Quenching and Suppression of Limit Cycles in 3x3 Nonlinear Systems

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Abstract: - For several decades, the importance and weight-age of prediction of nonlinear self-sustained oscillations or Limit Cycles (LC) and their quenching by signal stabilization have been discussed which is confined to Single Input and Single Output (SISO) system. However, for the last five to six decades, the analysis of 2x2 Multi Input and Multi Output (MIMO) Nonlinear Systems gained importance in which a lot of literature available. In recent days few literatures are available which addresses the exhibition of LC and their quenching/suppression in 3x3 MIMO Nonlinear systems. Poor performances in many cases like Load Frequency Control (LFC) in multi area power system, speed and position control in robotics, automation industry and other occasions have been observed which draws attention of Researchers. The complexity involved, in implicit non-memory type and memory type nonlinearities, it is extremely difficult to formulate the problem in particular for 3x3 systems. Under this circumstance, the harmonic linearization/ harmonic balance reduces the complexity considerably. Still the analytical expressions are so complex which loses the insight into the problem particularly for memory type nonlinearity in 3x3 system. Hence in the present work a novel graphical method has been developed for prediction of limit cycling oscillations in a 3x3 nonlinear system. The quenching of such LC using signal stabilization technique using deterministic (Sinusoidal) and random (Gaussian) signals has been explored. Suppression LC using pole placement technique through arbitrary selection and optimal selection of feedback Gain Matrix K with complete state controllability condition and Riccati Equation respectively. The method is made further simpler assuming a 3x3 system exhibits the LC predominantly at a single frequency, which facilitates clear insight into the problem and its solution.

The proposed techniques are well illustrated with example and validated/substantiated by digital simulation (a developed program using MATLAB codes) and use of SIMULINK Tool Box of MATLAB software.

The Signal stabilization with Random (Gaussian) Signals and Suppression LC with optimal selection of state feedback matrix K using Riccati Equation for 3x3 nonlinear systems have never been discussed elsewhere and hence it claims originality and novelty.

The present work has the brighter future scope of:

i. Adapting the Techniques like Signal Stabilization and Suppression LC for 3x3 or higher dimensional nonlinear systems through an exhaustive analysis.

ii. Analytical/Mathematical method may also be developed for signal stabilization using both deterministic and random signals based on Dual Input Describing function (DIDF) and Random Input Describing Function (RIDF) respectively.

iii. The phenomena of Synchronization and De-synchronization can be observed/identified analytically using Incremental Input Describing Function (IDF), which can also be validated by digital simulations.

Key-Words: - Limit Cycles, Describing function, 3x3 non-linear systems, Pole placement technique, Suppression limit cycle, signal stabilization.

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1 Introduction

It has been observed for years long the importance and weightage on self-sustained oscillations or nonlinear oscillations or limit cycles (LC) [1], [2], [3], [4], [5].

For last several decades, the analysis of 2x2 multivariable nonlinear systems drawn attention of the researchers and good number of literature is available [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], cover this area of research. The prediction of LC in 2 x 2 system, in means of increasing the reliability of the describing function (DF) are well established [4], [5], [10], [13], [16], [23], [47], [48] and others used harmonic linearization/ harmonic balance, [13], [27], [31], [49].

In the event of existence of limit cycling oscillations, the possibility, of quenching the sustained oscillations using the method of signal stabilization has been investigated, [5], [28], [29], [47], [48], in 2X2 nonlinear systems with non-memory type nonlinear elements and in memory type nonlinear elements in [37] using deterministic signals whereas the same has been addressed with Gaussian signals [52].

Prediction and suppression of limit cycling oscillations in 2 x 2 memory type nonlinear systems using arbitrary pole placement has been discussed in [30], [41], [42], [50].

The present work follows the dynamics of general 3X3 nonlinear systems shown in Fig. 2, Fig.3 [6], which is an equivalent representation of the general multivariable system of Fig.1[26].

Having realized the importance of quenching/suppression of limit cycle oscillations the present work first establishes the exhibitions of limit cycles in 3X3 nonlinear systems following the similar procedure as depicted/illustrated [6].

2. Graphical Method of prediction of LC in a general 3x3 Nonlinear Systems

In order to avoid complexity, involved in this structure a graphical method is developed for prediction of limit cycles in 3x3 nonlinear systems [6], [53].

2.1 Graphical Method

Consider a system of Fig.1, a class of 3x3 nonlinear systems for simplicity it is assumed that the whole 3x3 system exhibits the LC predominantly of a single frequency sinusoid and harmonic linearization/harmonic balance leading to use of describing function methods have been opted.

The normalized phase diagrams [44] are drawn for 3x3 systems with three combinations such as:

Combination 1: For subsystems S1, S2 & S3: C1 (+ve), C2 (-ve) and C3 (+ve)

Combination 2: For subsystems S3, S2 & S1: C2 (+ve), C3 (-ve) and C1 (+ve).

Combination 3: For subsystems S1, S3 & S2: C3 (+ve), C1 (-ve) and C2 (+ve).

Example: Used for illustration of procedures of Normalized phase diagrams.

The linear elements are represented by $G_1(s) = \frac{2}{s(s+1)^2}$; $G_2(s) = \frac{1}{s(s+4)}$; $G_3(s) = \frac{1}{s(s+2)}$ and Nonlinear elements are taken, Ideal relays as shown in Fig.2.

