Performance Analysis of High-Rate Full-Diversity Space Time Frequency/Space Frequency Codes for Multiuser MIMO-OFDM

R. SHELIM[1], M.A. MATIN[2] AND A.U.ALAM[2]
[1]Department of Electrical Engineering and Computer Science
North South University, Dhaka, Bangladesh
[2]Department of Electrical and Electronic Engineering
Institut Teknologi Brunei , Negara Brunei Darussalam
Email:matin.mnt@gmail.com

Abstract: - MIMO-OFDM has been recognized as one of the most promising techniques to support high data rate and high performance wireless systems. In particular, coding over the space, time, and frequency domains provided by MIMO-OFDM enables a much more reliable and robust transmission over the harsh wireless environment. In this paper, the authors provide an overview of space-frequency (SF) coding and space-time-frequency (STF) coding for MIMO-OFDM systems and compare the performance of different coding schemes. Performance results show that STF coding can achieve better coding and diversity gain in an end-to-end MIMO-OFDM system over broadband wireless channels.

Key-Words: -multiple-input multiple- output systems, multipath fading channels, multiple access channel (MAC), multiuser space time frequency (STF) coding, multiuser space-frequency (SF) coding.

1 Introduction

Future broadband wireless systems require high data rate and high performance over fading channel that is time selective and frequency-selective fading which are quite challenging. To address these challenges, one promising solution is to combine two powerful technologies, namely, multiple-input multiple-output (MIMO) antennas and orthogonal frequency division multiplexing (OFDM) modulation [1]. It encodes a data stream across different transmit antennas and time slots, so that multiple redundant copies of the data stream can be transmitted through independent fading channels. As a result, more reliable detection can be obtained at the receiver. Therefore, this topic has drawn a lot of interests among the researchers to date, consequently, a large number of space-time (ST) or space-frequency (SF) coding have been designed for MIMO-OFDM system [2]-[10] to exploit the spatial diversity gain. However, most of the codes are designed for single user system utilizing single-user space-time codes for each of the users and separating the users in signal space or canceling multiuser interference. However, these approaches lead to reduce transmission rate or suboptimum performance significantly, if the number of users is high. Considering the above issue, Gärtner and Bölcskei designed a multi-user space-time/frequency code [11] based on Gallager’s idea in [12], which is derived from the dominant error mechanisms in two-user Additive White Gaussian noise (AWGN) multiple access channel (MAC) but detailed code design seems to be missing. Zang and Letaief [13] introduced a systematic design of full diversity multi-user space-frequency code. But, the symbol or code rate of their proposed design is 1 though some remarks have been given for high rate full diversity code design. Recently, a systematic high symbol rate full-diversity STF/SF codes for multi-user MIMO-OFDM system have been designed [14-16]. To increase the symbol rate of their proposed code design, the space-frequency and space-time-frequency layering concept with algebraic component code is utilized. For both ST and STF design, each component code is assigned to a thread in the space-frequency (for SF codes) and space-time-frequency (for STF codes) matrix that provide full access to the channel frequency, time and spatial diversity in the absence of other threads. Diophantine approximation theory is then used where the Diophantine numbers are,

$$\left\{\theta_1{}^0 = 1, \theta_1{}^1 = \gamma^{\frac{1}{W_t}}, \dots, \theta_1{}^{(w_t-1)} = \gamma^{\frac{w_t-1}{W_t}}\right\},$$

and is chosen such that the threads are transparent to each other. Furthermore, another Diophantine approximation is used where the Diophantine numbers are as follows so that users are transparent to each other.

$$\left\{\theta_2{}^0 = 1, \theta_2{}^1 = \theta^{\frac{1}{K}}, \dots, \theta_2{}^{(K-1)} = \theta^{\frac{K-1}{K}}\right\}$$

Depending on the pair wise-error probability of multi-user MIMO system, the full diversity performance of the multi-user STF codes is associated with the full-rank property of any codeword difference matrix and any set of users.

In this paper, we attempt to provide an overview of SF coding, and STF coding for MIMO OFDM wireless systems, in particular focusing on recent work on high rate and full diversity multi-user SF/STF code design. For better understanding of SF/STF code design, SF and STF codes have been discussed in two separate sections. Simulation section compares the performance of the proposed STF codes with SF codes.

The organization of this paper is as follows. The MIMO-OFDM system is briefly introduced in section 2. In section 3, the design of full diversity and high code rate multiuser SF code with an example is presented. In section 4, a design of STF code is provided with an example to verify the code design. The presented code can also achieve full diversity and high code rate. Section 5 compares the performance of the STF codes with multiuser SF code and other typical SF codes. Finally, a conclusion is given in section 6.

2 MIMO-OFDM System

MIMO-OFDM is the combination of MIMO and OFDM where MIMO can enhance the capacity and diversity and OFDM can mitigate the interference effects due to multipath fading. Suppose that the MIMO-OFDM system shown in Fig. 1 has Z users, where each user is equipped with A_t transmit antennas, a base station (BS) with A_r receive antennas and N-tone OFDM. The MIMO channels experience frequency selective fading induced by L independent paths between each pair of transmit and receive antennas and can be expressed as,

$$h_{i,j}^{(z)} = \sum_{a=0}^{L-1}\left[\alpha_{i,j}\;\; a\;\; \delta\;\; t-\tau_a\;\;\right],$$

where, τ_a is the delay of a^{th} path, $\alpha_{i,j}\;\; a$ denotes the channel coefficient of the a^{th} path from the transmit antenna j to the receive antenna i, where, i=1,… A_r; $j = 1,\cdots, A_t$; and a = 0, 1, 2 ,…,L-1.

The source in MIMO-OFDM system generates a block of N_s information symbols from the discrete alphabet $\mathcal{T}$, which is a quadrature amplitude modulation (QAM) constellation normalized into the unit power. Using a mapping, the information symbol vector $S \in \mathcal{T}^{N_s}$ is encoded into $N \times A_t$ code matrix $C^{(z)} \in \mathcal{T}^{N\times A_t}$ which is then sent through transmitting antennas. The symbol rate or code rate per channel use of the code matrix $C^{(z)}$ is given by, $\mathcal{R} = \frac{N_s}{N}$.

