Risk Hedging in a Supply Chain

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Abstract—**We study two risk hedging strategies, i.e., option contract and advance purchase discount contract, in a two-echelon supply chain. We study the optimal decisions under each contract for both the supplier and the retailer respectively. We derive the conditions under which either contract should be adopted from the supplier's perspective. We further demonstrate that supply chain coordination could be reached under the option contract.**

Keywords—risk hedging; option contract; advance purchase discount contract

I. INTRODUCTION

Long lead times are common in many industries. In the apparel industry, the lead time between retailer ordering and manufacturer delivering can be as long as 12 months (Fisher and Raman, 1996). In the toy industry, this gap can be as long as 18 months (Biyalogorsky and Koenigsberg, 2006). In these industries, long lead times and high uncertainty in consumer demand makes the matching between supply and demand even more complicated. To reconcile the mismatch, the supplier would encourage the retailer to place orders through the Advance Purchase Discount (APD) contract. Under the APD contract, the supplier offers the retailer with two wholesale prices: a discount price if the retailer orders before the selling season starts and a regular price if the retailer buys during the selling season. The retailer bears the cost on inventory ordered before the selling season starts and the opportunity cost on any lost margins as purchasing during the selling season is not guaranteed. On the other hand, the supplier bears the risk on any production in excess of retailer order. Under the APD contract, the supplier's capacity decision has a significant impact on the allocation of supply chain risks, as the APD contract allows for intermediate allocations of inventory risks between supply chain partners.

Option contract is another common practice to reconcile the mismatch. Under the option contract, the buyer of the option gains the right, but not the obligation, to engage in the transaction, while the seller incurs the corresponding obligation to fulfill the transaction. In our paper, the retailer uses option to gain lower exercising price (for a comparison, here we assume it is the discount price, afterwards we will optimize the option price) in advance. In the supply chain management area, under the option contract, the supplier bears the inventory risk and the retailer pays the option fee.

Our work brings together the research streams on option contract and APD contract. It has been shown that options (Barnes-Schuster et al. 2002) or options-like contracts, such as buy-back contract (Emmons and Gilbert 1998, Wu 2011), backup contract (Eppen and Iyer 1997) and quantity flexibility contract (Tsay 1999), can provide both suppliers and retailers with flexibility to share the demand risks thus improve supply chain performance. From the traditional newsvendor's perspective, Xu (2006) consider a class of multi-period flexible supply policies with options and characterize the optimal options ordering policy. Chen and Parlar (2007) explore the single-period inventory model with a put option and stochastic demand. In a supply chain context, Burnetas and Ritchken (2005) investigate the role of option contracts with downward sloping demand curve and show that the introduction of option contracts causes the wholesale price to increase and the volatility of the retail price to decrease. Wu and Kleindorfer (2005) analyze integrating contract procurement with capacity options and forwards. They characterize the existence and structure of market equilibrium. In view of supply chain coordination, Erkoc and Wu (2005) and Jin and Wu (2007) use capacity option contract to share supply chain risks and encourage high-tech manufacturers to expand capacity more. They both propose option-like contracts to coordinate the supply chain. Wang and Liu (2007) exam a retailer-led supply chain and find the conditions to coordinate such a supply chain. More recently, Zhao et al. (2010) take a cooperative game approach to study the coordination issue in a manufacturer-led supply chain using option contract. They find that option contracts can coordinate the supply chain and achieve Pareto-improvement.

The APD contract has also been investigated. Weng and Parlar (1999) is among the first ones to examine the effects of prior-sale discount decisions. They derive the optimal order quantity and optimal discount price. Prasad et al. (2010) extend the advance selling to a two-period setting and Zhao and Stecke (2010) consider loss averse consumer. They both characterized the optimal selling strategy. Our paper focuses on risk sharing between suppliers and retailers under APD contract. Many papers have shown APD contract could allow for an intermediate inventory risk sharing between supply chain partners and a better performance could be reached (Cachon 2004, Dong and Zhu 2007). Besides, He and Khouja (2011) evaluate the performance of Push, Pull, and APD contracts in a manufacturer–retailer supply chain with a satisficing objective. They find that a modified buy-back and profit guarantee contracts can provide significant Pareto improvement over Push or APD contracts. Lai et al. (2009) analyze the three contracts in a supply chain with financial constraint. They conclude that a financially unconstrained manufacturer prefers a Pull contract but a financially

constrained manufacturer prefers the APD contract. Davis and Katok (2011) experimentally investigates the three contracts and conclude that APD contract is superior to Push contract in terms of both retailer and supplier profits. Following this vein, we study APD contract but take a different perspective. Specifically, we study the optimal capacity planning for the suppliers while most of the existing literatures study the pricing of the contracts. Furthermore, we compare APD contract with option contract.