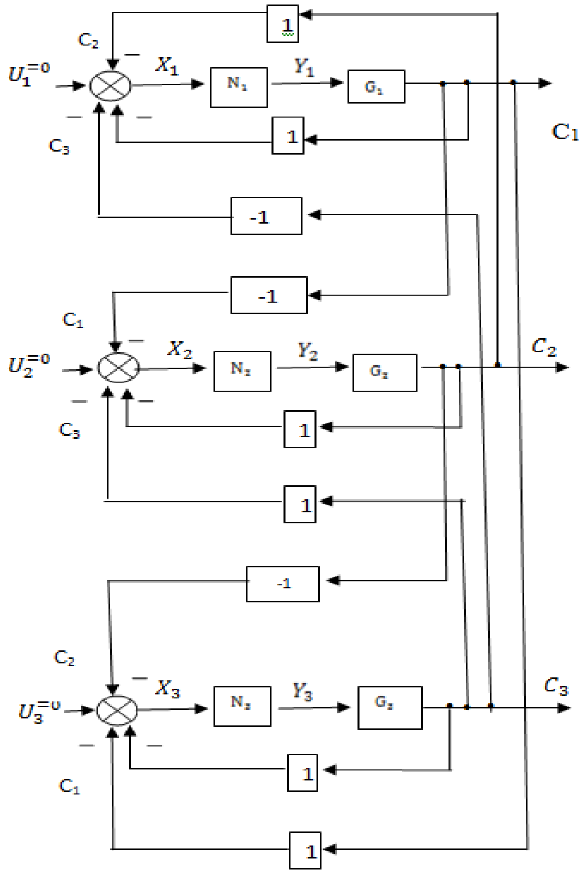


Fig. 1: A class of 3x3 multivariable nonlinear systems

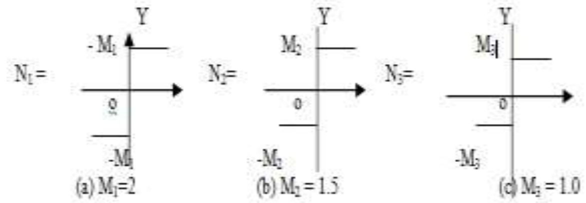


Fig. 2: All Ideal Relays

Assuming harmonic linearization these nonlinear elements can be equivalently represented by their describing functions which are real functions in these two examples and do not contribute any phase angles to the system. Hence the phase angles of the system are due to linear functions, $G_1(s)$, $G_2(s)$, $G_3(s)$ which are complex functions of complex variable s, the Laplace operator. *It may be noted that for frequency response, input is sinusoidal and outputs are steady state values considered, so that s (Laplace Operator) is replaced by jω[6].*

X_1, X_2 & X_3 are the amplitudes of respective sinusoidal inputs to the nonlinear elements. C_1, C_2 & C_3 are the amplitudes of sinusoidal output of subsystems S_1, S_2 & S_3 respectively. G_1, G_2 & G_3 are the magnitudes/absolute values of linear elements represented by their transfer functions of subsystems S_1, S_2 & S_3 respectively and N_1, N_2 & N_3 are the magnitudes/absolute values of linear elements represented by their describing functions of subsystems S_1, S_2 & S_3 respectively.

$\theta_{L1} = \text{Arg.}\,(G_1\,(j\omega)) = -90 - 2\tan^{-1}(\omega)$:

$\theta_{L2} = \text{Arg.}\,(G_2\,(j\omega)) = -90 - \tan^{-1}(\frac{\omega}{4})$:

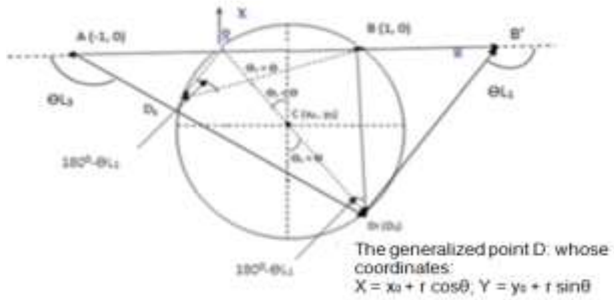
$\theta_{L3} = \text{Arg.}\,(G_3\,(j\omega)) = -90 - \tan^{-1}(\frac{\omega}{2})$:

$$N_2=(11-3\omega^2)\omega^2 \pm \sqrt{(11-3\omega^2)\omega^4 - 8(\omega^2+16)(1-\omega^2)^2\omega^2} \quad \text{.....(1)},$$

$$N_1=\frac{(\omega^2-1)}{8}N_2+\frac{9\omega^2-\omega^4}{8} \quad \cdots (2),$$

$$\frac{X_1}{X_2}=\frac{(1+\omega^2)\sqrt{[\omega^2(\omega^2+16-2N_2)+N_2^2]}}{2N_1\sqrt{\omega^2+16}} \quad \cdots (3),$$

With reference to Fig. 3(a),



$$\frac{X_1}{X_2}=\frac{BD_i}{AD_i}=\sqrt{\frac{(1-u_i)^2+(u_i)^2}{(1+u_i)^2+(u_i)^2}} \quad \text{....(4)},$$

For a fixed value of ω the combinations of subsystems 1, 2, and 3, Normalised Phase Diagrams are shown in Figure 3(a), (b), and (c) respectively. However, *any one* of these combinations can be used for the determination of limit cycling conditions and the related quantities of interest.

FIG.3 (a): Normalised phase diagram with C_1, C_2 & C_3 for the combination 1, where C_1 (+ve), C_2 (-ve) and C_3 (+ve).

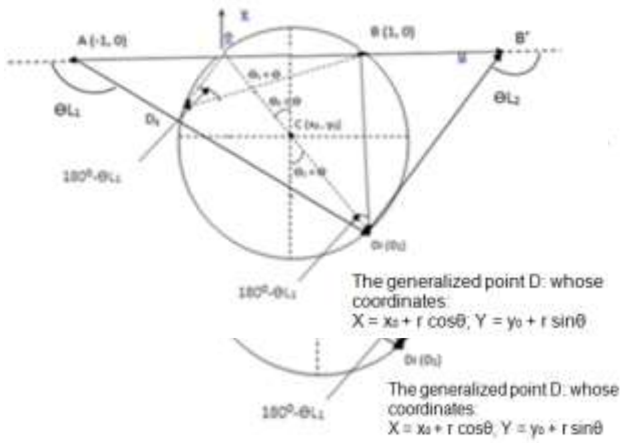


FIG.3 (b): Normalised phase diagram with C_1, C_2 & C_3 for the combination 2, where C_2 (+ve), C_3 (-ve) and C_1 (+ve).

FIG.3 (c): Normalised phase diagram with C_1, C_2 & C_3 for the combination 3, C_3 (+ve), C_1 (-ve) and C_2 (+ve).

