# Generalized Methodology Application for System Design 

ALEXANDER ZEMLIAK<br>Department of Physics and Mathematics<br>Autonomous University of Puebla<br>Av. San Claudio y 18 Sur, Puebla, 72570<br>MEXICO<br>Institute of Technical Physics<br>National Technical University of Ukraine UKRAINE


#### Abstract

The design process for analogue circuit design is formulated on the basis of the optimum control theory. The artificially introduced special control vector is defined for the redistribution of computational costs between network analysis and parametric optimization. This redistribution minimizes computer time. The problem of the minimal-time network design can be formulated in this case as a classical problem of the optimal control for some functional minimization. There is a principal difference between the new approach and before elaborated methodology. This difference is based on a higher level of the problem generalization. In this case the structural basis of design strategies is more complete and this circumstance gives possibility to obtain a great value of computer time gain. Numerical results demonstrate the effectiveness and prospects of a more generalized approach to circuit optimization. This approach generalizes the design process and generates an infinite number of the different design strategies that will serve as the structural basis for the minimal time algorithm construction. This paper is advocated to electronic systems built with transistors. The main equations for the system design process were elaborated.


Key-Words: - Circuit optimization, control theory formulation, controllable dynamic system, optimization strategies, generalized methodology.

Received: June 29, 2021. Revised: November 3, 2021. Accepted: November 17, 2021. Published: January 3, 2022.

## 1 Introduction

One of the sources of overall improvement in design quality is the reduction of CPU time in the design of large systems. This problem has a great significance because it has a lot of applications, for example on VLSI electronic circuit design. Any traditional system design strategy includes two main parts: the mathematical model of the physical system that can be defined by the algebraic equations or differentialintegral equations and optimization procedure that achieves the optimum point of the design objective function. Within the framework of this concept, it is possible to change the optimization strategy and use different models and different analysis methods, but at each stage of the optimization process of the circuit there is a fixed number of equations of the mathematical model and a fixed number of independent parameters when optimizing the circuit.

Some powerful techniques have been used to reduce the time required to analyze the circuit. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used
successfully for this purpose [1-2]. Other approach to reduce the amount of computational required for both linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution, algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macro model representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the
different optimization criterions [8-9]. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers [10-11].

The ideas of designing the system described above can be called the traditional approach or the traditional strategy, since the method of analysis is based on the laws of Kirchhoff. Other idea for the problem of optimizing the circuit were developed at a heuristic level several decades ago [12]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [13] and for the synthesis of high-performance analog circuits [14] in extremely case, when the total system model was eliminated. The authors of the last papers affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

At the same time, all these ideas can be generalized to reduce the total computer design time for system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [15]. The number of the different design strategies, which appear in the generalized theory of the first level, is equal to $2^{M}$ for the constant value of all the control functions, where $M$ is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm. One way to solve this problem is to use the Lyapunov function of the design process [16].

However, the developed theory [15] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of
the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

In this case the new objective function would be introduced to take into account the corresponding information about the system. The number of the different design strategies, which appear in this new generalized theory, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$ for the constant value of all the control functions, where $M$ is the number of dependent parameters and $K$ is the number of independent parameters. These strategies serve as the structural basis for other strategies construction with the variable control functions. The almost infinite number of the different design strategies appears for this methodology in contrast to the results [15] where only one particular case was studied. The characteristic curves of the transistor must be taken into account in order to obtain both a good and a real design.

## 2 Problem Formulation

In accordance with the last design methodology [15] the design process is defined by the optimization procedure, which can be determined in continuous form as:

$$
\begin{equation*}
\frac{d x_{i}}{d t}=f_{i}(X, U), \quad i=1,2, \ldots, N \tag{1}
\end{equation*}
$$

and by the analysis of the electronic system model in next form:

$$
\begin{equation*}
\left(1-u_{j}\right) g_{j}(X)=0, \quad j=1,2, \ldots, M \tag{2}
\end{equation*}
$$

where $N=K+M, K$ is the number of independent system parameters, $M$ is the number of dependent system parameters, $X$ is the vector of all variables $X=\left(x_{1}, x_{2}, \ldots, x_{N}\right) ; U$ is the vector of control variables $U=\left(u_{1}, u_{2}, \ldots, u_{M}\right)$, where $u_{j} \in \Omega$; $\Omega=\{0 ; 1\}$.