We study a two echelon supply chain consisting of one supplier and one retailer. We consider not only the retailer's purchasing decision but also the supplier's production decision under both the APD contract and the option contract. We show that supply chain coordination could be reached under the option contract. Moreover, to the best of our knowledge, this work is among the first to consider the strategy adoption between APD contract and option contract.

II. MODEL DESCRIPTION

We study a two echelon supply chain consisting of one supplier and one retailer. The supplier offers the retailer with either APD contract or option contract. Under APD contract, a lower purchasing price w_1 is offered to the retailer at a point of time before the selling season starts (i.e., at time *0*) and a higher purchasing price w_2 is offered during the selling season (i.e., after time *T*). At time *0*, the retailer places a firm order y at w_1. The supplier builds up capacity q in anticipation of the retailer's at-once orders afterwards. At time *T* , the retailer buys at-once products for w_2 each if its pre-order couldn't satisfy customer demand. In this scenario, the retailer optimizes order y and the supplier optimizes capacity q . Under the option contract, the retailer pays an option fee of c_0 per unit to the supplier at time *0* to lock the lower unit price w_1 in advance if the retailer exercises option at time *T* .The supplier plans capacity and commits to offer products up to a limit of the retailer's order quantity. Then the retailer could exercise option up to the limit at the lower price at time *T* . Similarly, we focus on the optimization of supplier's capacity q , retailer's order quantity y and option price c_0 .

We assume both the retailer and the supplier can purchase additional units from an emergency source at a more expensive price (Alfredsson and Verrijdt 1999, Lawson and Porteus 2000) if necessary. The emergency source emerges in terms of lateral transshipment and direct deliveries. Compared with the traditional Newsvendor model, emergency sourcing has a critical impact on the retailer's order decision under different contracts. Sequence of events is illustrated in *Fig. 1*.
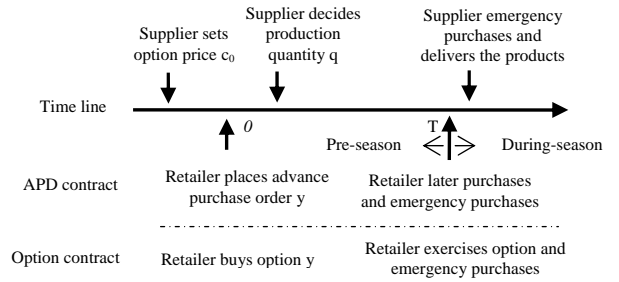

Fig. 1. Sequence of events.

Notations are summarized as follows

c : The supplier's production cost per unit.

p : The retailer's sales price per unit.

e : The emergency purchasing cost per unit.

c_0 : The option price per unit.

w_1 : The advanced purchase price per unit under the APD contract as well as the strike price per unit under the option contract.

w_2 : later purchase price under APD contract, $e \geq w_2 \geq w_1$

y : The retailer's order quantity at time *0*.

q : The supplier's reserved capacity at time *0*.

v : Salvage value per unit

D : The stochastic customer demand after T .

μ : Mean of customer demand.

$F(x)$: The cumulated distribution function of customer demand.

$f(x)$: The probability distribution function of customer demand.

π_j^i : Player j 's expected profit under strategy i , $j \in \{r, s\}$, where r and s represents retailer and supplier respectively; $i \in \{O, A\}$, where O and A represents Option contract and APD contract respectively.

q_i^* : Supplier's optimal capacity under strategy i , $i \in \{O, A\}$.

y_i^* : Retailer'S optimal order quantity under strategy i , $i \in \{O, A\}$.

q^i : The system's optimal capacity under strategy i , $i \in \{O, A\}$.

$q_{i\lambda}^*$: The loss averse supplier's optimal capacity under strategy i , $i \in \{O, A\}$.

III. RISK NEUTRAL SUPPLIER VS. RISK NEUTRAL RETAILER

When supplier and retailer are both risk neutral, they choose to maximize their own expected profit respectively.

A. The option contract

Under the option contract, a rational supplier will not produce more than the retailer's option order quantity, i.e. $q \le y$. Let $S(y)$ be the expected sales, then $S(y) = \min(D, y) = y - \int_0^y F(x)dx$. The expected leftover inventory, denoted as $I(q)$, is $I(q) = E(q - D)^+ = q - S(q)$. The supplier's expected order from the emergency source is

$$H(y, q) = E[\min(D, y) - q]^+ = E((D - q)^+ - (D - y)^+)$$
$$= S(y) - S(q).$$

Under the option contract, as a Stackelberg game follower, the retailer's expected profit is

$$\pi_r^o(y) = pED - c_0 y - w_1 S(y) - eE(D - y)^+$$
$$= p\mu - c_0 y - w_1 S(y) - e(\mu - S(y)) \quad (1)$$

The first term in (1) is the retailer's revenue. The next two terms are the retailer's option purchasing cost and exercising cost, and the last term is the retailers's purchasing cost from the emergency source.