Let, each user has N-tone OFDM. For user z, the codeword $C^{(z)}$ can be written as,

$$C^{(z)} = \left[X_1^{(z)} \dots X_{A_t}^{(z)}\right], \quad (1)$$

where, the OFDM symbol $X_n^{(z)}$ is assumed to be transmitted from the n^{th} transmit antenna. The OFDM symbols are sent simultaneously from all transmit antennas after IFFT and cyclic prefix (CP) insertion.

After passing through the MIMO Channels, the received signals will be first sent to the reverse OFDM block to remove CP and FFT demodulation and then sent it to the decoder.

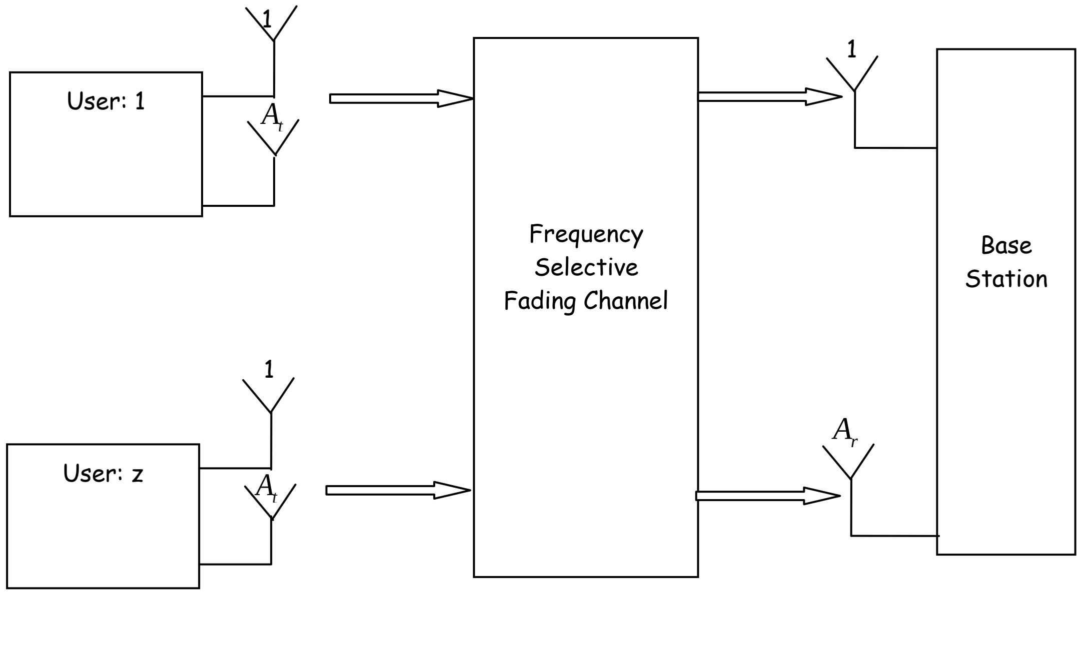

Fig.1: Multiuser MIMO OFDM System

3 Multiuser Space-Frequency Code Design

The design of full diversity $A_r A_t L$ and high code rate (i.e. rate-A_t) multiuser SF code design has been discussed in this section. Our SF code has been constructed based on threaded layering concept [14]. Exploiting the duality, space-frequency coding in frequency-selective channel is constructed from space-time coding in time-selective channel. The proposed design of full diversity $A_r A_t L$ and high rate A_t, SF codes have been shown in Fig. 2. An example of the presented multi-user SF codes has also been given.

Here, it is considered that the system has N-OFDM tones.

A block of NA_t information symbols $S^{(z)} = [S_1^{\,z}\, S_2^{\,z} \ldots S_{BK}^{(z)}]^T$ from the discrete alphabet $\mathcal{T}$, where $S^{(z)} \in \mathcal{T}^{NA_t}$, and S is a QAM constellation are normalized into the unit power and are parsed into $B = \frac{N}{K}$, subblocks, Each sub block $S_b^{(z)} \in \mathcal{T}^{W_Z A_t W_L W_t}, b = 1, 2, \ldots, B$ is composed of the signal vectors $S_w^{(z)} \in \mathcal{T}^{\tilde{w}}$, w=1,2,3,… w_t, and $\tilde{w} = W_Z W_L A_t$. The signal vector comprises of QAM symbols.

Each sub-block, $S_b^{(z)} \epsilon\, \mathcal{T}^{W_Z A_t W_L W_t}$ b=1, 2, 3… B is respectively, encoded into an SF code matrix C_b of size $K \times A_t$ matrix. Let

$$K = W_Z W_L W_t,$$

where, $W_L = 2^{\lceil \log_2 L \rceil}, W_t = 2^{\lceil \log_2 A_t \rceil}, W_Z = 2^{\lceil \log_2 Z \rceil}$, The SF code matrix can be expressed as,

,

$$c_b^{(z)} = \begin{pmatrix} \bar{X}_{1,1}^{(z)} \\ \vdots \\ \bar{X}_{1,W_Z}^{(z)} \\ \vdots \\ \bar{X}_{W_L,1}^{(z)} \\ \vdots \\ \bar{X}_{W_L,W_Z}^{(z)} \end{pmatrix}, \quad (4)$$

Next, for multi-user MIMO-OFDM MAC, the encoded codeword $\bar{C}_b^{(z)}$ is,

$$\bar{C}_b^{\ z} = C_b^{\ z} \circ \Phi_{2,z} \otimes 1_{1\times A_t} \quad , \tag{5}$$

Where, $\Phi_{2,z}$ is the z^{th} column of the $K \times W_Z$ matrix Φ_2, which is given by

$$\Phi_2 = \begin{pmatrix} 1 & \theta_2^{\ Q-1} & \dots & \theta_2^{\ K-Z+1} \\ \theta_2 & 1 & \dots & \theta_2^{\ K-Z+2} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \theta_2^{\ Q-1} & \theta_2^{\ Q-2} & \dots & \theta_2^{\left(1-\frac{Z}{K}\right)K} \end{pmatrix} \tag{6}$$

Where θ_2 is given by $\theta_2 = \gamma^{\frac{1}{K}}$ where γ is an algebraic number with degree of at least $K\, W_L$ over $\mathcal{A}$ where $\mathcal{A}$ is the field extension of $\mathbb{Q}$ which contains all the entries of Θ the signal alphabet, $\mathcal{T} \subset \mathbb{Z}\ j$ and $e^{\frac{-j2\pi\tau_l}{T_s}}, (l = 0,1,\dots,\mathrm{L}-1)$.