The functions of the right part of the system (1) can be determined for the gradient method for instance as:
$f_{i}(X, U)=-b \frac{\delta}{\delta x_{i}}\left\{C(X)+\frac{1}{\varepsilon} \sum_{j=1}^{M} u_{j} g_{j}^{2}(X)\right\}$
for $i=1,2, \ldots, K$,

$$
\begin{align*}
f_{i}(X, U)= & -b \cdot u_{i-K} \frac{\delta}{\delta x_{i}}\left\{C(X)+\frac{1}{\varepsilon} \sum_{j=1}^{M} u_{j} g_{j}^{2}(X)\right\} \\
& +\frac{\left(1-u_{i-K}\right)}{d t}\left\{-x_{i}^{\prime}+\eta_{i}(X)\right\} \tag{3'}
\end{align*}
$$

for $\quad i=K+1, K+2, \ldots, N$,
where $C(X)$ is the traditional objective function of the design process. By this formulation the initially dependent parameters for $i=K+1, K+2, \ldots, N$ can be transformed to the independent ones when $u_{j}=1$ and it is dependent when $u_{j}=0$. On the other hand the initially independent parameters are independent ones always. The optimal behavior of the control functions for the minimal-time problem can be found by means of some approximate methods of the control theory [17]-[19].

We develop the new approach that permits to generalize more the design methodology [15]. We suppose now that all system parameters can be independent or dependent ones. In this case we need to change the equations (2) and (3). The equation (2) is transformed to the next one:

$$
\begin{gather*}
\left(1-u_{i}\right) g_{j}(X)=0  \tag{4}\\
i=1,2, \ldots, N \text { and } j \in J
\end{gather*}
$$

where $J$ is the index set of all those functions for which. $u_{i}=0, J=\left\{j_{1}, j_{2}, \ldots, j_{z}\right\}, j_{s} \in \Pi$ with $s=1$, $2, \ldots, z$, where $\Pi$ is the set of indexes from 1 to $M$, $\Pi=\{1,2, \ldots, M\}, z$ is the number of equations that will be left in the system (4), $z \in\{0,1 \ldots, M\}$. The right hand side of the system (1) is defined now as:

$$
\begin{align*}
& f_{i}(X, U)=-b \cdot u_{i} \frac{d}{d x_{i}} F(X, U)  \tag{5}\\
&+\frac{\left(1-u_{i}\right)}{d t}\left\{-x_{i}^{\prime}+\eta_{i}(X)\right\} \\
& i=1,2, \ldots, N
\end{align*}
$$

where $F(X, U)$ is the generalized objective function and it is defined as:

$$
\begin{equation*}
F(X, U)=C(X)+\frac{1}{\varepsilon} \sum_{j \in \Pi \backslash J} u_{j} g_{j}^{2}(X) \tag{6}
\end{equation*}
$$

This definition of the design process is more general than in [15]. It generalizes the methodology for the system design and produces more
representative structural basis of different design strategies.

## 3 Numerical Results

### 3.1 Example 1

In Fig. 1 there is a circuit that has 3 independent variables as admittance $y_{1}, y_{2}, y_{3}(K=3)$ and 3 dependent variables as nodal voltages $V_{1}, V_{2}, V_{3}$ $(M=3)$ at the nodes 1,2 , and 3 respectively.


