a quadratic in terms of the actuator fault to be optimized $\widehat{F}_{N_f}(k)$.

In the case where constraints on the actuator fault are taken into account, the optimized actuator fault is given as follows:

$$\widehat{F}(k-1) = \min_{F(k-1) \in S_f} J_f(k) \tag{33}$$

where the criterion $J_f(k)$ is given by relation (32) and $\widehat{F}(k-1)$ is defined as follows:

$$\widehat{F}(k-1) = \begin{bmatrix} \widehat{f}(k-N_f+1) & \ldots & \widehat{f}(k-1) \end{bmatrix}^T \tag{34}$$

with S_f is the admissible set of actuator fault constraints defined as:

$$S_f = \left\{ \widehat{F}(k) \ / \ \Gamma\, \widehat{F}(k-1) \leq V \right\} \tag{35}$$

where Γ and V are defined as:

$$\Gamma = \begin{bmatrix} I_{N_f} & -I_{N_f} \end{bmatrix}^T, \quad V = \begin{bmatrix} F_{max} & -F_{min} \end{bmatrix}^T \tag{36}$$

with I_{N_f} is a N_f dimensional identity matrix and F_{min} and F_{max} are N_f dimensional vectors defined as follows:

$$F_{min} = \begin{bmatrix} f_m, & \ldots, & f_m \end{bmatrix}, \; F_{max} = \begin{bmatrix} f_M, & \ldots, & f_M \end{bmatrix} \tag{37}$$

f_m and f_M are the bounds of actuator fault that can be chosen from the physical constraints.

5 Numerical Example

To validate the proposed moving horizon fault estimation of hybrid systems using SARX-Laguerre model, we consider a linear hybrid system composed of three linear submodels. The system is given by a switched ARX model (SARX) as follows:

$$y(k) = a_1(\lambda_k)\, y(k-1) + a_2(\lambda_k)\, y(k-2) + a_3(\lambda_k)\, y(k-3) + b_1(\lambda_k)\, u(k-1) + b_2(\lambda_k)\, u(k-2) + b_3(\lambda_k)\, u(k-3) + \varepsilon(t) \tag{38}$$

with $\lambda_k \in \{1, 2, 3\}$ and the parameters a_1, a_2, a_3, b_1, b_2 and b_3 are given as follows:

$$\begin{cases} a_1 = [\ -0.4 \quad 0.2 \quad 0.55\ ] \\ a_2 = [\ 0.25 \quad 0.35 \quad 0.2\ ] \\ a_3 = [\ 0.25 \quad -0.2 \quad -0.15\ ] \\ b_1 = [\ 0.85 \quad 1.15 \quad 0.5\ ] \\ b_2 = [\ 0.8 \quad 0.5 \quad 0.6\ ] \\ b_3 = [\ 0.25 \quad 0.5 \quad 0.4\ ] \end{cases} \tag{39}$$

The SARX model corresponding to (38) contains 18 parameters. In the goal to reduce the parameters number, we model this system bay a SARX-laguerre where we decompose every ARX submodel on two Laguerre bases with a truncated order ($N_a^{\lambda_k} = N_b^{\lambda_k} = 2$) for ($\lambda_k = 1,\ 2,\ 3$). Then, the obtained SARX-Laguerre model is characterised by $N_L = 12$ Fourier coefficients to be identified and 6 poles to be optimized. To obtain the SARX-Laguerre model, we applied the proposed identification procedure using a genetic algorithm to optimise the poles and a recursive method to identify the Fourier coefficients. To generate the identification data we use an excitation input $u(t)$ for $M = 2000$ observations as given in Figure 2. In Figure 3 we draw the evolution of the arbitrary switching signal and in Figure 4 we plot the evolution of the SARX model output. For $\alpha = 0.5$, the optimal values of SARX-Laguerre poles and the Fourier coefficients are given in Table 1. To validate the capability of the moving horizon fault estimation of hybrid systems using SARX-Laguerre model in fault estimation, we applied to the SARX system given by (38) a constant input $u = 1$ and we added to the input a constant fault at every case of switching signal as given in Figure 5. By considering constraints to the fault where $f_m = 0$ and $f_M = 0.85$ and by applying the proposed fault estimation procedure, we obtain the optimal fault as given in Figure 6.

Table 1. SARX-Laguerre parameters

λ_k	Poles		Fourier coefficients
	$\xi_a^{\lambda_k}$	$\xi_b^{\lambda_k}$	
1	-0.7478	0.5203	$C_1 = [0.0541,\ 0.0218,\ 1.1794,\ -0.0007]$
2	0.0202	0.4113	$C_2 = [0.3895,\ 0.0016,\ 1.2869,\ 0.0030]$
3	-0.0580	0.4414	$C_3 = [0.5733,\ -0.0029,\ 1.3113,\ -0.0023]$