Here, the SF code for one sub block $\bar{C}_b^{(z),}$ is illustrated as in Fig.2 and the same phenomena is applied for other encoded sub blocks. Thus, the presented rate - A_t SF codes $C^{(z)} \in \mathcal{T}^{N\times M_t}$ for the z^{th} user is of the form:-

$$C^{(z)} = \left[\ \bar{C}_1^{(z)\ T}\ \ \bar{C}_2^{(z)\ T} \dots\ \bar{C}_B^{(z)\ T} \right]^T \ . \tag{7}$$

Note that as TASF code confirms that N is the integer multiple of K, no zero-padding matrix is required in our proposed code structure. Thus the rate-A_t can be always guaranteed. Detailed code design is given in [14].

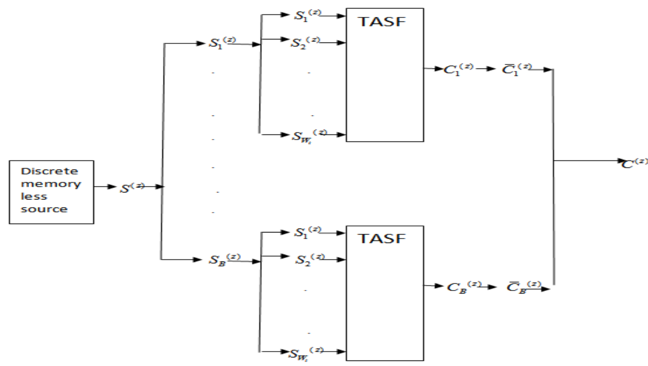

Fig. 2.SF coding structure in MIMO-OFDM system [14]

Example of SF codes:

$$For, A_t = 3, L = 2, Z = 2,$$

For z=1,

$$\bar{C}_1^{(1)} = \begin{pmatrix}
X_1^{(1)}(1) & \theta_1^3 X_4^{(1)}(1) & \theta_1^2 X_3^{(1)}(1) \\
\theta_2\theta_1 X_2^{(1)}(1) & \theta_2 X_1^{(1)}(2) & \theta_2\theta_1^3 X_4^{(1)}(2) \\
\theta_2^2\theta_1^2 X_3^{(1)}(2) & \theta_2^2\theta_1 X_2^{(1)}(2) & \theta_2^2 X_1^{\ 1}(3) \\
\theta_2^3\theta_1^3 X_4^{(1)}(3) & \theta_2^3\theta_1^2 X_3^{(1)}(3) & \theta_2^3\theta_1 X_2^{(1)}(3) \\
\theta_2^4 X_1^{(1)}(7) & \theta_2^4\theta_1^3 X_4^{(1)}(7) & \theta_2^4\theta_1^2 X_3^{(1)}(7) \\
\theta_2^5\theta_1 X_2^{(1)}(7) & \theta_2^5 X_1^{(1)}(8) & \theta_2^5\theta_1^3 X_4^{(1)}(8) \\
\theta_2^6\theta_1^2 X_3^{(1)}(8) & \theta_2^6\theta_1 X_2^{(1)}(8) & \theta_2^6 X_1^{(1)}(9) \\
\theta_2^7\theta_1^3 X_4^{(1)}(9) & \theta_2^7\theta_1^2 X_3^{(1)}(9) & \theta_2^7\theta_1 X_2^{(1)}(9) \\
\theta_2^8 X_1^{(1)}(4) & \theta_2^8\theta_1^3 X_4^{(1)}(4) & \theta_2^8\theta_1^2 X_3^{(1)}(4) \\
\theta_2^9\theta_1 X_2^{(1)}(4) & \theta_2^9 X_1^{(1)}(5) & \theta_2^9\theta_1^3 X_4^{(1)}(5) \\
\theta_2^{10}\theta_1^2 X_3^{(1)}(5) & \theta_2^{10}\theta_1 X_2^{(1)}(5) & \theta_2^{10} X_1^{(1)}(6) \\
\theta_2^{11}\theta_1^3 X_4^{(1)}(6) & \theta_2^{11}\theta_1^2 X_3^{(1)}(6) & \theta_2^{11}\theta_1 X_2^{(1)}(6) \\
\theta_2^{12} X_1^{(1)}(10) & \theta_2^{12}\theta_1^3 X_4^{(1)}(10) & \theta_2^{12}\theta_1^2 X_3^{(1)}(10) \\
\theta_2^{13}\theta_1 X_2^{(1)}(10) & \theta_2^{13} X_1^{(1)}(11) & \theta_2^{13}\theta_1^3 X_4^{(1)}(11) \\
\theta_2^{14}\theta_1^2 X_3^{(1)}(11) & \theta_2^{14}\theta_1 X_2^{(1)}(11) & \theta_2^{14} X_1^{(1)}(12) \\
\theta_2^{15}\theta_1^3 X_4^{(1)}(12) & \theta_2^{15}\theta_1^2 X_3^{(1)}(12) & \theta_2^{15}\theta_1 X_1^{(1)}(12)
\end{pmatrix}$$

