Analysing the Swing Equation using MATLAB Simulink for Primary Resonance, Subharmonic Resonance and for the case of Quasiperiodicity

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Abstract: The swing equation plays a significant role in the analysis of stability and frequency response various power systems and mechanical systems. MATLAB Simulink simulates and analyses different systems, including synchronous generators with various excitation methods. This research aims to study the swing equation by modelling the system in Simulink. Swing equation analysis can be applied to tackle power instabilities in the electrical grid, to avoid power outages by monitoring the small disturbances that occur within the system. This paper shows the time series, phase portraits, and Poincaré maps generated using data from the simulated model. It highlights the occurrence of period doublings which lead to loss of synchronisation and the resulting instability in the system that descends into chaos when the variables are changed in the Simulink model. The integrity diagrams were also identified for primary resonance, subharmonic resonance, and quasiperiodicity, offering valuable information to understand the system's nonlinear behaviour. Using the swing equation in MATLAB Simulink provides a robust tool for analysing, simulating, and optimising systems. Hence this study provides an enhanced understanding of the system's behaviour in Simulink for primary resonance, subharmonic resonance and for the case of quasiperiodicity. Additionally, it validates the analytical and numerical findings from prior works by the same authors.

Key-Words: nonlinear dynamics, swing equation, Simulink, power system, chaos.

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1 Introduction

The swing equation, which represents the nonlinear behaviour of synchronous generators, has been thoroughly examined in numerous research studies in recent years. These models are key for analysing the stability of complex synchronous machines in dynamical systems. Researchers have used Simulink to create enhanced swing equation models to remove simplifying assumptions, resulting in a more precise depiction of system dynamics, [1], [2]. The improved models have been employed to examine the performance of synchronous generators in various situations, such as when coupled to steady loads or infinite buses, yielding valuable information on stability and frequency regulation, [3], [4]. Furthermore, MATLAB Simulink provides clear visualisation when the different excitation frequencies are then considered within the systems, [5].

The swing equation studies and examines the dynamical behaviour of the rotor of the machine and small external disturbances, [2], [6]. Also studies have shown that modifying and altering certain variables in the equation results in different behavioural patterns within the system. Therefore, the system faces difficulty coming to its original condition, exhibiting little alterations that eventually lead to chaos within the structure, [7]. Studying the core principles of chaos theory will yield crucial insights for managing the nonlinear system, [8].

Initially, the swing equation is modelled on Matlab Simulink whereby primary resonance, subharmonic resonance and quasiperiodicity are examined when changing the excitation frequency of the system. The produced results were then compared and validated to the analytical and numerical results obtained from previous works of the same authors, [2], [6], [9], [10]. Different values of Ω are considered and Poincaré maps were plotted to compare the analytical methods with Simulink model to obtain strong conclusions for this study.

This work aims to comprehend the process of modelling the swing equation using Simulink and provide validation for the analytical methodologies used to better the understanding of academics and scholars. Hence, the objective is to highlight advancements in this specific study of the swing equation using a Matlab model and to focus on understanding this model to offer new perspectives on ongoing concerns regarding the stability of dynamical systems.

1.1 Brief Literature Review

MATLAB Simulink models are key in studying intricate behaviour of the nonlinear systems. It allows for modelling, simulation and thorough analysis of complex power systems, [11]. These provides engineers and researchers within the electrical field to visualise the system on the digital computer before implementing the methods on the actual power grids, [12], [13]. Storage plants also use Simulink models to understand the process at different speeds and at variable loads providing assurance with the safety within the systems, [14]. It helps in the modelling and simulation of models, which in turn enables the development of novel chaotic systems with a wide range of dynamic behaviours, [15].

Within the domain of modelling on Simulink, numerous investigations have then examined diverse applications and methodologies. The Simulink models have been used in diagnosing faults in control systems and also simulate power systems by breaking down the complex components, [16], [17], [18], [19]. These models also been utilised in studying the vehicle gearbox and memristors in chaotic systems, [20],[21], [22].

