A Tutorial-Based Approach to Teaching the Mathematics and Strategies behind Gambling Payouts

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Abstract: – Roulette is a popular gambling game due to its simplicity and attraction of high payouts and it is purely based on chance. It is essential to know about the various payout strategies adopted by gambling houses before deciding to venture into any game of gambling. This tutorial paper mathematically analyzes the payout strategies adopted by the gambling houses for the game Roulette based on simple probability concepts. From the presented mathematical analysis, it can be easily verified that the payout strategy is fixed in favor of gambling houses and there is rarely any chance for a gambler to win on a long-term average. Finally, to maintain the neutrality of chance between the gambler and the gambling house, appropriate payouts are proposed for one set of common bets and relaxations for another set of common bets in Roulette.

Key-Words: - Roulette, Probability, Gambling, Long term average, Payout, Casinos.

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1 Introduction

Gambling games have a rich and diverse past that spans across cultures and centuries, developing from ancient rites to modern-day entertainment. The roots of gambling can be traced back to ancient civilizations. Dice, one of the initial known gambling tools, dates to about 3000 BCE in Mesopotamia. The Chinese were also initial gamblers, developing rudimentary forms of lottery games and games of chance as early as 200 BCE.

In ancient Rome and Greece, gambling was both a widespread pastime and a root of concern, leading to rules and prohibitions in certain times. Card games appeared in China all through the 9th century and progressively spread to Europe by the 14th century, developing into numerous forms including modern poker and blackjack. The 17th century saw the founding of the first casinos in Italy, with the Ridotto in Venice opening in 1638 to offer skillful gambling environments. The 19th and early 20th centuries marked the upsurge of gambling in the United States, where games like roulette and slot machines added popularity in saloons and later in Las Vegas, which became the gambling capital of the world by the mid-20th century. At present, gambling has extended into the digital realm, with online casinos contributing a wide range of games accessible globally. Notwithstanding its development, the primary appeal of gambling remains unaffected by the excitement of chance and the confidence of winning.

Roulette has been a widespread gambling game since 1863 and it is pervasive everywhere in the world, [1]. It is purely a chance-based game. The game is very simple and cool to track, [2]. Gamblers throng to Casinos as the game seems to be a charm and mystery with enthusiasm but not for a person who can think rationally and precisely. The rules for playing Roulette are almost alike everywhere in the world. Since gambling strategy is always designed in favor of the gambling house, there certainly cannot be any method to reveal the winning strategy. However, mathematicians have widely used Roulette to teach probability theory [3] and or propose strategies to increase the chances of winning the games, [4]. A model with better visualization of the betting game Roulette was carried out in [5]. Roulette wheel game strategy finds wide applications while solving various optimization problems in engineering like test suite minimization problems [6], declarative programming [7], crowdsourcing [8], metabolic pathway design [9], classification tasks [10], smart building [11]. This paper is organized as follows: Section 2 gives the playing methodology with a Roulette wheel. In section 3, a mathematical analysis is made which proves that playing with

Roulette never pays off in the long run and gambling strategies are fixed to favor gambling houses. Section 4 discusses the results obtained and section 5 concludes the paper and future research avenues are indicated.

2 Roulette Wheel Game

Roulette is a wheel that houses 37 slots (numbered from 0 to 36 randomly) as shown in Figure 1. Out of 37 slots 18 are red, 18 are black and the remaining 1 is green. In some places, the Roulette wheel houses 38 randomly numbered slots with 18 red, 18 black, and 2 green. We consider only the former and not the latter in the analysis. However, the playing strategy is the same in both types. In this game, a dealer spins the wheel in a particular direction (clockwise or anticlockwise) and at the same instant, a tiny metallic ball is made to roll in the opposite direction, exactly around the wheel rim. In this process, the ball moves fast initially but after a time gap, comes to rest due to gravity. The number and color at the resting place holds the key to the game. A gambler is a winner if he is able to predict exactly the number or color of the resting place. Else, the gambler becomes a loser. That is, the gambler can make a bet on the number or color. Possible bets include predicting the color (either red or black), odd or even number, low value (1-18), or high-valued number (19-36), [3]. A gambler is not permitted to bet for the number '0' (or 'green') and if a ball lands here, the gambler loses. This is the very first game strategy fixed to favor the casino or the gambling house.