For z=2,

$$\bar{C}_1^{(2)} = \begin{pmatrix} \theta_2^{15}X_1^{(2)}(1) & \theta_2^{15}\theta_1^{3}X_4^{(2)}(1) & \theta_2^{15}\theta_1^{2}X_3^{(2)}(1) \\ \theta_1 X_2^{(2)}(1) & X_1^{(2)}(2) & \theta_1^{3}X_4^{(2)}(2) \\ \theta_2^{1}\theta_1^{2}X_3^{(2)}(2) & \theta_2^{1}\theta_1 X_2^{(2)}(2) & \theta_2^{1}X_1^{\ 2}\ (3) \\ \theta_2^{2}\theta_1^{3}X_4^{(2)}(3) & \theta_2^{2}\theta_1^{2}X_3^{(2)}(3) & \theta_2^{2}\theta_1 X_2^{(2)}(3) \\ \theta_2^{3}X_1^{(2)}(7) & \theta_2^{3}\theta_1^{3}X_4^{(2)}(7) & \theta_2^{3}\theta_1^{2}X_3^{(2)}(7) \\ \theta_2^{4}\theta_1 X_2^{(2)}(7) & \theta_2^{4}X_1^{(2)}(8) & \theta_2^{4}\theta_1^{3}X_4^{(2)}(8) \\ \theta_2^{5}\theta_1^{2}X_3^{(2)}(8) & \theta_2^{5}\theta_1 X_2^{(2)}(8) & \theta_2^{5}X_1^{(2)}(9) \\ \theta_2^{6}\theta_1^{3}X_4^{(2)}(9) & \theta_2^{6}\theta_1^{2}X_3^{(2)}(9) & \theta_2^{6}\theta_1 X_2^{(2)}(9) \\ \theta_2^{7}X_1^{(2)}(4) & \theta_2^{7}\theta_1^{3}X_4^{(2)}(4) & \theta_2^{7}\theta_1^{2}X_3^{(2)}(4) \\ \theta_2^{8}\theta_1 X_2^{(2)}(4) & \theta_2^{8}X_1^{(2)}(5) & \theta_2^{8}\theta_1^{3}X_4^{(2)}(5) \\ \theta_2^{9}\theta_1^{2}X_3^{(2)}(5) & \theta_2^{9}\theta_1 X_2^{(2)}(5) & \theta_2^{9}X_1^{(2)}(6) \\ \theta_2^{10}\theta_1^{3}X_4^{(2)}(6) & \theta_2^{10}\theta_1^{2}X_3^{(2)}(6) & \theta_2^{10}\theta_1 X_2^{(2)}(6) \\ \theta_2^{11}X_1^{(2)}(10) & \theta_2^{11}\theta_1^{3}X_4^{(2)}(10) & \theta_2^{11}\theta_1^{2}X_3^{(2)}(10) \\ \theta_2^{12}\theta_1 X_2^{(2)}(10) & \theta_2^{12}X_1^{(2)}(11) & \theta_2^{12}\theta_1^{3}X_4^{(2)}(11) \\ \theta_2^{13}\theta_1^{2}X_3^{(2)}(11) & \theta_2^{13}\theta_1 X_2^{(2)}(11) & \theta_2^{13}X_1^{(2)}(12) \\ \theta_2^{14}\theta_1^{3}X_4^{(2)}(12) & \theta_2^{14}\theta_1^{2}X_3^{(2)}(12) & \theta_2^{14}\theta_1 X_1^{(2)}(12) \end{pmatrix}$$

Where, $\left[X_w\ 1\ \ldots\ldots\ldots\ldots X_W(12)\right]^T = \Theta\left[S_{12\ W-1\ +1}\ldots\ldots S_{12W}\right]^T, W=1,2,3,4.$

4 Multiuser Space-Time-Frequency Code Design

In this section, we have presented a design of STF code that is capable to achieve full diversity $A_tA_rA_uL$ and high code rate (i.e. rate-A_t) over MIMO frequency selective block fading channel. The design of the presented multiuser STF is illustrated in Fig. 3. It is assumed that the source generates a block of $N_s = NA_tA_u$ information symbols $S^{(z)}$ for user z, where z=1,2,…,Z and $S^{(z)} \in \mathcal{T}^{NA_tA_u}$, which are QAM from the discrete alphabet τ and the system has N-OFDM tones.

The block of NA_tA_u information symbols is as

$$S^{(z)} = [S_1{}^{(z)}S_2{}^{(z)} \ldots S_{BK}{}^{(z)}]^T, \qquad (8)$$

which are normalized into the unit power and are evenly split into $B = \frac{N}{K}$ sub blocks,

$$S^{(z)} = \left[\left(S_1{}^{(z)}\right)^T\left(S_2{}^{(z)}\right)^T \ldots \left(S_B{}^{(z)}\right)^T\right]^T. \qquad (9)$$

Then each sub-block $S_b{}^{(z)}$, b=1,…, B is encoded into an STF code matrix $c_b{}^{(z)}$ of size $K \times A_tA_u$ matrix.Let, $K = W_zW_LW_t$, where, $W_L = 2^{\lceil \log_2 L \rceil}$, $W_t = 2^{\lceil \log_2 A_t \rceil}$, $W_z = 2^{\lceil \log_2 Z \rceil}$.It is obvious that K is always a power of two. The $K \times A_tA_u$, matrix $c_b{}^{(z)}$ is structured as

$$c_b{}^{(z)} = \begin{bmatrix} \bar{X}_{1,1}^{1}{}^{(z)} & \ldots & \bar{X}_{1,1}^{A_u}{}^{(z)} \\ \vdots & \ddots & \vdots \\ \bar{X}_{1,W_z}^{1}{}^{(z)} & \ldots & \bar{X}_{1,W_z}^{A_u}{}^{(z)} \\ \vdots & \ddots & \vdots \\ \bar{X}_{W_L,1}^{1}{}^{(z)} & \ldots & \bar{X}_{W_L,1}^{A_u} \\ \vdots & \ddots & \vdots \\ \bar{X}_{W_L,W_z}^{1}{}^{(z)} & \ldots & \bar{X}_{W_L,W_z}^{A_u}{}^{(z)} \end{bmatrix}. \qquad (10)$$

For multiuser MIMO-OFDM MAC, the encoded codeword $\bar{C}_b{}^{(z)}$ is given by,

$$\bar{C}_b{}^{(z)} = \left(\Phi_{2,z} \otimes 1_{1\times A_tA_u}\right) \circ C_b{}^{(z)}, \qquad (11)$$

where, $\Phi_{2,z}$ is the z^{th} column of the $K \times W_z$ matrix Φ_2,

$$\Phi_2 = \begin{pmatrix} 1 & \theta_2{}^{(K-1)} & \cdots & \theta_2{}^{(K-W_z)+1} \\ \theta_2 & 1 \quad \cdots & & \theta_2{}^{(K-W_z)+2} \\ \vdots & \vdots \quad \cdots & & \vdots \\ \vdots & \vdots \quad \ddots & & \vdots \\ \theta_2{}^{(K-1)} & \theta_2{}^{(K-2)} & \cdots & \theta_2{}^{\left(1-\frac{W_z}{K}\right)K} \end{pmatrix} \quad (12)$$

Where, $\theta_2 = \gamma^{\frac{1}{K}}$ where γ is an algebraic number with degree of at least KW_L over , where $\mathcal{A}$ is the field extension of $\mathbb{Q}$ which contains the signal alphabet $\mathcal{T} \subset \mathbb{Z}[j]$, $e^{\frac{-j2\pi\tau_l}{T_s}}, (l = 0,1,\dots,L-1)$ and all the entries of Θ.