With reference to a normalized phase diagram [44], the phase representing X_2 would lie along a straight line drawn at an angle θ_{L2} with the phase C_2 (C_2 = -R_1). The intersections of this straight line with the circle drawn with respect to θ_{L1} would represent possible self-oscillations. The concept has been extended for 3 x 3 as:

(i) Consider Fig. 3(a) the phase representing X_2 and X_3 would lie along straight lines drawn at angles θ_{L2} and Θ_{L3} with the phase C_2 ($C_2 = - R_1$) and C_3 ($C_3=R_1$) respectively. The intersections of these straight lines with the circle drawn with respect to θ_{L1} would represent possible self-oscillations.
(ii) Consider Fig. 3(b), the phase representing X_3 and X_1 would lie along straight lines drawn at angles θ_{L3} and θ_{L1} with the phase C_3 ($C_3= - R_2$) and C_1 ($C_1=R_2$) respectively. The intersections of these straight lines with the circle drawn with respect to θ_{L2} would represent possible self-oscillations.
(iii) Consider Fig. 3(c) the phase X_1 and X_2 would lie along straight lines drawn at angles θ_{L1} and θ_{L2} with phase C_1(C_1=-R_3) and C_2(C_2 = R_3) respectively. The intersections of these straight lines with the circle drawn with respect to θ_{L3} would represent possible self-oscillations.
Table 1: Shows the θ_{L1}, θ_{L2}, θ_{L3}, r (radius), and the intersection points of the straight lines and circle for combination 1 corresponding to the *example*. It may be noted that Table 1: Contains $\frac{X_1}{X_2}$ obtained from Eqn.3 and Eqn.4 are matched at a limit cycling frequency.

2.2 Digital Simulation

The Example is revisited:

A program has been developed [6] with the use of MATLAB code for digital simulation.

The equivalent canonical form of Fig. 1 for the example is shown in Fig. 4(a) and digital representation is shown in Fig. 4(b) respectively.

Numerical results obtained from different methods are compared in Table 2 for the example.

The results/images for the example obtained from digital simulation (using the developed program) and that of obtained using SIMULINK Toolbox of MATLAB software are shown in Fig. 5 for comparison.

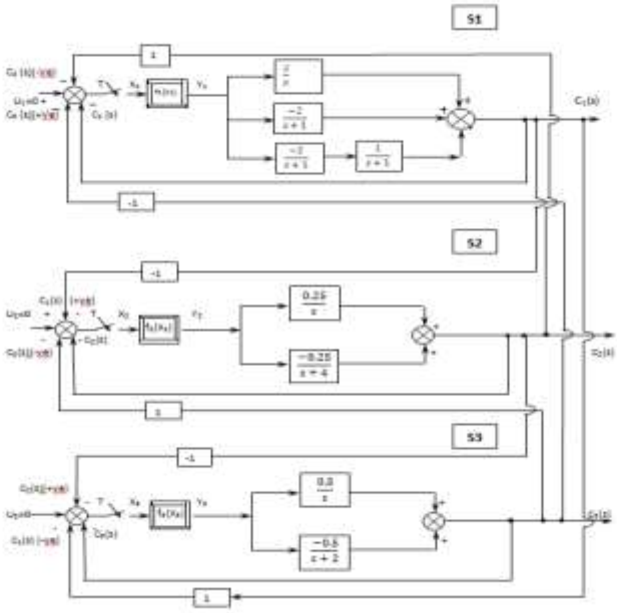


Fig. 4(a): Equivalent Canonical form of Fig.1 for the Example

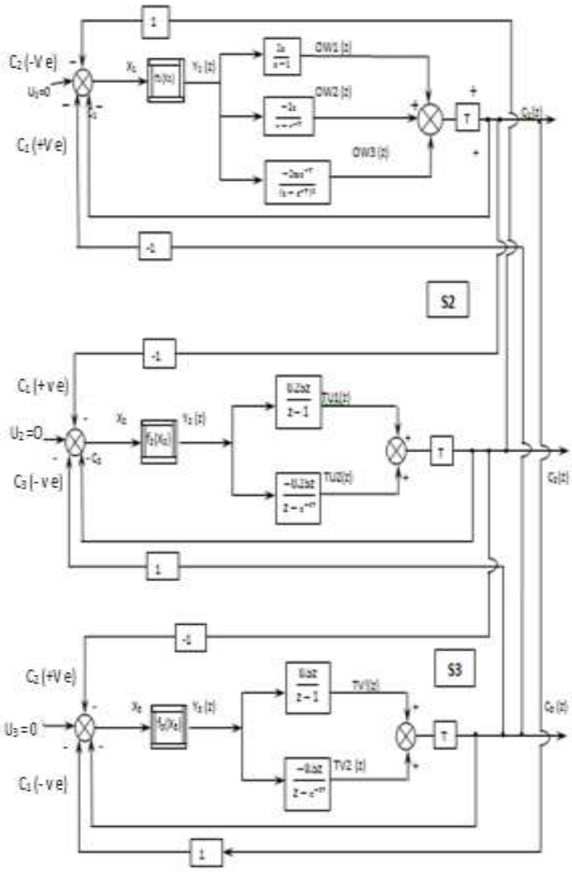
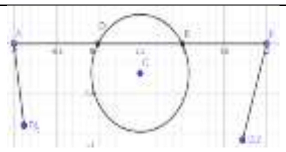
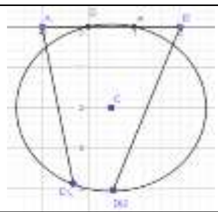
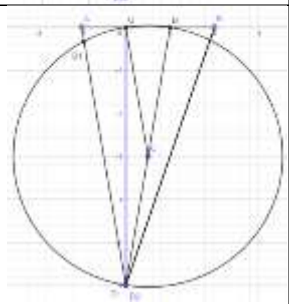
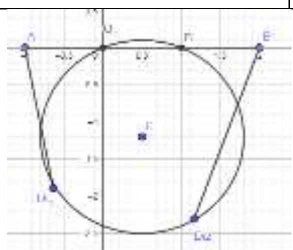


Fig. 4(b): The Digital representation of Fig.1 for the Example

Table 1: Shows the θ_{L1}, θ_{L2}, θ_{L3}, r (radius), and the intersection points of the straight lines and circles for combination 1 corresponding to the Example (with reference to Fig. 3(a)).

