

Mesh Refinement with Finite Elements and Artificial Neural Networks

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Abstract: - In this paper, we present a new modeling for Mesh-size refinement with finite elements and artificial neural networks adopted by standards actual videos based on the SOM for image one domain, in the form of a structure. We developed in this study a mesh based on the object of interest by finite elements method and reduce the effort required to apply finite element analysis to image, this presentation that allows the identification of edges is a good representation of the movement of network nodes, and then we approach the follow-up of objects on sequences of Mesh-size refinement images. The algorithm of SOM the Kohonen is one of the important methods; it is a biologically inspired data clustering technique. It is a question of determining the Mesh adapte of an object nets, from one image to another. For that we used the algorithm allowing following a deformable plane object. On the one hand, we improve its performance, and then we study the optimization of the error function by error the Mesh-size refinement object simplification of our model, among the different meshes associated with images references. At the end of this work, we present simulation results.

Key-Words: - Refinement mesh-size, Learning Kohonen SOM, Mesh-size by Finite Elements, deformation the Mesh-size refinement, interpolation.

1 Introduction

Finite element analysis is a powerful computational tool for modeling the deformation objects. Getting place on a domain that corresponds to the representation space of the physical problem. To simplify the presentation, we will constraint to the case where is a differential variety of dimension 2D, and the Kohonen algorithm is an automatic classification method which is the origin of Self-Organizing Maps (SOM)[9]. This famous method falls within the framework of algorithms quantification vector and the method of k-means algorithm. More precisely, the SOM can be seen as an extension of the algorithm of pattern recognition and the automatic classification algorithms [5]. The latter algorithms detail the possible methodologies for introducing expertise that follows the learning unsupervised stage [1].

Because its interesting and specific quality, Kohonen is efficient tool in the important domain which is the unsupervised classification. Indeed this latter is considered as a useful tool in data mining because it accelerates the research of relevant information. In fact the access to this

latter is in suitable time has become a daily need and difficult task. This is due to the huge amount of information available in the website. An unsupervised classifier groups the similar information that refer to the same topic in same cluster and these which are dissimilar in the distinct ones. This avoid the search of the desired information in a lot of clusters, consequently an important time is economized. The Kohonen algorithm is unsupervised partitioned classifier i.e. it treat with unlabeled inputs and provides classes with no overlap. Beside its robustness i.e. its ability to resist to the noise, the Kohonen algorithm possesses other interesting properties. Indeed the self-organizing map is an unsupervised neural network which projects high-dimensional data onto a low-dimensional grid which called a topological map [9]. This projection must preserve the topology of inputs (more details of this point is given after in this paper). This lead to an organization and representation in map. This latter provides a visualization of the data. Furthermore the distances between classes are identified and visualized. This propriety is the most attracting quality of this algorithm. For this reason many huge efforts are performed aiming to overcome the shortcoming of

this algorithm and improve the performance of Kohonen algorithm.

A mesh is therefore determined by geometry and topology [3]. A mesh size is said to be consistent if the intersection of two of these distinct triangles is either an arc or a node, or null. In other words it prevents a summit to be specified only at the end of an edge. Property compliance is critical to ensure the continuity of function interpolating thereof. We distinguish different types of meshes. A priori triangles can be, but it often imposes constraints on the number of edges of these triangles. It often sets the number of edges by triangles (triangular meshes) or (quadrilateral meshes). Triangular meshes are however a number of advantages. On the one hand, triangles can more accurately model the border of any domain. Indeed, any area polygon border is triangularly. On the other hand triangular meshes interest to provide continuous representation modeled objects and the associated interpolation functions are often easier [5][32].

Generally, the size of the topological map is randomly chosen. Indeed such choice effect the performance of the Kohonen algorithm. Consequently, the labeling phase becomes difficult. The weight vectors are also randomly selected, hence the result is affected too. To facilitate the selection phase, we propose in this work a learning method which allows selecting the size of the topological map. In order to attempt this goal, we add to learning Kohonen algorithm a phase called selection stage. This phase is based on a function called selection function [17][15]. To construct this latter, we use a sub-set of the data set. In addition, we divide the topological map on two parts. The first contains the used neurons; the second part is formed by the unused ones. It should be noted that the size of the first and the second parts are modified by the use of the selection function. This method can be also used as a mean of neural architecture optimization.