The presented STF coding applies the same coding phenomena to all encoded sub blocks, $\bar{C}_b{}^{(z)}$, $b = 1,2,\dots,B$. But for ease, STF coding for one subblock $\bar{C}_b{}^{(z)}$, is illustrated in Fig.3.

Thus, the presented STF codes $C^{(z)} \in \mathcal{T}^{N \times A_t A_u}$ for the z^{th} user is of the form,

$$C^{(z)} = \left[\left(\bar{C}_1{}^{(z)}\right)^T \left(\bar{C}_2{}^{(z)}\right)^T \dots \left(\bar{C}_B{}^{(z)}\right)^T\right]^T. \quad (13)$$

Note that as no zero-padding matrix is required in the presented code structure, the rate-A_t can always be guaranteed.Also note that the difference of the STF codes between two users is made by the selection of different columns of $K \times W_z$ matrix Φ_2 given in (12). As Φ_2 can be known in advance and each transmitter can know its fixed selected column $\Phi_{2,z}$ before the data transmission, the cooperation among the different users is not necessary in the uplink process. Detailed code design is given in [15]

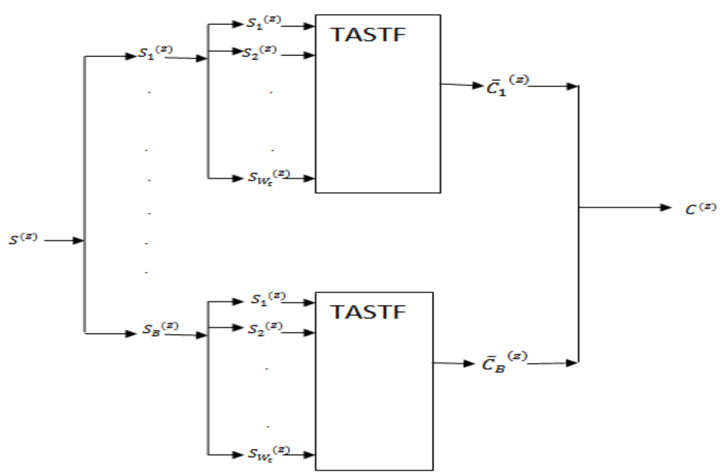

Fig.3. STF coding structure in MIMO-OFDM system [15]

Code Design Example:

Let,

$A_u = 2$, $A_t = 3$, $L = 2$, $Z = 2$,

The STF coding applies the same coding strategy to every sub-block. For convenience, we just consider the sub-block $\bar{C}_1{}^{(z)}$ and $\bar{C}_2{}^{(z)}$ as an example for user z=1 and z=2, respectively. Here, $\phi_2 = \theta_2$ and $\phi_1 = \theta_1$ which are defined earlier.

user-1,

$$
\bar{C}_1^{(1)} = \begin{pmatrix}
X_1^{(1)}(1) & \phi_1^3X_4^{(1)}(1) & \phi_1^2X_3^{(1)}(1) & X_1^{(1)}(13) & \phi_1^3X_4^{(1)}(13) & \phi_1^2X_3^{(1)}(13) \\
\phi_2\phi_1X_2^{(1)}(1) & \phi_2X_1^{(1)}(2) & \phi_2\phi_1^3X_4^{(1)}(2) & \phi_2\phi_1X_2^{(1)}(13) & \phi_2X_1^{(1)}(14) & \phi_2\phi_1^3X_4^{(1)}(14) \\
\phi_2^2\phi_1^2X_3^{(1)}(2) & \phi_2^2\phi_1X_2^{(1)}(2) & \phi_2^2X_1^{(1)}(3) & \phi_2^2\phi_1^2X_3^{(1)}(14) & \phi_2^2\phi_1X_2^{(1)}(14) & \phi_2^2X_1^{(1)}(15) \\
\phi_2^3\phi_1^3X_4^{(1)}(3) & \phi_2^3\phi_1^2X_3^{(1)}(3) & \phi_2^3\phi_1X_2^{(1)}(3) & \phi_2^3\phi_1^3X_4^{(1)}(15) & \phi_2^3\phi_1^2X_3^{(1)}(15) & \phi_2^3\phi_1X_2^{(1)}(15) \\
\phi_2^4X_1^{(1)}(7) & \phi_2^4\phi_1^3X_4^{(1)}(7) & \phi_2^4\phi_1^2X_3^{(1)}(7) & \phi_2^4X_1^{(1)}(19) & \phi_2^4\phi_1^3X_4^{(1)}(19) & \phi_2^4\phi_1^2X_3^{(1)}(19) \\
\phi_2^5\phi_1X_2^{(1)}(7) & \phi_2^5X_1^{(1)}(8) & \phi_2^5\phi_1^3X_4^{(1)}(8) & \phi_2^5\phi_1X_2^{(1)}(19) & \phi_2^5X_1^{(1)}(20) & \phi_2^5\phi_1^3X_4^{(1)}(20) \\
\phi_2^6\phi_1^2X_3^{(1)}(8) & \phi_2^6\phi_1X_2^{(1)}(8) & \phi_2^6X_1^{(1)}(9) & \phi_2^6\phi_1^2X_3^{(1)}(20) & \phi_2^6\phi_1X_2^{(1)}(20) & \phi_2^6X_1^{(1)}(21) \\
\phi_2^7\phi_1^3X_4^{(1)}(9) & \phi_2^7\phi_1^2X_3^{(1)}(9) & \phi_2^7\phi_1X_2^{(1)}(9) & \phi_2^7\phi_1^3X_4^{(1)}(21) & \phi_2^7\phi_1^2X_3^{(1)}(21) & \phi_2^7\phi_1X_2^{(1)}(21) \\
\phi_2^8X_1^{(1)}(4) & \phi_2^8\phi_1^3X_4^{(1)}(4) & \phi_2^8\phi_1^2X_3^{(1)}(4) & \phi_2^8X_1^{(1)}(16) & \phi_2^8\phi_1^3X_4^{(1)}(16) & \phi_2^8\phi_1^2X_3^{(1)}(16) \\
\phi_2^9\phi_1X_2^{(1)}(4) & \phi_2^9X_1^{(1)}(5) & \phi_2^9\phi_1^3X_4^{(1)}(5) & \phi_2^9\phi_1X_2^{(1)}(16) & \phi_2^9X_1^{(1)}(17) & \phi_2^9\phi_1^3X_4^{(1)}(17) \\
\phi_2^{10}\phi_1^2X_3^{(1)}(5) & \phi_2^{10}\phi_1X_2^{(1)}(5) & \phi_2^{10}X_1^{(1)}(6) & \phi_2^{10}\phi_1^2X_3^{(1)}(17) & \phi_2^{10}\phi_1X_2^{(1)}(17) & \phi_2^{10}X_1^{(1)}(18) \\
\phi_2^{11}\phi_1^3X_4^{(1)}(6) & \phi_2^{11}\phi_1^2X_3^{(1)}(6) & \phi_2^{11}\phi_1X_2^{(1)}(6) & \phi_2^{11}\phi_1^3X_4^{(1)}(18) & \phi_2^{11}\phi_1^2X_3^{(1)}(18) & \phi_2^{11}\phi_1X_2^{(1)}(18) \\
\phi_2^{12}X_1^{(1)}(10) & \phi_2^{12}\phi_1^3X_4^{(1)}(10) & \phi_2^{12}\phi_1^2X_3^{(1)}(10) & \phi_2^{12}X_1^{(1)}(22) & \phi_2^{12}\phi_1^3X_4^{(1)}(22) & \phi_2^{12}\phi_1^2X_3^{(1)}(22) \\
\phi_2^{13}\phi_1X_2^{(1)}(10) & \phi_2^{13}X_1^{(1)}(11) & \phi_2^{13}\phi_1^3X_4^{(1)}(11) & \phi_2^{13}\phi_1X_2^{(1)}(22) & \phi_2^{13}X_1^{(1)}(23) & \phi_2^{13}\phi_1^3X_4^{(1)}(23) \\
\phi_2^{14}\phi_1^2X_3^{(1)}(11) & \phi_2^{14}\phi_1X_2^{(1)}(11) & \phi_2^{14}X_1^{(1)}(12) & \phi_2^{14}\phi_1^2X_3^{(1)}(23) & \phi_2^{14}\phi_1X_2^{(1)}(23) & \phi_2^{14}X_1^{(1)}(24) \\
\phi_2^{15}\phi_1^3X_4^{(1)}(12) & \phi_2^{15}\phi_1^2X_3^{(1)}(12) & \phi_2^{15}\phi_1X_1^{(1)}(12) & \phi_2^{15}\phi_1^3X_4^{(1)}(24) & \phi_2^{15}\phi_1^2X_3^{(1)}(24) & \phi_2^{15}\phi_1X_1^{(1)}(24)
\end{pmatrix} \quad (14)
$$