ω	**θ_{L1}**	**θ_{L2}**	**θ_{L3}**	**r**	**X_1/X_2 from eqn. 3**	**X_1/X_2 from eqn. 4**	**Normalized Phase Diagrams**	**Remark**
0.600	-151.93	-98.531	-106.7	-0.55257	-	-		No intersection of straight lines and circle
0.650	-156.05	-99.23	-108	0.58256	-	-		No intersection of straight lines and circle
0.700	-159.98	-99.926	-109.29	-2.128	-	-		No intersection of straight lines and circle
0.701	**-160.06**	**-99.94**	**-109.32**	**-3.1323**	**1.0**	**1.02 (matched)**		The intersection of st. lines & circle found: Confirms the occurrence of limit cycles ω=0.701, C1 = OD2 = 6 C2 = 1 C3 = 1 X1=BD2=6.08 X2=AD2=6.08 X3=B’D2= 6.32
0.750	-163.74	-100.62	-110.56	-1.3583	-	-		No intersection of straight lines and circle

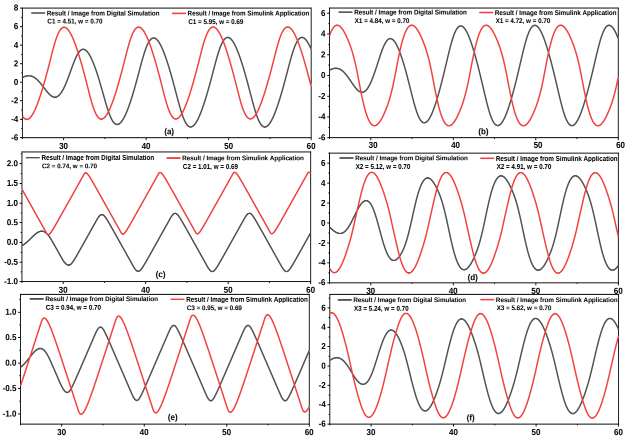


Fig. 5: Results/Images from digital simulation and SIMULINK for C_1, C_2, C_3, X_1, X_2 and X_3 of the Example (relay type nonlinearities).

Table 2 Results obtained using different methods corresponding to Ideal Relay Example

Sl. No	Methods	C_1	C_2	C_3	X_1	X_2	X_3	ω
1	Graphical	6.0	1.0	1.0	6.08	6.08	6.32	0.701
2	Digital Simulation (developed program)	4.83	0.74	0.95	4.72	4.91	5.23	0.70
3	Using SIMULINK TOOL BOX OF MATLAB	5.95	1.01	0.96	4.84	5.12	5.62	0.70

3. Signal Stabilization in 3x3 Nonlinear System

The System exhibits limit cycles (LC) in the autonomous state, the possibility of quenching the LC by injecting a suitable high frequency signal, preferably, at least 10 times of the limit cycling frequency.

3.1 Using Deterministic Signal

The forced oscillations can be realised by feeding deterministic or random signals of high frequency, at least greater than 10 times the limit cycling frequency at any one /all input points of the subsystems S_1, S_2, S_3.

If the amplitude B of the high frequency signal is gradually increased, the system would exhibit complex oscillations before the synchronization takes place. On the reverse operation, if the amplitude B is gradually reduced at certain value of B the self-oscillations i.e. the Limit cycle would reappear and the system would exhibit complex oscillations again which can be called the de-synchronisation. The phenomena of synchronization and de-synchronization can be observed / identified analytically using Incremental Input Describing function (IDF) [44].

However, the forced oscillation can also be analysed using the Equivalent Gain/Dual input Describing Function (DIDF) [44] in case of a deterministic forcing signal in particular with a sinusoidal signal. Taking the second option i.e. all three inputs are same as B $\sin\omega_f t$ at 3 input points U_1, U_2, & U_3, shown in Fig.6. Amplitude B is gradually increased, the frequency of self-oscillation, ω_s would gradually change, the system will synchronize to forcing frequency i.e. the self-oscillation would be quenched and the system would exhibit forced oscillations at frequency ω_f.

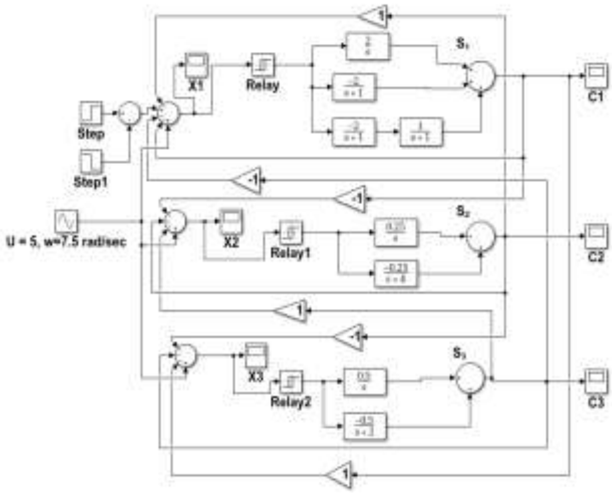


Fig 6: Equivalent System of Fig. 1 for forced oscillations (Signal Stabilization) with deterministic signal for the Example.

The results/images from digital simulation for signal stabilization with deterministic (sinusoidal signal) for the Example shown in Fig.7.

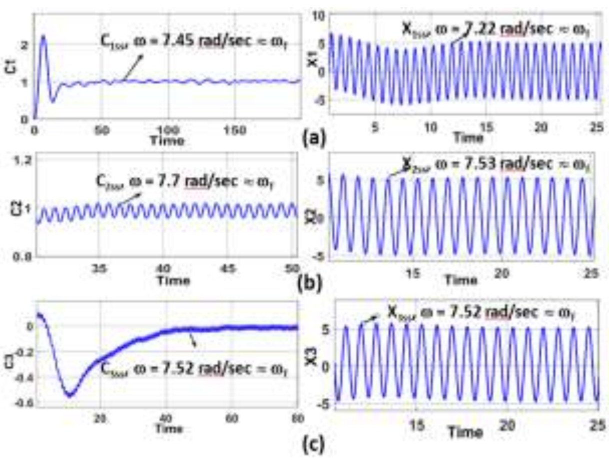


Fig. 7: Forced Oscillations by Signal Stabilization with deterministic signal for the Example Forcing Signal U = $5\sin\omega_f t$ (ω_f = 7.5 rad / sec)

3.2 Using Gaussian signal:

The forced oscillation is analysed with Equivalent Gain (similar to DIDF - Random Input Describing Function) (RIDF) in case of the Random Signals in particular with Gaussian Signals [52].

Consider the *Example*. The system is exhibiting LC under autonomous state, a Gaussian signal with specified *mean* and *variance* is injected at U_1, U_2 & U_3 of subsystems for stabilizing the system / quenching the self-sustained oscillations. At a suitable value of mean (μ) and variance (φ), the self-sustained oscillations are vanished/the system is synchronised to high frequency forcing input.

The results/images are shown in Fig. 8, which is obtained from digital simulation by signal stabilization with Gaussian signals for the *Example* replacing B sin $\omega_f t$ with a suitable random signals in Fig.7.