This paper is organized as follow: The first paragraph relates of the Mesh-Size Finite Elements Method, that is to say, approximation by piecewise, the second consists introduced Mesh –size refinement objects by models of the SOM, the third consists in modeling a model of the Optimum Adaptation of a Mesh Size by finite Elements [6] and SOM. Simulation results are also available.

2 Mesh-Size Finite Elements Method

2.1 Finite Element Method (FEM)

The principle of finite elements solution is to show values calculated from variable approach taken in a number of end points properly chosen. We then rebuilt solution across the field by interpolation [10]. Extends in the method discretization involves setting up a domain partition considers in simple elements. This partition is the mesh of the domain and must possess properties that depend on the application. The finite element methods provide approximations of exact solutions and we try to get the error associated with the smallest possible [7].

We present generally how can be approximated by piecewise that are to say mesh-size by finite elements. The method finite element (FEM) uses a representation based nodes by estimating the values in the facet by functions the nodal values weighting. Polynomial representation is also widely used in the field of image deformation or coefficients are obtained by identifying the nodal values to a polynomial approximation of piecewise. Mesh-size is used for modeling by discretization of partial differential equations. It is also possible, by a judicious choice of weighting functions of the FEM, these two performances are the same, and however, the nodes based representations have several advantages:

- Numerical analysis more efficient and more stable.
- The definition of positions and values of the nodes require less accuracy that polynomial coefficients estimation (this last point is especially important in applications of coding that all these parameters are subject to the quantification).
- It is easier to visualize a function or deformity interpolated from the positions of the nodes from polynomial coefficients.

2.2 Mesh size by models of surface finite elements

This method is to evaluate values approached the field looked only at certain points in the domain, and then deduct these values by interpolation, the solution at any point. A first step will be to achieve a Mesh size of the field that will define, among other things, points. This pre-treatment is an essential phase of the method, the mesh nodes, will determine the convergence of the method to a good solution[34]. Also, the domain can have a complex geometry, which implies that the mesh is not trivial. We focus only, in this section triangular mesh [20]. The algorithms presented here for automating finite

element analysis are potentially applicable to a variety of objects videos that require deformation modeling. In this paper, the conception of the finite elements method of the mesh can be realized by three phases:

- The first phase is the analysis of the problems, which consist the studying of domain geometry that is very complicated problem will be broken down into simple forms problems.
- The second phase is the formal construction the mesh: Takes into account the analysis results, and defines simple objects allowing division the total work into phase.
- The third phase is the realization of the mesh, which includes the previous phases.

3 Refinement Mesh-Sizes with Maps SOM AND FINITE ELEMENTS

3.1 Self-Organizing Map (SOM) and Finite Elements.

The Self-Organizing Map (SOM), proposed by Kohonen, consists projecting high dimensional data onto a low-dimensional grid [9]. The projected data preserves the topological relationship of the original data; therefore, this ordered grid can be used as a convenient visualization surface for showing various features of the training data, for example, cluster structures [8]. The Kohonen network has one single layer, let name this one the output layer. The additional input layer just distributes the inputs to output layer. The neurons of this latter are arranged in a matrix. We consider, in this work, that the map is in two dimensions. The goal of self-organizing maps is to associate with each neuron a referent of a vector space data; see figure 1. The number of neurons on input layer is equal to the dimension of input vector. Kohonen has proposed various alternatives for the automatic classification, and presented the Kohonen topological map [9]. Basing on the structure graph defined on the topological map, Kohonen has defined the discrete distance δ . For any pair of neurons (c ; r), it calculates a discrete distance $\delta_{c,r}$, where $\delta_{c,r}$ is defined as the length of the shortest path between c and r on the graph. For each neuron c , this discrete distance can define the concept of a neighborhood as follows: $V_c = \{r / \delta_{c,r} \leq d\}$, where d represents the ray of the neighborhood of the neuron c .

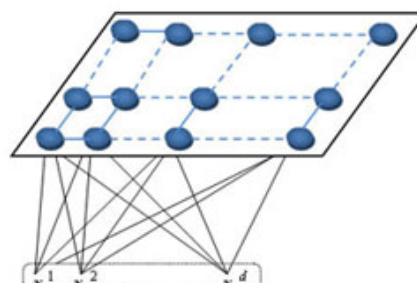


Fig. 1 Kohonen topological map

3.2 The Proposed Method

In this part, we describe with some detail the main component of our approach.

It should be noted that this method consists integrating a new phase in the Kohonen algorithm.

This later is called Interpolation learning phase and has as role to facilitate the refinement Mesh.