for user-2,

$$
\bar{C}_1^{(2)} = \begin{pmatrix}
\phi_2^{15}X_1^{(2)}(1) & \phi_2^{15}\phi_1^3X_4^{(2)}(1) & \phi_2^{15}\phi_1^2X_3^{(2)}(1) & \phi_2^{15}X_1^{(2)}(13) & \phi_2^{15}\phi_1^3X_4^{(2)}(13) & \phi_2^{15}\phi_1^2X_3^{(2)}(13) \\
\phi_1X_2^{(2)}(1) & X_1^{(2)}(2) & \phi_1^3X_4^{(2)}(2) & \phi_1X_2^{(2)}(13) & X_1^{(2)}(14) & \phi_1^3X_4^{(2)}(14) \\
\phi_2\phi_1^2X_3^{(2)}(2) & \phi_2\phi_1X_2^{(2)}(2) & \phi_2X_1^{(2)}(3) & \phi_2\phi_1^2X_3^{(2)}(14) & \phi_2\phi_1X_2^{(2)}(14) & \phi_2X_1^{(2)}(15) \\
\phi_2^2\phi_1^3X_4^{(2)}(3) & \phi_2^2\phi_1^2X_3^{(2)}(3) & \phi_2^2\phi_1X_2^{(2)}(3) & \phi_2^2\phi_1^3X_4^{(2)}(15) & \phi_2^2\phi_1^2X_3^{(2)}(15) & \phi_2^2\phi_1X_2^{(2)}(15) \\
\phi_2^3X_1^{(2)}(7) & \phi_2^3\phi_1^3X_4^{(2)}(7) & \phi_2^3\phi_1^2X_3^{(2)}(7) & \phi_2^3X_1^{(2)}(19) & \phi_2^3\phi_1^3X_4^{(2)}(19) & \phi_2^3\phi_1^2X_3^{(2)}(19) \\
\phi_2^4\phi_1X_2^{(2)}(7) & \phi_2^4X_1^{(2)}(8) & \phi_2^4\phi_1^3X_4^{(2)}(8) & \phi_2^4\phi_1X_2^{(2)}(19) & \phi_2^4X_1^{(2)}(20) & \phi_2^4\phi_1^3X_4^{(2)}(20) \\
\phi_2^5\phi_1^2X_3^{(2)}(8) & \phi_2^5\phi_1X_2^{(2)}(8) & \phi_2^5X_1^{(2)}(9) & \phi_2^5\phi_1^2X_3^{(2)}(20) & \phi_2^5\phi_1X_2^{(2)}(20) & \phi_2^5X_1^{(2)}(21) \\
\phi_2^6\phi_1^3X_4^{(2)}(9) & \phi_2^6\phi_1^2X_3^{(2)}(9) & \phi_2^6\phi_1X_2^{(2)}(9) & \phi_2^6\phi_1^3X_4^{(2)}(21) & \phi_2^6\phi_1^2X_3^{(2)}(21) & \phi_2^6\phi_1X_2^{(2)}(21) \\
\phi_2^7X_1^{(2)}(4) & \phi_2^7\phi_1^3X_4^{(2)}(4) & \phi_2^7\phi_1^2X_3^{(2)}(4) & \phi_2^7X_1^{(2)}(16) & \phi_2^7\phi_1^3X_4^{(2)}(16) & \phi_2^7\phi_1^2X_3^{(2)}(16) \\
\phi_2^8\phi_1X_2^{(2)}(4) & \phi_2^8X_1^{(2)}(5) & \phi_2^8\phi_1^3X_4^{(2)}(5) & \phi_2^8\phi_1X_2^{(2)}(16) & \phi_2^8X_1^{(2)}(17) & \phi_2^8\phi_1^3X_4^{(2)}(17) \\
\phi_2^9\phi_1^2X_3^{(2)}(5) & \phi_2^9\phi_1X_2^{(2)}(5) & \phi_2^9X_1^{(2)}(6) & \phi_2^9\phi_1^2X_3^{(2)}(17) & \phi_2^9\phi_1X_2^{(2)}(17) & \phi_2^9X_1^{(2)}(18) \\
\phi_2^{10}\phi_1^3X_4^{(2)}(6) & \phi_2^{10}\phi_1^2X_3^{(2)}(6) & \phi_2^{10}\phi_1X_2^{(2)}(6) & \phi_2^{10}\phi_1^3X_4^{(2)}(18) & \phi_2^{10}\phi_1^2X_3^{(2)}(18) & \phi_2^{10}\phi_1X_2^{(2)}(18) \\
\phi_2^{11}X_1^{(2)}(10) & \phi_2^{11}\phi_1^3X_4^{(2)}(10) & \phi_2^{11}\phi_1^2X_3^{(2)}(10) & \phi_2^{11}X_1^{(2)}(22) & \phi_2^{11}\phi_1^3X_4^{(2)}(22) & \phi_2^{11}\phi_1^2X_3^{(2)}(22) \\
\phi_2^{12}\phi_1X_2^{(2)}(10) & \phi_2^{12}X_1^{(2)}(11) & \phi_2^{12}\phi_1^3X_4^{(2)}(11) & \phi_2^{12}\phi_1X_2^{(2)}(22) & \phi_2^{12}X_1^{(2)}(23) & \phi_2^{12}\phi_1^3X_4^{(2)}(23) \\
\phi_2^{13}\phi_1^2X_3^{(2)}(11) & \phi_2^{13}\phi_1X_2^{(2)}(11) & \phi_2^{13}X_1^{(2)}(12) & \phi_2^{13}\phi_1^2X_3^{(2)}(23) & \phi_2^{13}\phi_1X_2^{(2)}(23) & \phi_2^{13}X_1^{(2)}(24) \\
\phi_2^{14}\phi_1^3X_4^{(2)}(12) & \phi_2^{14}\phi_1^2X_3^{(2)}(12) & \phi_2^{14}\phi_1X_2^{(2)}(12) & \phi_2^{14}\phi_1^3X_4^{(2)}(24) & \phi_2^{14}\phi_1^2X_3^{(2)}(24) & \phi_2^{14}\phi_1X_2^{(2)}(24)
\end{pmatrix}
$$