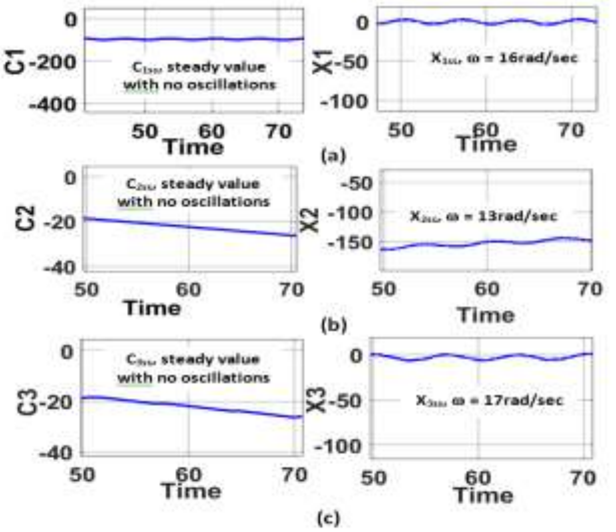


Fig. 8: Forced Oscillations by signal stabilization with Gaussian Signal of mean 50 and variance 0.05 for the Example

4. Suppression of limit cycle in 3x3 nonlinear system using pole placement technique

The System of the *Example* exhibits Limit Cycles which can be suppressed by pole placement technique [46]. The closed loop poles or Eigen values of the closed loop systems can be placed at the desired location through state feedback using an appropriate feedback gain matrix K [k_1, k_2, k_3]. Necessary and sufficient condition for arbitrary pole placement is that the system be completely state controllable [46]. This can also be done by optimal selection of feedback gain matrix K using Riccati Equation [46].

4.1 Suppression of Limit Cycles in 3x3 Nonlinear system using arbitrary Pole Placement by state feedback:

Pole placement technique by state feedback is done by determining the Eigen values or poles of the system. These Eigen values cause the limit cycles in the system, and as the complete removal of these self-oscillations may not be possible, the location of the poles must be changed from its original position so as to bring about suppression of the limit cycle. The most general multivariable nonlinear system [53] is shown in Fig. 9 (a). For existence of limit cycles, an autonomous system (input U=0) Fig. 9(a) can be represented in simplified form as shown in Fig. 9(b). Making use of the first harmonic linearization of the nonlinear elements, the matrix equation for the system of Fig. 9(b) can be expressed as

X = -HC, where C = GN(x) X. Hence,

$$X = -HGN(x) = AX \quad \cdots \quad (5)$$

Where, A = -HGN(x)

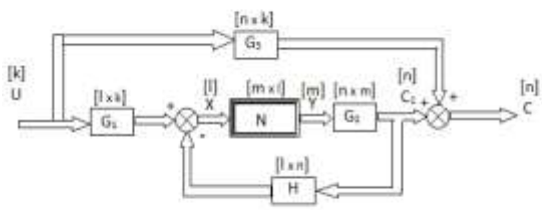


Fig. 9(a): Block diagram representation of a most general nonlinear multivariable system

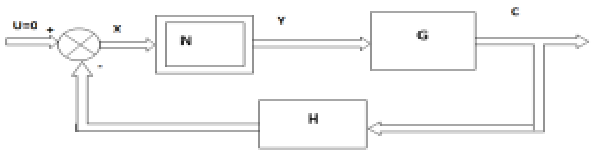


Fig. 9(b): Equivalent of the system of Fig. 9 (a) with input U= 0

Realizing Eqn. (5) as a transformation of the vector X onto itself, it is noted that for a limit cycle to exist the following two conditions should be satisfied, [6], [53]:

(i) For every non-trivial solution of X, the matrix A must has an Eigen value λ equal to unity, and
(ii) The Eigen vector of "A" corresponding to this unity Eigen value must be coincident with X.

4.1.1: Arbitrary Pole Placement for suppression of limit cycles in the Example with all ideal relays

In order to suppress the limit cycles, arbitrary pole placements may be possible if the system is completely state controllable [46].

The controllability matrix $S = [B \quad AB \; A^2B \ldots \ldots]$ … (6)

Where,

$$A = \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 & N_2G_2 & -N_3G_3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

$$H = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix};$$

$$G(\omega) = \begin{bmatrix} G_1(\omega) & 0 & 0 \\ 0 & G_2(\omega) & 0 \\ 0 & 0 & G_3(\omega) \end{bmatrix};$$

$$N(X) = \begin{bmatrix} N_1(X_1) & 0 & 0 \\ 0 & N_2(X_2) & 0 \\ 0 & 0 & N_3(X_3) \end{bmatrix};$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}; C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

From Table 1 for the *Example*,

$\omega = 0.701$ radian/sec, $X_1 = 6.08$, $X_2 = 6.08$, $X_3 = 6.32$

$N_1(X_1) = \frac{4M_1}{\pi X_1} = \frac{4x2}{\pi x 6.08} = 0.419$; $N_2(X_2) = \frac{4M_2}{\pi X_2} = \frac{4x1.5}{\pi x 6.08} = 0.314$, $N_3(X_3) = \frac{4M_3}{\pi X_3} = \frac{4x1}{\pi x 6.32} = 0.202$

$$|G_1(j\omega)| = \frac{2}{\sqrt{(\omega-\omega^3)^2+(2\omega^2)^2}} = \frac{2}{\omega(\omega^2+1)} = 1.913$$

$$|G2(j\omega)| = \frac{1}{\sqrt{(\omega^2)^2+(4\omega)^2}} = \frac{1}{\omega\sqrt{16+\omega^2}} = 0.351$$

$$|G_3| = \frac{1}{\omega\sqrt{\omega^2+4}} = 0.673$$

On substitution of the numerical values:

$-N_1G_1$ = -0.419 X 1.913 = - 0.802,

$-N_2G_2$ = - 0.314 X 0.351 = -0.110,

$-N_3G_3$ = -0.202 X 0.673 = -0.136

$$A = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix}; \quad AB = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.136 \\ -0.136 \\ -0.136 \end{bmatrix};$$

$$A^2B = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix} \begin{bmatrix} 0.136 \\ -0.136 \\ -0.136 \end{bmatrix} = \begin{bmatrix} -0.1125 \\ 0.1425 \\ -0.1055 \end{bmatrix}$$

Hence $S = \begin{bmatrix} 0 & 0.136 & -0.1125 \\ 0 & -0.136 & 0.1425 \\ 1 & -0.136 & -0.1055 \end{bmatrix} = 0.0215 \neq 0$ (*The system is completely state controllable*)

Hence arbitrary pole placement is possible [46]

$\frac{d}{dt}[x(t)] = AX + Bu$ … (7)

The system under autonomous state is represented as shown in Fig. 10.