Fig. 1 Three-node circuit
Kirchhoff's law applied to this circuit includes three equations that can be written as follows:

$$
\begin{align*}
& g_{1}=I_{B}-\left(E_{1}-V_{1}\right) \cdot y_{1}=0 \\
& g_{2}=I_{E}-V_{2} \cdot y_{2}=0  \tag{7}\\
& g_{3}=I_{C}-\left(E_{2}-V_{3}\right) \cdot y_{3}=0
\end{align*}
$$

The $X$ vector includes seven components defined by the following formulas: $x_{1}^{2}=y_{1}, x_{2}^{2}=y_{2}$, $x_{3}^{2}=y_{3}, x_{4}=V_{1}, x_{5}=V_{2}$ and $x_{6}=V_{3}, I_{\mathrm{E}}, I_{\mathrm{C}}$ and $I_{\mathrm{B}}$ can be obtained in four regions by Ebers-Moll static model, implemented in SPICE2 [20]. The system model is determined by the following equations:

$$
\begin{align*}
& g_{1}(X)=I_{B}-\left(E_{1}-x_{4}\right) \cdot x_{1}^{2} \\
& g_{2}(X)=I_{E}-\left(x_{2}^{2} \cdot x_{5}\right)  \tag{8}\\
& g_{3}(X)=I_{C}-\left(E_{2}-x_{6}\right) \cdot x_{3}^{2}
\end{align*}
$$

The optimization procedure includes six equations in this case:

$$
\begin{align*}
f_{i}(X, U)= & -b \cdot u_{i} \frac{\delta}{\delta x_{i}} F(X, U)  \tag{9}\\
& +\frac{\left(1-u_{i}\right)}{d t}\left\{-x_{i}(t-d t)+\eta(X)\right\}
\end{align*}
$$

$i=1,2, \ldots, 6$.
3.1.1 Strategy (111000). This is the traditional design strategy. Only three first equations of the system (10) compose the optimization procedure with objective function $F(X)=C(X)$ and with three equations (8) that permit to calculate all the coordinates of the vector $X$. Equations (8) are solved by the Newton-Raphson method. Having characterized the transistor, selecting one operation point (e.g. $\mathrm{V}_{\mathrm{BC}}=-2.2 \mathrm{~V} ., \mathrm{V}_{\mathrm{CE}}=2.9 \mathrm{~V}$. and $\mathrm{V}_{\mathrm{BE}}=$ 0.7 V.$)$, the characteristic for this amplifier is to has the Collector voltage similar to a constant value then the function objective is defined as $C(x)=\left(x_{6}-m m_{1}\right)^{2}$, but in order to study all the trajectories arriving to the same final point, we add the terms $\left(x_{4}-x_{5}-m m_{2}\right)^{2}$ and $\left(x_{4}-x_{6}-m m_{3}\right)^{2}$, $m m_{2}$ and $m m_{3}$ are the voltages of union of the transistor, therefore the function ordinary objective $C(X)$ is defined by the following formula:

$$
C(x)=\left(x_{6}-m m\right)^{2}+\left(x_{4}-x_{5}-m m_{2}\right)^{2}+\left(x_{4}-x_{6}-m m_{3}\right)^{2} .
$$

3.1.2 Strategy (111111). This is the modified traditional design strategy. The six equations of the system (9) compose the optimization procedure with the objective function $F(X)$ but the equations (8) disappear from the system's model. The objective function $F(X)$ is defined by the following form:

$$
\begin{equation*}
F(X)=C(X)+\sum_{j=1}^{3} g_{j}^{2}(x) \tag{10}
\end{equation*}
$$

3.1.3 Intermediate strategies. Others strategies are intermediate ones. Some of these are the strategies that appear in the previously developed methodology and the others are the strategies that appear inside the new generalized approach. Only some of the total number of the different design strategies are shown in Table 1, because of the number of strategies for this example are equal to $\sum_{i=0}^{3} C_{6}^{i}=32$ strategies. Table 1 corresponds to the "old" strategies that have been analyzed in previous papers. Table 2 corresponds to the new strategies that appear in limits of the proposed approach.