5 Simulation Results

In the simulation experiments, we consider two users scenario where each user has two transmit antennas and the base station has two receive antennas with equal power gain. The MIMO frequency selective fading channels are simulated from a two ray channel model in which the second path delay is assumed to be 0.5μs that is, 10 times the sampling interval for each pair of transmit-receive antennas. The length of the cyclic prefix is chosen as ¼ of N where N=64 OFDM tones is used for each transmit antenna. In the simulation, the channel coefficients are independent from one OFDM block to other block but are remain constant during one OFDM block. The following three codes are simulated and compared in symbol error rate (SER) performance with baseband symbols, multiuser SF codes with 16-QAM, proposed multiuser space-frequency with 16-QAM and the proposed multiuser space-time-frequency codes with 16-QAM. From Fig. 4, it is observed that in the low-to-middle SNR region, presented SF code shows the similar performance as [13]. But, in the high SNR region, the proposed multi-user SF codes have better performance than [13]. Therefore, a higher coding gain is achieved compared with multi-user SF code in [13] in the high SNR region. Again it can be seen from the simulation result that the proposed multiuser STF codes have larger slope curve, compared with the two SF codes (i.e. multiuser SF codes [13], proposed multiuser space-frequency) and with the baseband signal.

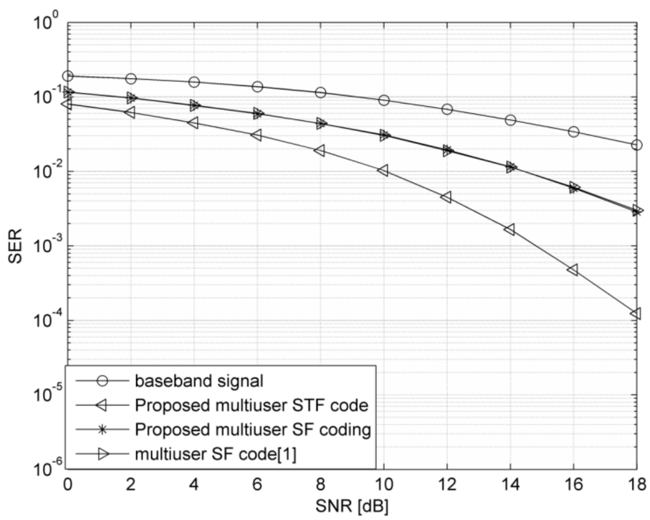

Fig:4: SER performance comparison between the proposed multiuser SF, multiuser STF code, the multiuser SF code in [13] and the baseband symbol.

This implies that the proposed STF codes achieve a larger diversity gain than both SF codes. From the analysis, it is clear that the proposed full-diversity multiuser STF code can achieve a diversity gain $A_t A_r A_u L$ =16, whereas the proposed full-diversity multiuser SF and the multiuser SF codes [13] codes achieves $A_t A_r L$ =8 only. Moreover, it is observed that the proposed multiuser rate-2 STF code achieves the best SER performance among the simulated cases, and outperforms the rate-1 SF code [13] and proposed rate-2 multiuser SF codes with a gap of about 4 dB over all the examined SNR values. This indicates that the rate-2 STF code from 16-QAM has a better coding and the diversity gain than the rate-1 SF code [13] and proposed rate-2 SF from 16QAM. The simulation result also shows that our presented high rate multiuser SF and STF codes achieve much better performance than the baseband symbols that justifies our proposed schemes. Therefore, the presented multiuser SF and STF codes do not affect the code performance while the symbol rate is increased to A_t.

7 Conclusion

The authors present two coding schemes for multi-user MIMO-OFDM system in this paper. The high-rate (rate-A_t) and full-diversity $A_t A_r L$ (for multi-user SF codes) and $A_t A_r A_u L$ (for multi-user STF codes) for each user is achieved without bandwidth expansion. The presented schemes are bandwidth efficient and always ensure rate-A_t as no zero padding is needed. Moreover, the presented coding schemes do not require the co-operation of multiple transmitters in the uplink process. Few examples of the presented code designs are given. Simulation result proves that the presented SF codes has a better coding gain in compare to [13] in the high SNR region whereas STF codes has a better coding and diversity gain in compare to [13] in both low and high SNR regions. Furthermore, the presented coding schemes do not affect the code performance while the symbol rate is increased to rate-A_t.