Given a sequence of images, we seek to refine a Mesh-size each pixel based on the principle of conservation of intensity [13]. Our method is based on a specific architecture of neural networks this architecture allows to refine the mesh in two steps. The first is to refine the mesh in each pixel using the algorithm of the SOM [20] winner. The second step is used to adapt the mesh according to the movement range of each pixel, the results of the previous phase, in which the displacement of a pixel configuration takes into account the movement in the vicinity [19]. The approach we propose is based on two phases: an initialization phase is to assign the weight of each neuron-image value of the gray level of the pixel corresponding to the images of the first part. During the second phase, after building the learning that includes all the images of all cycles together With interpolation finite elements, we present the elements of this set iteratively to neuron-pixel to deduce what the neuron-image of the input values. For updating the weights of the neuron, called "winner neuron" we can not change is that the weights of the winner without changing the weights of the neighboring neuron-images (SOM algorithm) or modify the weights of the neuron-picture winner and the weight of the neighboring neuron-images (SOM algorithm.) the purpose of the change is to allow different weights converge to values that represent the corresponding image appropriately according to measurements from the acquisition of several cycles. At the end of the process, the neuron-map images form a sequence of images.

The network we propose is inspired by Kohonen maps in the principle of self-organization. It consists two layers. An input the number of neurons which is equal to the number of pixels of a block layer of the mesh of predetermined size, a self-organizing map having the same size as an image where each pixel represents a neuron of an image of the sequence and a self-organizing is a Kohonen map of input picture size. The images of the sequence are considered one by one by presenting, after cutting into blocks of fixed size, to the input of the network. Figure 3 shows the network topology that we propose for the mesh refinement.

- o The input layer E: composed of $n * m$ neurons, with $m * n r$ representing the size of the vicinity.
- o The self-organizing map consists of N neurons having the size of an image sequence in which each neuron is connected to all the neurons of the input layer.

The solution of finite elements of type winner is interpolated on the structured cloud of mesh builds in the steps. To do it, we proceed as follows:

- The network of points mesh calculated in the previous step is planned on the mesh of the face not deformable.
- These coordinate are used as functions to calculate the point surface of interpolation to deform; by using the functions of interpolation; calculated the solution finite elements by type movements with the algorithm of the learning a method [5].

3.3 Algorithm

The algorithm we propose for our method consists of two stages: the first allows initializing the mesh of each pixel based on the principle of the Kohonen SOM algorithm. The second which may be recursive or non-recursive can refine the mesh in the case of displacement obtained in the previous phase.

Initialization phase:

The initializing phase for the Kohonen maps generally involves initializing the weight vectors with random values or, when these are known a priori. In the approach we propose, the weight vectors are initialized with mesh images, supposedly close to the other images of the periodic and orderly sequence, which requires the sequence of neural network (image 1 mesh -> neuron 1 , mesh image 2 - > neuron 2 ...). This phase is therefore to present

sequentially input to the network, T-meshes corresponding image (Equation 3.1). The network is initialized by writing to each neuron-mesh t an image of a CAD w_t weight vector each of whose components $n, i = 1 \dots n$, is equal to the grayscale

noted $w_n^t = I_n^t, i = 1 \dots n$, each of the n pixels in the input image. The operation is repeated for T-meshes neuron-images using pictures successively T cycle used for initialization.

$$w_1^1 = I_1^1 \quad w_1^t = I_1^t \quad w_1^T = I_1^T \quad (1)$$

$$w_n^1 = I_n^1 \quad w_n^t = I_n^t \quad w_n^T = I_n^T$$

With the vector associated with each neuron-cell-picture:

$$w^1 = (w_1^1, \dots, w_n^1), w^t = (w_1^t, \dots, w_n^t) \text{ AND} \quad (2)$$

$$w^T = (w_1^T, \dots, w_n^T) \quad (3)$$

Learning phase

The sequence we have is composed of L cycles, the first cycle is used to initialize the network, other cycles form our learning together. During the learning phase, we present successively the elements of the training set, $L-1$ cycles of the input image grid. The first values, presented to neuron-pixels of the input layer are the intensity of the first image of the second mesh-cycle values, and then the second-grid image and so on until the meshing of the picture of the last cycle $T L$. The entrance is mesh mesh image image I_i amplitude of pixel i of the image being analyzed is assigned to the input neuron-pixel e_i or $e_i = I_i$. We are looking for each entry, w_t component corresponding to the neuron-mesh-ct picture of CAD that minimizes the gap with the values shown in entry. This neuron-mesh-image is called neuron-mesh-picture "winner" or << simply "winner."