Fig. 1: A Roulette Wheel

3 Mathematical Analysis

Table 1 shows the payouts for various schemes of this game. In this section, mathematical analysis is done for the cases of betting on a number and betting on a color. First, assume that a bet is made on a number (the bet common name is straight up). If the game is played, there are two possible outcomes: either the ball lands on the bet number (success/win) or the ball does not land on the bet number (failure/loss). The payout for this type is 35:1 as dictated by the gaming house. In this succeeding mathematical analysis, the initial registration fee paid to become a member and enter a gambling house or casino is ignored. In the case of winning the bet, the gambler gets \$35 and in case of losing the bet, the loss is \$1 for the gambler. Let the sample space associated with this experiment be $\{0,1,...,36\}$. This sample space can be mapped to a Bernoulli trial with two possible outcomes, one for success and another for failure. It is to be noted that all Bernoulli trials are independent trials. Thus, the outcome of this experiment can be mapped onto a random variable which can take a value '-1' for a loss and '+35' for a win. Hence, Probability of win = 1/37 and Probability of loss = 36/37. Also, the Probability of win = (Number of wins) / (Number of games) which needs to be equal to 1/37. For an easy mathematical analysis, consider a long-term average with 3700 games, then a number of wins = 100 and a number of losses = 3600, based on the above probability values. The total amount likely to be gained after playing 3700 games is (100 x 35)-(3600x1) = 3500-3600 = -100, which will eventually be a loss for gamblers. It is to be noted that for any number of games, there will always be a loss for a gambler with this payout. Therefore, the conclusion is ***to stop gambling*** as the gambling houses can only end up making a profit in the long term!

Table 1. Wages and Payouts for Roulette, [3]

Bet common name	Winning spaces	Payout	Odds against winning
Straight up	Any single number including 0	35 to 1	36 to 1
Split	any two adjoining numbers vertical or horizontal	17 to 1	17.5 to 1
Basket	0, 1, 2 or 0, 2, 3	11 to 1	11.33 to 1
Street	any three numbers horizontal (1, 2, 3 or 4, 5, 6 etc.)	11 to 1	11.33 to 1
Corner	any four adjoining numbers in a block (eg 17, 18, 20, 21)	8 to 1	8.25 to 1
Six Line	any six numbers from two rows (eg 28, 29, 30, 31, 32, 33)	5 to 1	5.167 to 1
1st Column	1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34	2 to 1	2.083 to 1
2nd Column	2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35	2 to 1	2.083 to 1
3rd Column	3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	2 to 1	2.083 to 1
1st Dozen	1 through 12	2 to 1	2.083 to 1
2nd Dozen	13 through 24	2 to 1	2.083 to 1
3rd Dozen	25 through 36	2 to 1	2.083 to 1
Odd	1, 3, 5, ..., 35	1 to 1	1.056 to 1
Even	2, 4, 6, ..., 36	1 to 1	1.056 to 1
Red	Red nos	1 to 1	1.056 to 1
Black	Black nos	1 to 1	1.056 to 1
1 to 18	1, 2, 3, ..., 18	1 to 1	1.056 to 1
19 to 36	19, 20, 21, ..., 36	1 to 1	1.056 to 1

What if the payout is changed to 36:1, which may logically look truthful as the sample space has 37 elements out of which one element corresponds to win and the rest corresponds to loss? In such a scenario, the total amount likely to be gained after playing 3700 games would be (100 x 36)-(3600x1) = 0, which leads to neither a win nor a loss for both the gambler and the gambling house. However, a gambler has a certainty to win on a long-term average, only if the payout is 37:1, as (100 x 37)-(3600x1) = 100.

Now, assume that a bet is made on the color (let the bet's common name be Red, without loss of generality). Payout for this type is 1:1 as dictated by the gaming house and shown in Table 2. If the game is played, there are two possible outcomes: either the ball lands on the Red color (success/win) or the ball does not land on the Red color (failure/loss). It is to be noted that the gambling house has fixed the payout safely in its favor. That is, for a win, the ball needs to land on a Red color but if the ball lands on either Green or Black color, it is interpreted as a loss. After the gambler bets on color and if the gambler wins the game, he gets $1. If the gambler loses, his loss is $1. The sample space is {Red, Not a Red}. However, this sample space can be mapped to a Bernoulli trial with two possible outcomes, one for success and another for failure. Thus, the outcome of this experiment can be mapped onto a random variable which can take a value '-1' for a loss and '1' for a win. Hence,
Probability of win = 18/37
Probability of loss = 19/37
Probability of win = (Number of wins) / (Number of games) = 18/37.