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APPENDIX

STF CODE DESIGN EXAMPLE IN DETAILS

An example of our recently proposed STF code [15] design has been given. We have just shown the sub-block $\bar{C}_1^{(z)}$ for convenience because the same coding strategies can be applied to every sub-block$\bar{C}_b^{(z)}$, b=1,..., B.

Let, $A_u = 1$, $A_t = 2$, $L = 2$, $Z = 2$. So, $W_L = 2$, $W_t = 2$, $W_z = 2$.

Thus, Φ_1 is 2× 2 matrix and given by, $\Phi_1=\begin{pmatrix} 1 & \phi_1 \\ \phi_1 & 1 \end{pmatrix}$ and Φ_2 is 8×2 matrix and given by,

$$\Phi_2=\begin{pmatrix} 1 & {\phi_2}^7 \\ \phi_2 & {\phi_2}^6 \\ {\phi_2}^2 & {\phi_2}^5 \\ {\phi_2}^3 & {\phi_2}^4 \\ {\phi_2}^4 & {\phi_2}^3 \\ {\phi_2}^5 & {\phi_2}^2 \\ {\phi_2}^6 & \phi_2 \\ {\phi_2}^7 & 1 \end{pmatrix}.$$

Now, for i=1, l=1, d=1, ${P_{1,1}}^1$=0;

i=1, l=2, d=1, ${P_{1,1}}^2$=4;

i=2, l=1, d=1, ${P_{2,1}}^1$=2;

i=2, l=2, d=1, ${P_{2,1}}^2$=6;

For z=1, $${\bar{X}^1_{1,1}}^{(1)} = \begin{pmatrix} {X_1}^{(1)}(1) & \phi_1 {X_2}^{(1)}(1) \\ \phi_1 {X_2}^{(1)}(2) & {X_1}^{(1)}(2) \end{pmatrix}, \quad {\bar{X}^1_{2,1}}^{(1)} = \begin{pmatrix} {X_1}^{(1)}(3) & \phi_1 {X_2}^{(1)}(3) \\ \phi_1 {X_2}^{(1)}(4) & {X_1}^{(1)}(4) \end{pmatrix},$$

$${\bar{X}^1_{1,2}}^{(1)} = \begin{pmatrix} {X_1}^{(1)}(5) & \phi_1 {X_2}^{(1)}(5) \\ \phi_1 {X_2}^{(1)}(6) & {X_1}^{(1)}(6) \end{pmatrix}, \quad {\bar{X}^1_{2,2}}^{(1)} = \begin{pmatrix} {X_1}^{(1)}(7) & \phi_1 {X_2}^{(1)}(7) \\ \phi_1 {X_2}^{(1)}(8) & {X_1}^{(1)}(8) \end{pmatrix}.$$

Now, $${c_1}^{(z)} = \begin{pmatrix} {\bar{X}^1_{1,1}}^{(1)} \\ {\bar{X}^1_{1,2}}^{(1)} \\ {\bar{X}^1_{2,1}}^{(1)} \\ {\bar{X}^1_{2,2}}^{(1)} \end{pmatrix}$$

$$= \begin{pmatrix} {X_1}^{(1)}(1) & \phi_1 {X_2}^{(1)}(1) \\ \phi_1 {X_2}^{(1)}(2) & {X_1}^{(1)}(2) \\ {X_1}^{(1)}(5) & \phi_1 {X_2}^{(1)}(5) \\ \phi_1 {X_2}^{(1)}(6) & {X_1}^{(1)}(6) \\ {X_1}^{(1)}(3) & \phi_1 {X_2}^{(1)}(3) \\ \phi_1 {X_2}^{(1)}(4) & {X_1}^{(1)}(4) \\ {X_1}^{(1)}(7) & \phi_1 {X_2}^{(1)}(7) \\ \phi_1 {X_2}^{(1)}(8) & {X_1}^{(1)}(8) \end{pmatrix}.$$

Thus,

$$\bar{C}_1^{(1)} = \left(\Phi_{2,1} \otimes 1_{1\times 2}\right) \circ C_1^{(1)} = \begin{pmatrix} X_1^{(1)}(1) & \phi_1 X_2^{(1)}(1) \\ \phi_2\phi_1 X_2^{(1)}(2) & \phi_2 X_1^{(1)}(2) \\ {\phi_2}^2 X_1^{(1)}(5) & {\phi_2}^2\phi_1 X_2^{(1)}(5) \\ {\phi_2}^3\phi_1 X_2^{(1)}(6) & {\phi_2}^3 X_1^{(1)}(6) \\ {\phi_2}^4 X_1^{(1)}(3) & {\phi_2}^4\phi_1 X_2^{(1)}(3) \\ {\phi_2}^5\phi_1 X_2^{(1)}(4) & {\phi_2}^5 X_1^{(1)}(4) \\ {\phi_2}^6 X_1^{(1)}(7) & {\phi_2}^6\phi_1 X_2^{(1)}(7) \\ {\phi_2}^7\phi_1 X_2^{(1)}(8) & {\phi_2}^7 X_1^{(1)}(8) \end{pmatrix}$$

Similarly, for z=2,

$$\bar{C}_1^{(2)} = \begin{pmatrix} {\phi_2}^7 X_1^{(2)}(1) & {\phi_2}^7\phi_1 X_2^{(2)}(1) \\ \phi_1 X_2^{(2)}(2) & X_1^{(2)}(2) \\ \phi_2 X_1^{(2)}(5) & \phi_2\phi_1 X_2^{(2)}(5) \\ {\phi_2}^2\phi_1 X_2^{(2)}(6) & {\phi_2}^2 X_1^{(2)}(6) \\ {\phi_2}^3 X_1^{(2)}(3) & {\phi_2}^3\phi_1 X_2^{(2)}(3) \\ {\phi_2}^4\phi_1 X_2^{(2)}(4) & {\phi_2}^4 X_1^{(2)}(4) \\ {\phi_2}^5 X_1^{(2)}(7) & {\phi_2}^5\phi_1 X_2^{(2)}(7) \\ {\phi_2}^6\phi_1 X_2^{(2)}(8) & {\phi_2}^6 X_1^{(2)}(8) \end{pmatrix}$$