As a Stackelberg game leader, the supplier's expected profit function is

$$\pi_s^o(q \mid y) = c_0 y + w_1 S(y) + vI(q) - eH(y, q) - cq$$
$$= c_0 y + (w_1 - v)S(y) - (e - v)[S(y) - S(q)] - (c - v)q \quad (2)$$

The first two terms in (2) are the supplier's sales margins and the last two terms are the supplier's cost margins.

Proposition 1. Under the option contract, the supplier's optimal capacity is $q_o^* = F^{-1}(\frac{e-c}{e-v})$ and the retailer's optimal option order quantity is $y_o^* = F^{-1}(\frac{e-w_1-c_0}{e-w_1})$.

It's obvious that the supplier's optimal capacity is irrelevant to the option's price, so the supplier's production and pricing decisions can be made independently. Further, if the emergency source doesn't exist, results in proposition 1 turns out to be the same as in the traditional Newsvendor problem and the retailer's optimal order quantity is $y_o^{NV} = F^{-1}(\frac{p-w_1-c_0}{p-w_1})$. Since $e < p$, we have $y_o^* < y_o^{NV}$. Hence the existence of the emergency source makes the retailer order less.

Lemma 1. y_o^* is decreasing in c_0.

The more expensive the option price is the less order the retailer places.

Proposition 2. The supplier's optimal option price is $c_0^* = \frac{(c-v)(e-w_1)}{e-v}$, at which the retailer's order quantity y_o^* equals the supplier's planning capacity q_o^* i.e. $y_o^* = q_o^*$.

From the system's perspective, the system's expected profit is

$$\pi^o(q) = pED - cq - eE(D - q)^+ + vE(q - D)^+$$
$$= p\mu - cq - e(\mu - S(q)) + v(q - S(q)) \quad (3)$$

Proposition 3. The system-wide optimal capacity is $q^o = F^{-1}(\frac{e-c}{e-v})$.

Given Proposition 1 and Proposition 3, we conclude that the supplier's optimal capacity is always system-wide optimal no matter what the option price is i.e. $q_o^* = q^o$.

Proposition 4. $y_o^* = q_o^* = q^o$ iff $c_0 = c_0^*$.

From Proposition 4 we know that when $c_0 = c_0^*$, the retailer's order quantity and supplier's capacity both equal to the system-wide optimal capacity which makes supply chain coordination reached.

B. The APD contract

Under the APD contract, the retailer's expected profit is

$$\pi_r^A(y) = pED - w_1 y - w_2 E\min[(D - y)^+, q - y] + vE(y - D)^+ - eE(D - q)^+$$
$$= (p - e)\mu - (w_1 - v)y - (w_2 - v)[S(q) - S(y)] + (e - v)S(q) \quad (4)$$

The supplier's expected profit is

$$\pi_s^A(q \mid y) = w_1 y + w_2 E\min[(D - y)^+, q - y] + vE(q - y - \min[(D - y)^+, q - y]) - cq$$
$$= w_1 y + w_2[S(q) - S(y)] + v[q - y - S(q) + S(y)] - cq$$
$$= (w_1 - v)y + (w_2 - v)[S(q) - S(y)] - (c - v)q \quad (5)$$

Proposition 5. Under the APD contract, the supplier's optimal capacity is $q_A^* = F^{-1}(\frac{w_2-c}{w_2-v})$ and the retailer's optimal option order quantity is $y_A^* = F^{-1}(\frac{w_2-w_1}{w_2-v})$.

Different from the option contract, the retailer's optimal order quantity y_A^* is independent of e under the APD contract, so the existence of emergency source has no impact on the retailer's order quantity.

Lemma 2. q_A^* and y_A^* are increasing in w_2, y_A^* is decreasing in w_1.

From lemma 2 we know that the supplier's optimal capacity is positively influenced by w_2 and the retailer's optimal order quantity is negatively influenced by w_1 but positively influenced by w_2.

The system's expected profit is

$$\pi^A(q) = pED - cq - eE(D - q)^+ + vE(q - D)^+$$
$$= p\mu - cq - e(\mu - S(q)) + v(q - S(q)) \quad (6)$$

Proposition 6. The system-wide optimal capacity is $q^A = F^{-1}(\frac{e-c}{e-v})$.