Table 1. Strategies of the "old" structural basis.

|  | Strategy | Iterations | Time $(\mathrm{ms})$ |
| :---: | :---: | :---: | :---: |
| 1 | 111000 | 9311 | 7977.00 |
| 2 | 111001 | 7514 | 4989.11 |
| 3 | 111010 | 75635 | 43053.10 |
| 4 | 111011 | 324 | 60.10 |


| 5 | 111100 | 25079 | 10970.1 |
| :---: | :---: | :---: | :---: |
| 6 | 111101 | 243 | 40.11 |
| 7 | 111110 | 10232 | 2398.5 |
| 8 | 111111 | 2418 | 196.21 |

Table 2. Some strategies of the "new" structural basis.

|  | Strategy | Iterations | Time $(\mathrm{ms})$ |
| :---: | :---: | :---: | :---: |
| 1 | 101111 | 30 | 5.00 |
| 2 | 110111 | 778 | 139.10 |
| 3 | 101110 | 55992 | 25094.21 |
| 4 | 011100 | 12850 | 10992.33 |
| 5 | 011110 | 30015 | 10998.24 |
| 6 | 011101 | 47 | 19.73 |
| 7 | 110011 | 174 | 60.01 |
| 8 | 110101 | 606 | 220.21 |

### 3.2 Example 2

In Fig. 2 there is a circuit that has 5 independent variables as admittance $y_{1}, y_{2}, y_{3}, y_{4}, y_{5}(K=5)$ and 5 dependent variables as nodal voltages $V_{1}, V_{2}, V_{3}, V_{4}, V_{5} \quad(M=5)$. The total number of variables are $N=M+K=10$.


Fig. 2 Five-node circuit
Kirchhoff's law applied to this circuit includes three equations that can be written as follows:

$$
\begin{align*}
& g_{1}=\left(E_{1}-V_{1}\right) \cdot y_{1}-I_{B 1}=0 \\
& g_{2}=V_{2} \cdot y_{2}-I_{E 1}=0 \\
& g_{3}=\left(E_{2}-V_{3}\right) \cdot y_{3}-I_{C 1}-I_{B 2}=0  \tag{11}\\
& g_{4}=V_{4} \cdot y_{4}-I_{E 2}=0 \\
& g_{5}=\left(E_{2}-V_{5}\right) \cdot y_{5}-I_{C 2}=0
\end{align*}
$$

The $X$ vector includes ten components defined by the following formulas: $\quad x_{1}^{2}=y_{1}, \quad x_{2}^{2}=y_{2}$, $x_{3}^{2}=y_{3}, x_{4}^{2}=y_{4}, \quad x_{5}^{2}=y_{5}, x_{6}=V_{1}, x_{7}=V_{2}, x_{8}=$ $V_{3}, x_{9}=V_{4}, x_{10}=V_{5}$. The model of the system includes five equations:

$$
\begin{align*}
& g_{1}(X)=\left(E_{1}-x_{6}\right) \cdot x_{1}^{2}-I_{B 1}=0 \\
& g_{2}(X)=x_{7} \cdot x_{2}^{2}-I_{E 1}=0 \\
& g_{3}(X)=\left(E_{2}-x_{8}\right) \cdot x_{3}^{2}-I_{C 1}-I_{B 2}=0  \tag{12}\\
& g_{4}(X)=x_{9} \cdot x_{4}^{2}-I_{E 2}=0 \\
& g_{5}(X)=\left(E_{2}-x_{10}\right) \cdot x_{5}^{2}-I_{C 2}=0
\end{align*}
$$

$I_{\mathrm{E}}, I_{\mathrm{C}}$ and $I_{\mathrm{B}}$ can be obtained in four regions by Ebers-Moll static model, implemented in SPICE2. The optimization procedure includes ten equations in this case:

$$
\begin{align*}
f_{i}(X, U)= & -b \cdot u_{i} \frac{\delta}{\delta x_{i}} F(X, U)  \tag{13}\\
& +\frac{\left(1-u_{i}\right)}{d t}\left\{-x_{i}(t-d t)+\eta(X)\right\}
\end{align*}
$$