A measure of the difference between T-neuron meshes images of the self-organizing map, construction and meshes the input image (the values of which are carried by the neuron-pixels of the input layer) is defined as equation 1:

$$Em = \text{Ming}(w^t, e) \quad (4)$$

With $t=1..T$.

Em is the minimum value for the neuron-mesh-image $t * t$ of the self-organizing map, considered the winner among other neurons-mesh-map images. Then we proceed to the modification of the weights of neuron-mesh-picture winner and the weight of

neighboring neurons-images according to the following formula:

$$w^j(t) = w^j(t-1) + \beta_{t,k}(e - w^j(t-1)) \quad (5)$$

$$\beta_{t,k} = \exp\left(\frac{\|r_i - r_k\|}{(\sigma(t)^2)}\right) \quad (6)$$

Where $\|r_i - r_k\| \approx \|w^j - w^k\|$ and r_i the vector represents the coordinates of weights and functions are decreasing. The main important parameter in our method is the selection set. In this context, the choice of the later must be done carefully, taking into account the quality of learning and reduced complexity of the proposed method.

Interpolation phase

The image \hat{I} of the surface that we seek to approximate \hat{I} by a surface defined a polynomial in two variables: $\hat{I}(x,y) = P^n(x,y)$ we know that it is preferable to perform interpolation piecewise rather than global phenomena that lead to instability, but this involves studying between the different areas in order to ensure the consistency of the interpolate[3].

- We restrict ourselves by triangular elementary domains, also called elements. To triangulate and cover domain Ω image elementary domains support, triangle offers flexibility
- This is particularly useful for approaching the boundaries whatsoever such as the edges of object. Image border Ω is a polygon and in this case Ω is fully and accurately covered by triangles Basic fields defining a partition Ω , assembled them so that they have:
 - An intersection is empty.
 - Either an intersection reduced to a common vertex.
 - Either an intersection reduced to one side common.
- Should therefore define a global numbering of elementary domain e_i , global numbering for all S_i summits and a relationship that each elementary domain associates list heights.

Given a triangulation $T_\Omega = \cup_{i=1}^N e_i$ formed by triangles of a closed polygon domain Ω . Knowing the value S_i to the surface to approximate in each vertex of a triangle, I_{S_i} , it seeks to build \hat{I} such as: Restricting Court an e triangle is a polynomial of

degree $P_T \leq 1$.

- Determining P_T on e triangle vertices S_1, S_2 and S_3 knowing $I(S_1), I(S_2)$, and $I(S_3)$, conditions are imposed:

$$P_T(S_i) = I(S_i) \quad i \in \{1,2,3\} \quad (7)$$

To do this, define 3 basic functions are commanded local ψ_i checking for an e triangle vertices S_1, S_2 and S_3 :

$$\psi_i(S_j) = \delta_{ij} \quad i, j = 1,2,3 \quad (8)$$

- Interpolated point value expression (x, y) owned by an Element therefore corresponds Fig. 2 and Fig. 3.

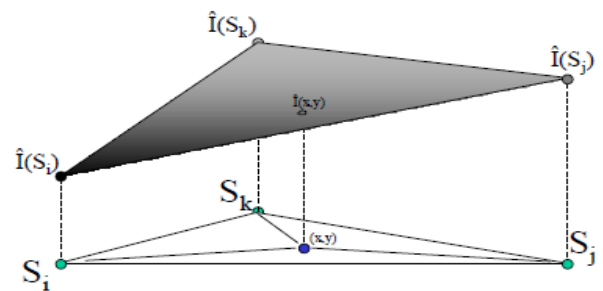


Fig2. Interpolation of value depending on the model of Lagrange, equivalent to the projection of the point (x, y) on the plan.