Proposition 7. $y_A^* < q_A^* \le q^A$ and the equality holds iff $w_2 = e$.

Given Proposition 7, the retailer's optimal order quantity is less than the supplier's optimal capacity and the supplier's optimal capacity is no greater than the system-wide optimal capacity. Moreover, since y_A^* is strictly less than q_A^*, supply chain coordination could not be reached.

C. Comparison between option and APD contracts

Proposition 8. $y_A^* < q_A^* \le q^A = q^o = q_o^* \le y_o^*$.

Given proposition 8, we conclude that the supply chain's system-wide optimal capacity is the same under both the APD contract and the option contract, i.e. $q^A = q^o$. In addition, the supplier tends to reserve less capacity under the APD contract than under the option contract i.e. $q_A^* \le q_o^*$.

Though APD contract could not coordinate the supply chain, but as a supply chain dominator whether option contract is always beneficial to the supplier needs to be considered. If we let $w_2 = e$ and $c_0 = \frac{(c-v)(e-w_1)}{e-v}$, then $y_A^* < q_A^* = q^A = q^o = q_o^* = y_o^*$, the supplier produces the system-wide optimal quantity under both contracts. In what follows, we compare the supplier's expected profit to see which contract is better for the supplier.

Let $G(x) = \frac{x\bar{F}(x) - S(x)}{F(x)} = \frac{-\int_0^x tf(t)dt}{F(x)}$ and $y_m = G^{-1}(G(q^o)F(q^o))$. For the purpose of simplicity, we define y_m as the threshold value.

Lemma 3. y_m is increasing in q^o

By Lemma 3 we conclude that the bigger the system capacity q^o is, the bigger the threshold value is.

Proposition 9

$$\begin{cases} \pi_s^o(q^o \mid y_o^*) > \pi_s^A(q^A \mid y_A^*), & when \quad y_A^* > y_m \\ \pi_s^o(q^o \mid y_o^*) < \pi_s^A(q^A \mid y_A^*), & when \quad y_A^* < y_m \\ \pi_s^o(q^o \mid y_o^*) = \pi_s^A(q^A \mid y_A^*), & when \quad y_A^* = y_m \end{cases}$$

Lemma 3 and proposition 9 imply that when q^o is small enough, y_m also takes a small value and the probability for y_A^* to be less than y_m is small. In other words, y_A^* is greater than y_m in most of the cases, hence the option contract is more preferred. When q^o is not small enough, we plot the expected profits for the risk neutral supplier under both contracts in *Fig. 2*.
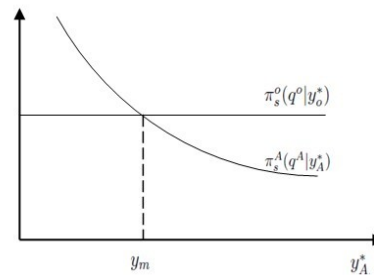

Fig. 2. Expected profits of a risk neutral supplier under APD and option contract

Given *Fig. 2*, we see that under the option contract, since $q^o = q_o^* = y_o^*$, the supply chain reaches its optimal system-wide profit and the supplier's profit, i.e. $\pi_s^o(q^o \mid y_o^*)$, is fixed. Therefore we set the supplier's expected profit under the option contract as the benchmark to further examine whether the supplier will be better off under the APD contract. Intuitively, the supplier's expected profit under the APD contract, i.e., $\pi_s^A(q^A \mid y_A^*)$, is decreasing in y_A^*, because the larger y_A^* is, the less order the retailer will purchase over the later purchase chance after the selling season starts, thus the less profit the supplier earns.

From proposition 9, we know that when the retailer's advance purchase quantity is small enough, i.e. $y_A^* < y_m$, the supplier prefers the APD contract. Otherwise, the supplier prefers the option contract. In other words, if the retailer chooses to follow the option contract, then the supplier can set a quantity threshold value y_m to make sure that the retailer purchases at least y_m in advance.

IV. CONCLUSION

We study two risk hedging strategies, i.e., the option contract and the APD contract, within a two-echelon supply chain context. We summarize the optimal decisions under each contract for both the supplier and the retailer respectively and derive the conditions under which the option/APD contract should be adopted by the supplier. Specifically, when both the supplier and the retailer are risk neutral, then under the option contract, the supplier always reserves the system-wide optimal capacity no matter how much the option price is. When the optimal system-wide capacity is too small, the option contract is more preferred by the supplier. If not, only when the retailer's advance purchasing quantity is smaller than the threshold value should the APD contract be adopted. Moreover, we demonstrate supply chain coordination could be reached under the option contract, but not under the APD contract.

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