$$
i=1,2, \ldots, 10
$$

3.2.1 Strategy (1111100000). This is the traditional design strategy. Only five first equations of the system (13) compose the optimization procedure with objective function $F(X)=C(X)$ and with five equations (12) that permit to calculate all of the coordinates of the vector $X$. Equations (12) are solved by the Newton-Raphson method. Having characterized the transistor, selecting one operation point (e.g. $\mathrm{V}_{\mathrm{BC} 1}=-2.2 \mathrm{~V} ., \mathrm{V}_{\mathrm{CE} 1}=2.9, \mathrm{~V}_{\mathrm{BE} 1}=0.7 \mathrm{~V} .$, $\mathrm{V}_{\mathrm{BC} 2}=-2.2 \mathrm{~V} ., \mathrm{V}_{\mathrm{CE} 2}=2.9 \mathrm{~V}$. and $\mathrm{V}_{\mathrm{BE} 2}=0.7 \mathrm{~V}$.), the characteristic for this amplifier is to has the Collector voltage similar to a constant value then the function objective it is defined as $C(X)=\left(x_{10}-m m_{1}\right)^{2}$, but in order to study all the trajectories arriving to the same final point, we add the terms $\quad\left(x_{6}-x_{7}-m m_{2}\right)^{2}, \quad\left(x_{6}-x_{8}-m m_{3}\right)^{2}$, $\left(x_{8}-x_{9}-m m_{4}\right)^{2}$ and $\left(x_{8}-x_{10}-m m_{5}\right)^{2}$ then the traditional objective function $C(X)$ is defined by the following form:

$$
\begin{align*}
C(x) & =\left(x_{10}-m m_{1}\right)^{2}+\left(x_{6}-x_{7}-m m_{2}\right)^{2} \\
& +\left(x_{6}-x_{8}-m m_{3}\right)^{2}+\left(x_{8}-x_{9}-m m_{4}\right)^{2}  \tag{14}\\
& +\left(x_{8}-x_{10}-m m_{5}\right)^{2}
\end{align*}
$$

where $m m_{1}=7.8,{m m_{2}}_{2}=V_{B E I}=0.7,{m m_{3}}=V_{B C I}=-$ 2.2, $\mathrm{mm}_{4}=V_{B E 2}=0.7$ and $m m_{5}=V_{B C 2}=-2.2$
3.2.2 Strategy (1111111111). This is the modified traditional design strategy. The ten equations of the system (16) compose the optimization procedure with the objective function $F(X)$ but the equations (12) disappear from the system's model. The
objective function $F(X)$ is defined by the following form:

$$
\begin{equation*}
F(X)=C(X)+\sum_{j=1}^{5} g_{j}^{2}(x) \tag{15}
\end{equation*}
$$

3.2.3 Intermediate strategies. Others strategies are intermediate ones. Some of these are the strategies that appear in the previously developed methodology and the others are the strategies that appear inside the new generalized approach. Only some of the total number of the different design strategies are shown in Table 3, and Table 4 because of the number of strategies for this example are equal to $\sum_{i=0}^{5} C_{10}^{i}=512$ strategies. Table 3 corresponds to the "old" strategies that have been analyzed in previous papers. Table 4 corresponds to the "new" strategies that appear in limits of the proposed approach.

Table 3. Some "old" strategies.

|  | Strategy | Iterations | Time (s) |
| :---: | :---: | :---: | :---: |
| 1 | 1111100000 | 83402 | 333.6 |
| 2 | 1111100011 | 6695 | 8.990 |
| 3 | 1111100111 | 3395 | 4.007 |
| 4 | 1111101111 | 253 | 1.290 |
| 5 | 1111110001 | 70887 | 125.994 |
| 6 | 1111110011 | 93677 | 92.018 |
| 7 | 1111110111 | 588 | 2.700 |
| 8 | 111111001 | 148299 | 158.038 |
| 9 | 1111111011 | 24678 | 15.945 |
| 10 | 111111100 | 56464 | 57.015 |
| 11 | 1111111101 | 496 | 2.400 |
| 12 | 111111110 | 5583 | 2.007 |
| 13 | 1111111111 | 614 | 1.699 |

Table 4. Some "new" strategies.