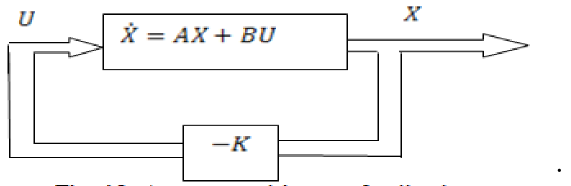


Fig. 10: A system with state feedback

Consider Fig. 10:

The control law u = -KX (8)

Where K= [$k_1 \quad k_2 \quad k_3$] is the feedback matrix.

Replacing K in Eqn. (7) by Eqn. (8), we get,

$\frac{d}{dt}[x(t)]$= (A-BK) X … (9)

Substituting the values of A, B and K, we get: The Characteristic Equation as

$$[\lambda I - (A - BK)] = 0 \text{ or}$$

$$\left|\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix} - \begin{bmatrix}-N_1G_1 & -N_2G_2 & N_3G_3\\ N_1G_1 & -N_2G_2 & -N_3G_3\\ -N_1G_1 & N_2G_2 & -N_3G_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}[k_1 \quad k_2 \quad k_3]\right| = 0$$

Hence

$$\begin{bmatrix}(\lambda + N_1G_1) & N_2G_2 & -N_3G_3\\ -N_1G_1 & (\lambda + N_2G_2) & N_3G_3\\ N_1G_1 + k_1 & -N_2G_2 - k_2 & \lambda + N_3G_3 + k_3\end{bmatrix} =$$

$$(\lambda + N_1G_1)\begin{vmatrix}\lambda + N_2G_2 & N_3G_3\\ -N_2G_2 - k_2 & \lambda + N_3G_3 + k_3\end{vmatrix}$$

$$-N_2G_2 \begin{vmatrix}-N_1G_1 & N_3G_3\\ N_1G_1 + k_1 & \lambda + N_3G_3 + k_3\end{vmatrix} -N_3G_3$$

$$\begin{vmatrix}-N_1G_1 & (\lambda + N_2G_2)\\ N_1G_1 + k_1 & -N_2G_2 - k_2\end{vmatrix} =$$

$=\lambda^3+\lambda^2(N_1G_1+N_2G_2+N_3G_3+k_3)+\lambda(2N_1N_2G_1G_2+2N_1N_3G_1G_3+2N_2N_3G_2G_3+k_1N_3G_3+k_3N_1G_1+k_3N_2G_2+k_2N_3G_3)+(4N_1N_2N_3G_1G_2G_3+2k_3N_1N_2G_1G_2+2k_1N_2N_3G_2G_3)=0\cdots\cdots$ (10) (***Ch. Equation***)

On substitution of the values of N_1, G_1, N_2, G_2 and N_3, G_3 in Eqn. (10), we get,

$\lambda^3+\lambda^2(0.136+0.11+0.802+k_3)+\lambda\{0.177+0.218+0.030+k_1 x\, 0.136+k_3(0.80+0.11)+k_2 x\, 0.136\}+(0.048+0.522+k_1 x0.03)=0$ Or

$\lambda^3+\lambda^2(1.048+k_3)+\lambda(0.425+k_1 x0.136+k_2 x\, 0.136+k_3 x0.91)+(0.57+k_1 x0.03)=0\cdots$ (11)

If the poles are **selected arbitrarily at** $\lambda_1,\lambda_2,\lambda_3 = -1, -1 \,\&\, -2$ **respectively,** the characteristic equation becomes:

$(\lambda+1)\ (\lambda+1)\ (\lambda+2) = \lambda^3+4\lambda^2+5\lambda+2=0\ \cdots$ (12)

Comparing Eq. (12) with Eq. (11), and equating the coefficients of like powers of λ we get:

$4 = 1.048 + k_3$, whence $k_3 = 2.952\cdots$ (13)

$2 = (0.57 + k_1 \text{ x } 0.03)$, whence $k_1 = 47.67 \ldots$ (14)

$5 = (0.425 + k_1 \text{ x } 0.136 + k_2 \text{ x } 0.136 + k_3 \text{ x } 0.91)$

or

$5= (0.425 + 47.67\text{x } 0.136 + k_2 \text{ x } 0.136 + 2.952\text{x}91 \text{ x } 0.91)$ or

$5=(0.425 + 6.48 + k_2 \text{ x } 0.136 + 2.68)$, whence $k_2 = -33.71 \cdots\cdots$ (15)

Hence $K = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ k1 & k2 & k3\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 47.67 & -33.71 & +2.952\end{bmatrix}\cdots\cdots$ (16)

From Eqn. (9), $(A - BK) = A_1$, with shifted poles for Example 1. Or

$$A_1 = \begin{bmatrix}-N_1G_1 & -N_2G_2 & N_3G_3\\ N_1G_1 & -N_2G_2 & -N_3G_3\\ -N_1G_1 - k_1 & N_2G_2 - k_2 & -N_3G_3 - k_3\end{bmatrix} =$$

$$\begin{bmatrix}-0.802 & -0.11 & 0.136\\ 0.802 & -0.11 & -0.136\\ -48.472 & 33.6 & -3.088\end{bmatrix}\cdots\cdots (17)$$

The images/response $C = \begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix}$, $X = \begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}$ in the autonomous state obtained from digital simulation for A_1 of *Example*, are shown in Fig.11.

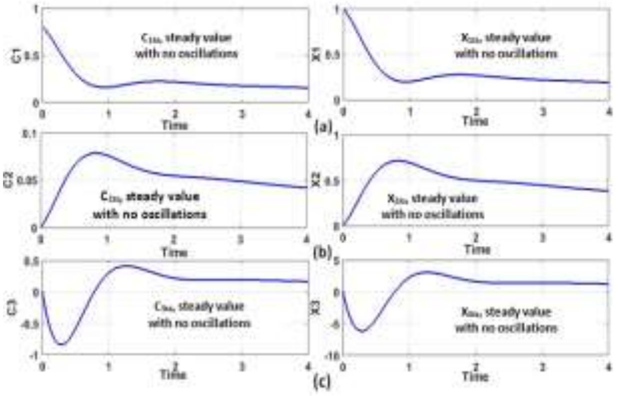


Fig. 11: Suppression of Limit Cycles by State Feedback with arbitrarily selection of feedback gain matrix for the *Example*.