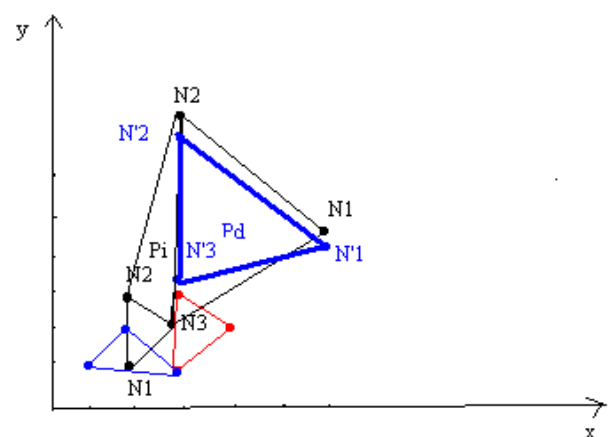


Fig.3: nodes the mesh deformable

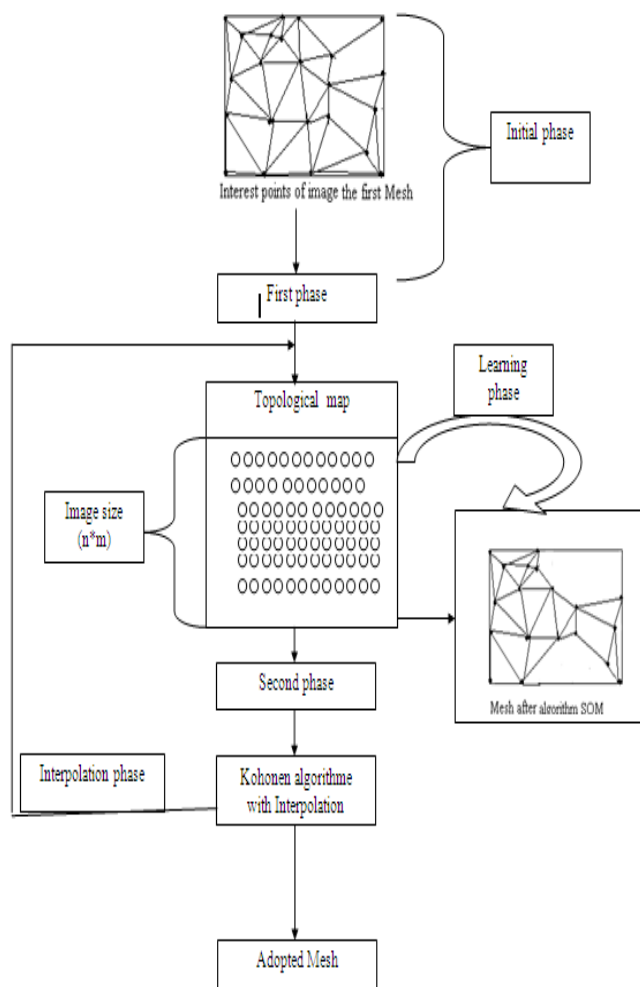


Fig2. Adapted Mesh Diagram

The diagram shows the different steps of our method.

- The choice of the selection set has great impact on the algorithm performance and on its complexity.
- The selection of neuron must reflect the most information of the population space. Since the interpolation function is defined on P , a big size of this set increases the complexity of the system. So, suitable choice is required [17][15].
- The dimension of the investigated inputs is typically huge. The neural network of kohonen is commonly used as an efficient mean to reduce this dimension. In the beginning no information about the class is available. So the choice of map size and the active neuron is made image size. Hence the performance will be affected and some problem will arise in the labeling phase. The update can be considered as a

correction of initialization phase and also as a way for searching a suitable architecture map. Thereby the performance of the system will be improved.

- The proposed method modelizes a behaviour used naturally by the human. Indeed, this latter when he wants accomplishes a task., at first he choose randomly a set of persons. After, this set is updated by liberating some persons and adding others basing on their competency. This behavior is also used by certain animals as ants [15].

4 Computational experiments

Experimental results are provided to compare the performance of the proposed refinement Mesh maps with finite elements by occlusion adaptive forward mesh tracking versus test sequence. The sequence starts with slow movements of the head of the Sequences Miss America, Foreman, Salesman, and Table Tennis were selected for the simulation and results can be seen in Table 1. However, the part between frames 150 and 151 is especially challenging and well suited to demonstrate the occlusion-adaptive mesh tracking concept. The processed video, the overall performance is improved in all test sequences at a fixed bitrate (0.2 bpp). We identified two results: refinement Mesh maps and refinement Mesh finite elements.

1) a new uniform mesh at each frame, and 2) a new content-based mesh at each frame. An algorithmic description of the refinement-Mesh by finite elements mesh approach is given in the Appendix. The case of redesigning a uniform mesh is expected to be a lower bound on the performance of mesh maps by the proposed forward tracking content-base mesh, since the structure of the mesh may not fit the motion boundaries well, leading to multiple motions within a single patch. However, it requires transmission of no overhead information about the mesh structure.