|  | Strategy | Iterations | Time (s) |
| :---: | :---: | :---: | :---: |
| 1 | 0000011111 | 55 | 0.159 |
| 2 | 0000111110 | 7912 | 23.985 |
| 3 | 0000111111 | 209 | 0.429 |
| 4 | 0001111100 | 57245 | 229.963 |
| 5 | 0001111111 | 420 | 0.560 |
| 6 | 0011111011 | 25884 | 52.022 |
| 7 | 0011111101 | 232 | 0.309 |
| 8 | 0011111110 | 138426 | 230.014 |
| 9 | 0011111111 | 381 | 0.319 |
| 10 | 0101010111 | 201 | 0.400 |
| 11 | 0101110100 | 47186 | 190.979 |


| 12 | 0101110111 | 242 | 0.329 |
| :---: | :---: | :---: | :---: |
| 13 | 0101111111 | 371 | 0.319 |
| 14 | 0110110111 | 338 | 0.440 |
| 15 | 0110111111 | 414 | 0.340 |
| 16 | 0111010111 | 156 | 0.209 |
| 17 | 0111011111 | 480 | 0.409 |
| 18 | 0111110110 | 8511 | 11.998 |
| 19 | 0111110111 | 68 | 0.080 |
| 20 | 0111111011 | 22381 | 26.012 |
| 21 | 0111111100 | 31525 | 55.060 |
| 22 | 0000011111 | 55 | 0.159 |
| 23 | 0000111110 | 7912 | 23.985 |
| 24 | 0000111111 | 209 | 0.429 |
| 25 | 0001111100 | 57245 | 229.963 |
| 26 | 0001111111 | 420 | 0.560 |
| 27 | 0011111011 | 25884 | 52.022 |
| 28 | 0011111101 | 232 | 0.309 |
| 29 | 0011111110 | 138426 | 230.014 |
| 30 | 0011111111 | 381 | 0.319 |
| 31 | 0101010111 | 201 | 0.400 |
| 32 | 0101110100 | 47186 | 190.979 |
| 33 | 0101110111 | 242 | 0.329 |
| 34 | 0101111111 | 371 | 0.319 |
| 35 | 0111111110 | 9264 | 8.961 |
| 36 | 011111111 | 205 | 0.0906 |
| 37 | 1000001111 | 98 | 0.290 |
| 38 | 1000011111 | 150 | 0.309 |
| 39 | 1001101100 | 40121 | 165.00 |
| 40 | 1001101111 | 286 | 0.379 |
| 41 | 1001111101 | 170 | 0.239 |
| 42 | 1001111111 | 547 | 0.479 |
|  |  |  |  |

### 3.3 Example 3

In Fig. 3 there is a circuit that has 7 independent variables as admittance $y_{1}, y_{2}, y_{3}, y_{4}, y_{5}, y_{6}, y_{7}$ ( $K=7$ ) and 7 dependent variables as nodal voltages $V_{1}, V_{2}, V_{3}, V_{4}, V_{5}, V_{6}, V_{7} \quad(M=7)$.


Fig. 3 Seven-node circuit

Kirchhoff law applying for this circuit the seven equations can be writing in following form:

$$
\begin{align*}
& g_{1}=\left(E_{1}-V_{1}\right) \cdot y_{1}-I_{B 1}=0 \\
& g_{2}=V_{2} \cdot y_{2}-I_{E 1}=0 \\
& g_{3}=\left(E_{2}-V_{3}\right) \cdot y_{3}-I_{C 1}-I_{B 2}=0 \\
& g_{4}=V_{4} \cdot y_{4}-I_{E 2}=0  \tag{16}\\
& g_{5}=\left(E_{2}-V_{5}\right) \cdot y_{5}-I_{C 2}-I_{B 3}=0 \\
& g_{6}=V_{6} \cdot y_{6}-I_{E 3}=0 \\
& g_{7}=\left(E_{2}-V_{7}\right) \cdot y_{7}-I_{C 3}=0
\end{align*}
$$