4.1.2 Optimal Selection of Feedback gain Matrix using Riccati Equation for Example 1

The Riccati Equation is $A'P+PA-PBR^{-1}B'P+Q=0$ …… (18)

And K = Feedback gain matrix = $R^{-1}B'P$ …… (19)

Assuming R = 1, $B=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $Q=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$, $A=\begin{bmatrix}-N_1G_1 & -N_2G_2 & N_3G_3\\ N_1G_1 & -N_2G_2 & -N_3G_3\\ -N_1G_1 & N_2G_2 & -N_3G_3\end{bmatrix}$

Let $P=\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}$, considering P to be symmetric matrix: $p_{21}=p_{12}, p_{31}=p_{13}, p_{32}=p_{23}$

Hence $P=\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{12}&p_{22}&p_{23}\\p_{13}&p_{23}&p_{33}\end{bmatrix}$

$$A'P=\begin{bmatrix}-N_1G_1 & N_1G_1 & -N_1G_1\\ -N_2G_2 & -N_2G_2 & N_2G_2\\ N_3G_3 & -N_3G_3 & -N_3G_3\end{bmatrix}\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}=$$

$$\begin{bmatrix}(-N_1G_1p_{11}+N_1G_1p_{21}-N_1G_1p_{31}) & (-N_1G_1p_{12}+N_1G_1p_{22}-N_1G_1p_{32}) & (-N_1G_1p_{13}+N_1G_1p_{23}-N_1G_1p_{33})\\ (-N_2G_2p_{11}-N_2G_2p_{21}+N_2G_2p_{31}) & (-N_2G_2p_{12}-N_2G_2p_{22}+N_2G_2p_{32}) & (-N_2G_2p_{13}-N_2G_2p_{23}+N_2G_2p_{33})\\ (+N_3G_3p_{11}-N_3G_3p_{21}-N_3G_3p_{31}) & (+N_3G_3p_{12}-N_3G_3p_{22}-N_3G_3p_{32}) & (+N_3G_3p_{13}-N_3G_3p_{23}-N_3G_3p_{33})\end{bmatrix}$$

…… (20)

$$PA=\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}\begin{bmatrix}-N_1G_1 & -N_2G_2 & N_3G_3\\ N_1G_1 & -N_2G_2 & -N_3G_3\\ -N_1G_1 & N_2G_2 & -N_3G_3\end{bmatrix}=$$

$$\begin{bmatrix}(-N_1G_1p_{11}+N_1G_1p_{12}-N_1G_1p_{13}) & (-N_2G_2p_{11}-N_2G_2p_{12}+N_2G_2p_{13}) & (+N_3G_3p_{11}-N_3G_3p_{12}-N_3G_3p_{13})\\ (-N_1G_1p_{21}+N_1G_1p_{22}-N_1G_1p_{23}) & (-N_2G_2p_{21}-N_2G_2p_{22}+N_2G_2p_{23}) & (+N_3G_3p_{21}-N_3G_3p_{22}-N_3G_3p_{23})\\ (-N_1G_1p_{31}+N_1G_1p_{32}-N_1G_1p_{33}) & (-N_2G_2p_{31}-N_2G_2p_{32}+N_2G_2p_{33}) & (+N_3G_3p_{31}-N_3G_3p_{32}-N_3G_3p_{33})\end{bmatrix}$$

…… (21)

$$PBR^{-1}B'P=\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix},$$

$$\begin{bmatrix}0&0&1\end{bmatrix}\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}=$$

$$\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}(0xp_{11}+0xp_{21}+1xp_{31}) & (0xp_{12}+0xp_{22}+1xp_{32}) & (0xp_{13}+0xp_{23}+1xp_{33})\end{bmatrix}=$$

$$\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}\begin{bmatrix}0xp_{31}&0xp_{32}&0xp_{33}\\0xp_{31}&0xp_{32}&0xp_{33}\\1xp_{31}&1xp_{32}&1xp_{33}\end{bmatrix}=$$

$$\begin{bmatrix}p_{11}&p_{12}&p_{13}\\p_{21}&p_{22}&p_{23}\\p_{31}&p_{32}&p_{33}\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\p_{31}&p_{32}&p_{33}\end{bmatrix}=$$

$$\begin{bmatrix}p_{13}p_{31}&p_{13}p_{32}&p_{13}p_{33}\\p_{23}p_{31}&p_{23}p_{32}&p_{23}p_{33}\\p_{33}p_{31}&p_{33}p_{32}&p_{33}p_{33}\end{bmatrix} \quad \cdots\cdots (22)$$

On substitution of numerical values, Eqn. 20 can be written as

$$\begin{bmatrix}(-0.802p_{11}+0.802p_{21}-0.802p_{31}) & (-0.802p_{12}+0.802p_{22}-0.802p_{32}) & (-0.802p_{13}+0.802p_{23}-0.802p_{33})\\ (-0.11p_{11}-0.11p_{21}+0.11p_{31}) & (-0.11p_{12}-0.11p_{22}+0.11p_{32}) & (-0.11p_{13}-0.11p_{23}+0.11p_{33})\\ (+0.136p_{11}-0.136p_{21}-0.136p_{31}) & (+0.136p_{12}-0.136p_{22}-0.136p_{32}) & (+0.136p_{13}-0.136p_{23}-0.136p_{33})\end{bmatrix}$$

---- (23)

On substitution of numerical values, Eqn. 21 can be written as

$$\begin{bmatrix}(-0.802p_{11}+0.802p_{12}-0.802p_{13}) & (-0.11p_{11}-0.11p_{12}+0.11p_{13}) & (+0.136p_{11}-0.136p_{12}-0.136p_{13})\\ (-0.802p_{21}+0.802p_{22}-0.802p_{23}) & (-0.11p_{21}-0.11p_{22}+0.11p_{23}) & (+0.136p_{21}-0.136p_{22}-0.136p_{23})\\ (-0.802p_{31}+0.802p_{32}-0.802p_{33}) & (-0.11p_{31}-0.11p_{32}+0.11p_{33}) & (+0.136p_{31}-0.136p_{32}-0.136p_{33})\end{bmatrix}$$

---- (24)

On substitution of these values of Eqns. (22), (23), (24) and the assumed value of Q in Riccati Eqn. 18 yields:

$(-1.604p_{11}+1.604p_{12}-1.604p_{13})+1-p^2 13 = 0$ …… (25)

$(-0.912p_{12}+0.802p_{22}-0.802p_{23}-0.11p_{11}+0.11p_{13}-p_{13}\,p_{23}=0$ …… (26)