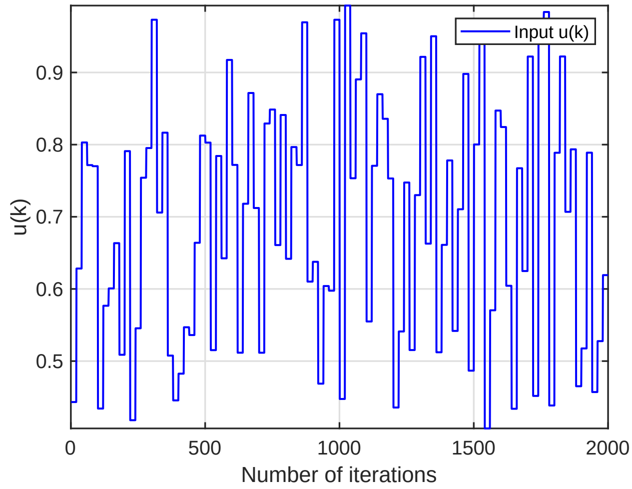

Fig. 2: Evolution of input signal

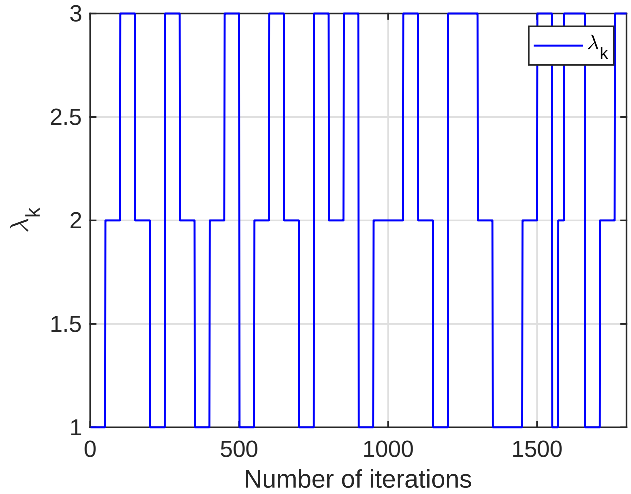

Fig. 3: Evolution of the arbitrary switching signal

6 Conclusion

In this paper, we propose a fault estimation scheme for a class of linear hybrid systems using moving horizon fault estimation. Linear hybrid systems are approximated by a reduced complexity SARX-Laguerre model obtained after expanding the SARX model on independent Laguerre orthonormal bases. To identify the SARX-Laguerre model, we propose in this paper to identify the Fourier coefficients using a recursive algorithm and a metaheuristic algorithm to optimize the poles. The identified model is used to develop an online moving horizon fault estimation algorithm. This algorithm can be used to develop a fault tolerant adaptive control algorithm for nonlinear hybrid systems. The proposed fault estimation scheme is evaluated on a numerical example and the performances are assessed in fault estimation.

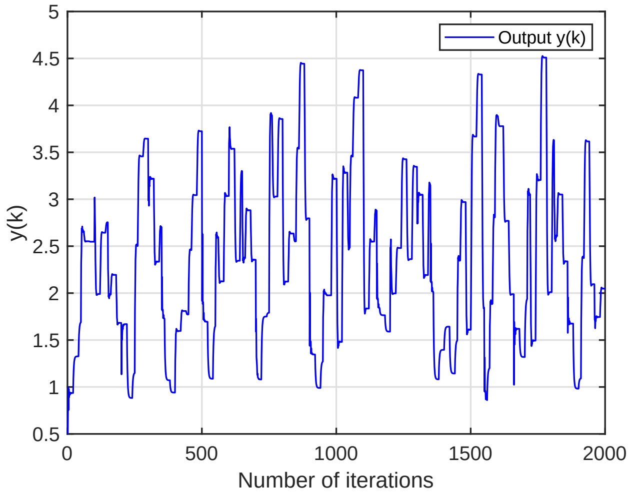

Fig. 4: Evolution of the SARX model output

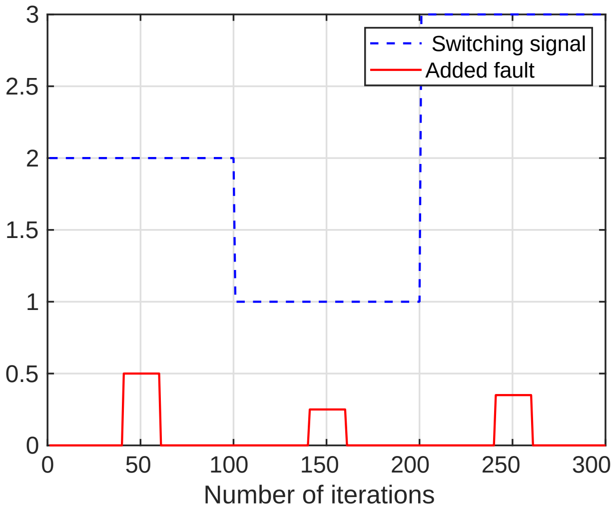

Fig. 5: Switching signal and added fault

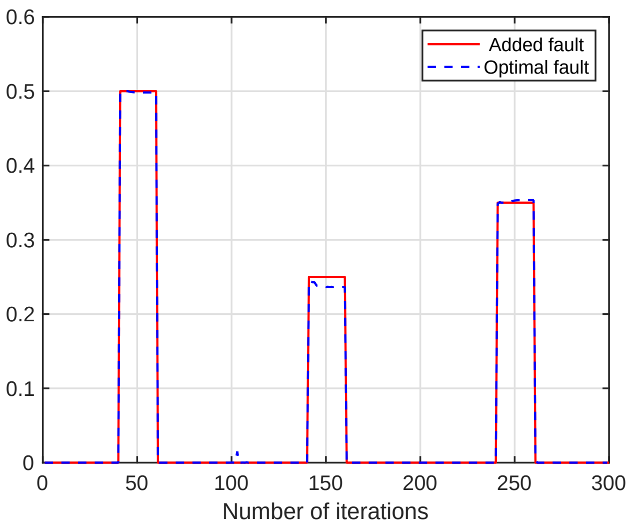

Fig. 6: Added fault and optimal fault

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Conflicts of Interest

The authors have no conflicts of interest to declare.