The $X$ vector includes fourteen components defined by the following formulas: $x_{1}^{2}=y_{1}, x_{2}^{2}=y_{2}$, $x_{3}^{2}=y_{3}, \quad x_{4}^{2}=y_{4}, x_{5}^{2}=y_{5}, x_{6}^{2}=y_{6}, x_{7}^{2}=y_{7}$, $x_{8}=V_{1}, x_{9}=V_{2}, x_{10}=V_{3}, x_{11}=V_{4}, x_{12}=V_{5}, x_{13}=$ $V_{6}, x_{14}=V_{7}, E_{1}=5 \mathrm{~V}$ y $E_{2}=10 \mathrm{~V}$, The model of the system is:

$$
\begin{align*}
& g_{1}(X)=\left(E_{1}-x_{8}\right) \cdot x_{1}^{2}-I_{B 1}=0 \\
& g_{2}(X)=x_{9} \cdot x_{2}^{2}-I_{E 1}=0 \\
& g_{3}(X)=\left(E_{2}-x_{10}\right) \cdot x_{3}^{2}-I_{C 1}-I_{B 2}=0 \\
& g_{4}(X)=x_{11} \cdot x_{4}^{2}-I_{E 2}=0  \tag{17}\\
& g_{5}(X)=\left(E_{2}-x_{12}\right) \cdot x_{5}^{2}-I_{C 2}-I_{B 3}=0 \\
& g_{6}(X)=x_{13} \cdot x_{6}^{2}-I_{E 3}=0 \\
& g_{7}(X)=\left(E_{2}-x_{14}\right) \cdot x_{7}^{2}-I_{C 3}=0
\end{align*}
$$

The optimization procedure includes fourteen equations in this case:

$$
\begin{align*}
f_{i}(X, U)= & -b \cdot u_{i} \frac{\delta}{\delta x_{i}} F(X, U)  \tag{18}\\
& +\frac{\left(1-u_{i}\right)}{d t}\left\{-x_{i}(t-d t)+\eta(X)\right\}
\end{align*}
$$

$i=1,2, \ldots, 14$.
3.3.1 Strategy (11111110000000). This is the traditional design strategy. Only seven first equations of the system (18) compose the optimization procedure with objective function $F(X)=C(X)$ and with five equations (17) that permit to calculate all of the coordinates of the vector $X$. Equations (17) are solved by the Newton-Raphson method. Having characterized the transistor, selecting one operation point (e.g. $\mathrm{V}_{\mathrm{BC} 1}=-1.7 \mathrm{~V}$., $\mathrm{V}_{\mathrm{BE} 1}=0.6 \mathrm{~V} ., \mathrm{V}_{\mathrm{BC} 2}=-1.0 \mathrm{~V} ., \mathrm{V}_{\mathrm{BE} 2}=0.6 \mathrm{~V} ., \mathrm{V}_{\mathrm{BC} 3}=$ -1.2 V . and $\left.\mathrm{V}_{\mathrm{BE} 3}=0.7 \mathrm{~V}.\right)$, .), the characteristic for this amplifier is to has the Collector voltage similar to a constant value then the function objective it is defined as $C(X)=\left(x_{14}-m m_{1}\right)^{2}$ but in order to study
all the trajectories arriving to the same final point, we add the terms $\left(x_{8}-x_{9}-m m_{2}\right)^{2},\left(x_{8}-x_{10}-m m_{3}\right)^{2}$, $\left(x_{10}-x_{11}-m m_{4}\right)^{2},\left(x_{10}-x_{12}-m m_{5}\right)^{2},\left(x_{12}-x_{13}-m m_{6}\right)^{2}$ y $\left(x_{12}-x_{14}-m m_{7}\right)^{2}$ then the traditional objective function $C(X)$ is defined by the following form:

$$
\begin{align*}
C(X)= & \left(x_{14}-m m_{1}\right)^{2}+\left(x_{8}-x_{9}-m m_{2}\right)^{2} \\
& +\left(x_{8}-x_{10}-m m_{3}\right)^{2}+\left(x_{10}-x_{11}-m m_{4}\right)^{2}  \tag{19}\\
& +\left(x_{10}-x_{12}-m m_{5}\right)^{2}+\left(x_{12}-x_{13}-m m_{6}\right)^{2} \\
& +\left(x_{12}-x_{14}-m m_{7}\right)^{2}
\end{align*}
$$