$(-0.938p_{13}+0.802\,p_{23}-0.802\,p_{33}+0.136\,p_{11}-0.136\,p_{12}-p_{13}\,p_{33}=0$ …… (27)

$(-0.22\ p_{12}-0.22\ p_{22}+0.22\ p_{23}-\ p^2_{23}=0$ …… (28)

$(-0.11\,p_{13}-0.246\,p_{23}+0.11\,p_{33}+0.136\,p_{12}-0.136\,p_{22}-p_{23}\,p_{33}=0$ …… (29)

$(0.272\ p_{13}-0.272\ p_{23}-0.272\ p_{33}-\ p_{33}\ p_{23})=0$ …… (30)

Further, subtracting Eqn. (29) from Eqn. (30), we get,

$0.382\,p_{13}-0.026\,p_{23}-0.382\,p_{33}-0.136\,p_{12}+0.136\,p_{22}$ …… (31)

The solution of these simultaneous Eqns. (26),(27),(28),(29),(30) & (31) yields :

$p_{11}=-116.68$, $p_{12}=-110.48$, $p_{13}=6.58$, $p_{22}=-93.24$, $p_{23}=-6.58$, $p_{33}=0$

From Eqn. (19), $K = R^{-1}\ B'P = 1\ [001]$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}$$

Or$[k_1 k_2 k_3]$=

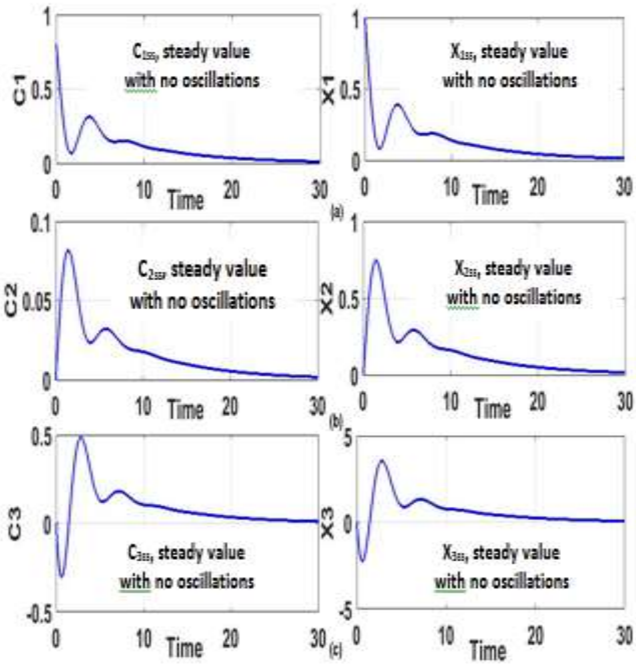
$[(0xp_{11}+0xp_{12}+1xp_{13})\ (0xp_{12}+0xp_{22}+1xp_{23})\ \ (0xp_{13}+0xp_{23}+1xp_{33})]$

Or $[k_1 k_2 k_3]$=$[(p_{13})\ (p_{23})\ (p_{33})]$

=$[6.58\ \ -6.58\ \ 0]$,

Whence, $k_1 = 6.58$, $k_2 = -6.58$ and $k_3 = 0$ …… (32)

Hence, A – BK =
A_2=



$$\begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1-k_1 & N_2G_2-k_2 & -N_3G_3-k_3 \end{bmatrix}$$

Fig. 12: Suppression of Limit Cycles by State Feedback with optimal selection of feedback gain matrix for the *Example*.

On substitution of numerical values for Example 1, A_2 becomes:

$A_2=$

$$\begin{bmatrix} -0.802 & -0.11 & 0.136 \\ 0.802 & -0.11 & -0.136 \\ -802-6.58 & 0.11+6.5 & -0.136-0 \end{bmatrix} =$$

$$\begin{bmatrix} -0.802 & -0.11 & 0.136 \\ 0.802 & -0.11 & -0.136 \\ -7.382 & 6.69 & -0.136 \end{bmatrix} \cdots\cdots (33)$$

The images/responses $C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$ and $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ in the autonomous state, obtained from digital simulation for the *Example*, are shown in Fig. 12.

5 Conclusion

In today's scenario, nonlinear self-sustained oscillations or Limit Cycles are the basic feature of instability. The existence /exhibition of such phenomena limit the performance of most of the physical systems such as the speed and position control in robotics, automation industry in particular. Quenching/Complete extinction of such LC has been a severe headache among the researchers for several decades. There are some methods, seen in the available literature which suggests the solution to this problem occurring in SISO or 2x2 systems. However, our present work explores the solution for 3x3 systems in the event of existence of LC problem and establishes the result graphically & validated by digital simulation. The novelty of the work claims in: (i) Quenching of LC exhibited in nonlinear systems by Signal Stabilization with deterministic as well as random (Gaussian) signals, (ii) Suppression of limit cycles in 3x3 nonlinear systems by Pole Placement using State feedback with arbitrary selection as well as optimal selection of feedback gain matrix K.

More importantly the poles of such 3x3 systems are shifted or placed suitably by State feedback so that the system do not exhibit limit cycles. This pole placement is done either by arbitrary selection satisfying the complete state controllability condition or by optimal selection of feedback gain matrix K using Riccati equation *which has not been attempted elsewhere*.

The present work has the brighter future scope of adopting the techniques like signal stabilization [44] and suppression of limit cycles [46] in the event of the existence of limit cycling oscillations for 3x3 higher dimensional systems through an exhaustive analysis.

Analytical/Mathematical procedures may also be developed for signal stabilization using both deterministic and random signals applying DIDF and RIDF respectively.

Backlash is one of the nonlinearities commonly occurring in physical systems which are an inherent characteristic of Governor, more popularly used for load frequency control (LFC) in power systems. The LFC shows poor performance due to the backlash characteristic of the governor. Similarly, the backlash characteristic limits the performance of speed and position control in the robotics, automation industry. The poor performance of LFC, speed and position control in robotics and in automation industries are happening since these systems exhibit limit cycles due to their backlash type of nonlinear characteristics. The proposed method of suppression of L.C. can be *extended and developed* for backlash type nonlinearity in 3x3 systems and used to completely eliminate the limit cycle to mitigate such problems.

The phenomena of synchronization and de-synchronization can be observed/identified analytically using Incremental Input Describing function (IDF).

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Kartik Chandra Patra has formulated the problem, methodology of analysis adopted and algorithm of computation presented.

Asutosh Patnaik has made the validation of the results using the geometric tools and SIMULINK toolbox of MATLAB software.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.