The proposed forward tracking content-based mesh is a compromise between these two, since it yields a mesh structure that fits the scene content without too much overhead transmission. To this effect, the efficacy of the proposed method has been evaluated based on how it compares against these benchmarks in terms of motion-compensation PSNR and the number of node points whose coordinates need to be transmitted at each frame. The PSNR values refer to the prediction PSNR of each frame based on the

original of the previous frame, using the affine motion field interpolated from the node-point motion vectors.

We have implemented the algorithms described in this paper in Java and employed them in a cloth simulation system. The tests were run on a standard Pentium 4 with 2.8GHz and 512 MB RAM, using the Java Runtime Environment 1.6.0.

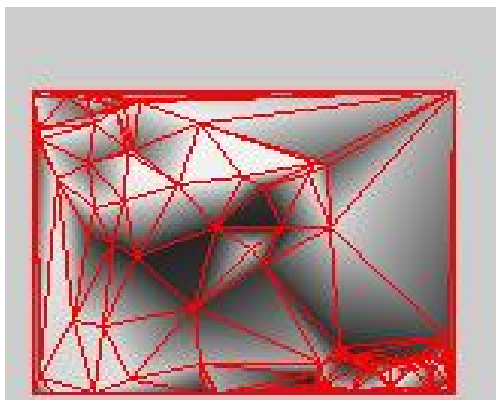
Table 1: optimization of neural architecture

Error after using SOM maps with finite elements "Foreman frame 150"	Error after using SOM maps finite elements « Foreman : frame 151»	Error after using SOM maps finite elements « Miss »
148.477301	166.729442	79.275862
158.679901	147.208811	75.773006
136.577586	144.424079	75.486486
134.106460	143.673505	74.850433
132.891856	141.990975	74.814356
140.155492	141.960921	85.000000
145.939469	141.951198	78.180812
138.193152	141.191111	73.644295
132.405510	140.778883	53.027019
143.106682	140.680321	52.146341
132.343431	140.111111	51.931646
132.515564	150.050847	52.792683
132.242105	139.948010	51.703582
131.612766	139.846154	51.388170

142.120000	139.283898	89.142857
135.786184	139.254237	51.317073
130.721947	146.205882	78.285714
131.190590	138.242820	50.319564
130.398340	138.120346	50.312821
134.925926	137.996516	57.581395
130.240469	137.703812	61.425000
146.929174	138.513292	49.628319
131.159716	135.872483	65.285968
129.073077	140.619495	49.790000
128.806897	139.276257	49.574820
134.695989	136.625000	49.500000



Original Image the Foreman



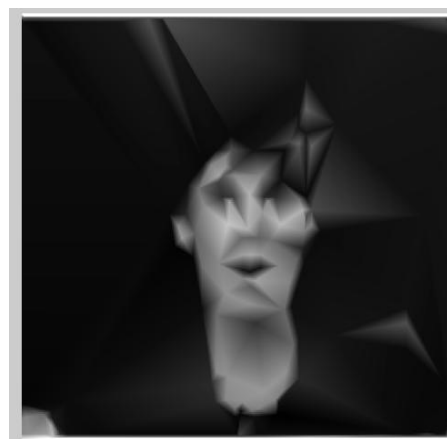
Refinement the mesh SOM maps with finite elements



Refinement the mesh SOM maps with finite elements



Error = 136.625000



Error = 49.500000

Table 1 presents the error de PSNR of the the Kohonen map with Finite Elements. It shows that only with 100 iterations, the proposed method lead to a reduction of the card.



Original Image the Miss

From the Table1, we see that the proposed method gives satisfying results, Kohonen algorithm will be well appreciated. The obtained amelioration can be explained as follows:

- The first term of the objective hybridation of our proposed model controls the geometric error on a given refinement.

The proposed method completes the Kohonen learning method. In fact, our approach realizes two tasks at the same time: the learning task and the regroup of the objet one which consists minimizing the size of the map.

5 Conclusion

In this paper, we have interested in Finite Elements with Artificial Neural Networks. We have chosen as a method of mesh refinement by self-organizing map of Kohonen SOM algorithm with Finite elements, in which we presented and its main techniques we have developed and implemented a method aims supervised learning, error minimization and mesh adaptation in either refine or looping mode. We have applied our algorithm to a widely used dataset, objects videos for refinement Mesh size. In this respect, our method produces good results in reasonable time in comparison with the recent.

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