3.3.2 Strategy (111111111111111). This is the modified traditional design strategy. The fourteen equations of the system (18) compose the optimization procedure with the objective function $F(X)$ but the equations (17) disappear from the system's model. The objective function $F(X)$ is defined by the following form:

$$
\begin{equation*}
F(X)=C(X)+\sum_{j=1}^{7} g_{j}^{2}(x) \tag{20}
\end{equation*}
$$

3.3.3 Intermediate strategies. Others strategies are intermediate ones. Some of these are the strategies that appear in the previously developed methodology and the others are the strategies that appear inside the new generalized approach. Only some of the total number of the different design strategies are shown in Table 3, because of the number of strategies for this example are equal to $\sum_{i=0}^{7} C_{14}^{i}=16384$ strategies. Table 5 corresponds to the old strategies that have been analyzed in previous papers. Table 6 corresponds to the new strategies that appear in limits of the proposed approach.

Table 5 Some "old" strategies.

|  | Strategy | Iterations | Time (s) |
| :---: | :---: | :---: | :---: |
| 1 | 11111110000000 | 38775 | 351456.6 |
| 2 | 11111110000001 | 100843 | 742993.0 |
| 3 | 11111110000100 | 45407 | 440014.0 |
| 4 | 11111110010000 | 2643 | 29002.0 |
| 5 | 11111110100000 | 82811 | 1163987.0 |
| 6 | 11111110111111 | 1127 | 1020.0 |
| 7 | 11111111000000 | 10454 | 89019.0 |
| 8 | 11111111011111 | 540 | 955.0 |
| 9 | 11111111101111 | 53880 | 61040.0 |
| 10 | 11111111110111 | 1008 | 1007.0 |
| 11 | 11111111111011 | 5647 | 6012.0 |


| 12 | 11111111111101 | 226 | 1885.0 |
| :---: | :---: | :---: | :---: |
| 13 | 11111111111110 | 7441 | 7999.0 |
| 14 | 11111111111111 | 3979 | 4005.0 |

Table 6 Some "new" strategies.

|  | Strategy | Iterations | Time (s) |
| :---: | :---: | :---: | :---: |
| 1 | 00000001111111 | 72 | 549.0 |
| 2 | 00000011111111 | 235 | 1030.0 |
| 3 | 00000111111111 | 506 | 1030.0 |
| 4 | 00001111111111 | 891 | 2980.0 |
| 5 | 00011111111111 | 660 | 1050.0 |
| 6 | 00111111111111 | 1262 | 2002.0 |
| 7 | 01111111111111 | 504 | 953.0 |
| 8 | 10111111111111 | 351 | 380.0 |
| 9 | 11011111111111 | 316 | 350.0 |
| 10 | 11101111111111 | 662 | 709.3 |
| 11 | 11110111111111 | 801 | 986.0 |
| 12 | 11111011111111 | 532 | 956.0 |
| 13 | 11111100000001 | 11993 | 129003.0 |
| 14 | 11111101111111 | 308 | 30.10 |

Table 7 summarizes the integral information about the computer gain for two levels of generalized optimization for all examples.

Table 7 Summary of Gain

| Example | Gain, Old <br> Strategy | Gain, New <br> Strategy |
| :---: | :---: | :---: |
| 1 | 198.8 | 1595.4 |
| 2 | 258.60 | 4170 |
| 3 | 368.01 | 11676 |

In Fig. 4 we show the behavior of gains of the first and second level of generalization for active circuits.


Fig. 4 Gain in time for active circuits

## 4 Conclusion

We developed a new and more complete approach to the electronic system design with transistors. We have checked that this approach generates more broadened structural basis of different design strategies. The total number of the different strategies, which compose the structural basis by this approach, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$ and the previous methodology produced $2^{M}$ strategies only. Some new strategies have better convergence and lesser computer time than the strategies that appeared in before developed methodology. We can observe that the new theory has a greater growth in the gain when the number of nodes increases. We can observe that the gains are greater when it is active circuit.

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