Consider a long-term average with 3700 games, then a number of wins = 1800 and a number of losses = 1900, based on the same probability values. So, the total amount likely to be earned after playing 3700 games would likely be -100 (1800 x 1 – 1900 x1) It is to be noted that for any number of games, there will always be a loss for a gambler with this payout. However, if the gambling houses ignore the scenario of the ball landing on Green color as neither success nor failure, then such a scenario puts both the gambler and the gaming house at par with each other as the probability of both success and failure are equally likely. But, gambling houses won't resort to such a payout as it may not favor the gambling house.

4 Results and Discussion

A mathematical analysis carried out in the previous section clearly reveals that payouts are fixed by the gambling houses in favor of them. It also shows that a gambler is more likely to lose on a long-term average. In a game of chance, playouts should favor neither gamblers nor the gambling house. Fair payouts are proposed for one set of the bets and relaxations required to maintain neutrality are proposed for another set of the bets in Table 2. Hence, it is to be clearly understood that *stopping gambling* is the only option for a gambler to avoid a loss as payouts favor only gambling houses.

Table 2. Existing payouts and proposed fair payouts

<table>
<tr><th>Bet Common Name</th><th>Winning Spaces</th><th>Existing Payout</th><th>Fair Payout proposed or relaxation required</th></tr>
<tr><td>Straight Up</td><td>Any Single Number Including '0'</td><td>35 to 1</td><td>36 to 1</td></tr>
<tr><td>Split</td><td>Any two adjoining numbers vertical or horizontal</td><td>17 to 1</td><td>18 to 1</td></tr>
<tr><td>Basket</td><td>(0, 1, 2) OR (0, 2, 3)</td><td>11 to 1</td><td>12 to 1</td></tr>
<tr><td>Street</td><td>Any 3 numbers horizontal</td><td>11 to 1</td><td>12 to 1</td></tr>
<tr><td>Corner</td><td>Any four adjoining numbers in a block</td><td>8 to 1</td><td>9 to 1</td></tr>
<tr><td>Six Line</td><td>Any 6 numbers from 2 rows</td><td>5 to 1</td><td>6 to 1</td></tr>
<tr><td>1st Column</td><td>1,4,7,10,13,16,19,22, 25,28,31,34</td><td rowspan="6">2 to 1</td><td rowspan="10">'0' needs to be placed under neither the win nor loss category</td></tr>
<tr><td>2nd Column</td><td>2,5,8,11,14,17,20,23, 26,29,32,35</td></tr>
<tr><td>3rd Column</td><td>3,6,9,12,15,18,21,24, 27,30,33,36</td></tr>
<tr><td>1st Dozen</td><td>1 through 12</td></tr>
<tr><td>2nd Dozen</td><td>13 through 24</td></tr>
<tr><td>3rd Dozen</td><td>25 through 36</td></tr>
<tr><td>Odd</td><td>1, 3, 5,…, 35</td><td rowspan="6">1 to 1</td></tr>
<tr><td>Even</td><td>2, 4, 6, …, 36</td></tr>
<tr><td>1 to 18</td><td>1, 2, 3,…, 18</td></tr>
<tr><td>19 to 36</td><td>19, 20, 21,…, 36</td></tr>
<tr><td>Red</td><td>Red numbers</td><td rowspan="2">Green numbers need to be placed under neither the win nor loss category.</td></tr>
<tr><td>Black</td><td>Black numbers</td></tr>
</table>

The mathematical analysis made in the preceding section is applicable only to straight and common bets in Roulette. Other common more risky betting strategies include D'Alembert systems, Martingale, and the Fibonacci. In the D'Alembert system type bet, bet value is increased or decreased by one unit based on the win or loss of the previous bet respectively. Another worse risky bet is the Fibonacci strategy, which follows the Fibonacci sequence to determine the bet size for the current bet based on the sum of the immediate past two bets. Finally, the worst risky bet is Martingale's strategy, which involves doubling the bet after each loss, aiming to recover all previous losses with a single win. The chance of the above risky versions of gambling strategies beneficial to gamblers may happen only in case of equally likely payouts for both gamblers and gambling houses.

5 Conclusion

In this paper, a mathematical study is carried out to show that playing Roulette never pays off for a gambler with the existing payouts. Further, it is shown that gambling payouts are fixed in favour of gambling houses and there is little chance for a gambler to earn on a long-term average. An interesting future work is analyzing other popular gambling games and revealing how the payouts are designed to favor gambling houses. Another interesting and much-needed future work is to review the application of Roulette and other gambling games in optimization techniques across multiple domains of science, engineering, and technology. A much-awaited work is the deployment of artificial intelligence and machine learning principles to bring out profits for the gambler with the existing payouts.

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