This study also examines the integrity diagrams for primary resonance, subharmonic resonance and for the case of quasiperiodicity when the variable is altered. An analytical approach based on the isolated resonance approximation can be used to obtain integrity diagrams and determine their boundaries prior to the occurrence of period doublings, [23], [24]. Stochastic Bifurcation limits are derived using this method considering different amplitudes and initial conditions, [25], [26]. It is also found that attractors lose the stability without chaos occurring within the system when a system goes into bistable state, [27], [28].

Primary resonance is when the excitation frequency of the system is approximately equal to the natural frequency of the system , while subharmonic resonance occurs when disturbances are multiples of the natural frequencies. Both of these resonances can result in system instability and equipment damage, [29], [30]. Methods such as the incremental averaging method and numerous scales yield precise analytical solutions that offer valuable insights into the resonant behaviour of nonlinear systems, including Duffing oscillators with different damping processes, [31], [32]. Moreover, the investigation of subharmonic resonance are used for diagnostic imaging withing ultrasonic contrast agents, [33]. Quasiperiodicity is when the ratio of the frequencies is an irrational number. All three cases are studied through the Simulink model. It is vital to study all three cases to get a better understanding of the behaviour of the swing equation system.

Bifurcation diagrams are a useful tool for analysing integrity diagrams in dynamics, as mentioned in various research publications. They provide crucial information about the dynamical behaviour of the system and the stability, [34], [35], [36]. Hamiltonian systems use bifurcation diagrams to study the intricate and chaotic behaviour within this domain, [37]. Hence it is important to consider and obtain bifurcation diagrams for nonlinear systems in order to understand the complexity of the structure to provide an in-depth analysis for future research.

2 Methodology

2.1 Analytical Work

The swing equation is formulated from the Law of Rotation which explains the motion of rotating systems. It derived with the help of Newton's second law of motion in synchronous generators and applied on the rotor of the swing equation. The analytical work shown below studies both mechanical and electrical torques on the rotor. Previous study done related to this concept are referenced by [2], [6], [9], [10].

The equation analysing the rotor's motion of the machine including a damping term is given by [23].

$$\frac{2H}{\omega_R}\,\frac{d^2\theta}{dt^2} + D\,\frac{d\theta}{dt} = P_m - \frac{V_G V_B}{X_G}\sin(\theta - \theta_B) \quad (1)$$

$$V_{\mathrm{B}} = V_{B0} + V_{\mathrm{B1}}\cos(\Omega t + \phi_v) \quad (2)$$

$$\theta_B = \theta_{B0} + \theta_{B1}\cos(\Omega t + \phi_0) \quad (3)$$

with
$\omega_R = Constant\ angular\ velocity$,
H= Inertia,
D= Damping,
P_m= Mechanical Power,
V_G= Voltage of machine,
X_G = Transient Reactance,
V_B= Voltage of busbar system,
θ_B = phase of busbar system,

V_{B1} and θ_{B1} magnitudes assumed to be small.

A deeper understanding of this equation is essential to understand the concept of stability and its dynamical behaviour. This analytical work uses Taylor expansion and algebraic methods to formulate the equation to obtain more results using the digital computers, [7], [8], [33]. Hence changes can be made to the variables of the swing equation to observe and analyse the intricate behaviour of this system.

2.1.1 The Swing Equation Model from MATLAB Simulink

The swing equation, equation (1), explains both the electrical and mechanical torque of the rotor of the machine and studies the behaviour of the angle of the rotor and speed when a small change is introduced. Analysing the acceleration of the machine and the torques provides a strong foundation for engineers to overcome difficulties within the systems, [23]. Hence modelling this concept to obtain real-time values will be ideal to study the equation in detail.

The rotor of the machine used by the swing equation, explains the intricate behaviour of both electrical and mechanical elements of the system. Hence studying the stability of this machine is vital to comprehend the abrupt alterations to the parameters of the equation. Stability can be observed through changing the load and inputs of the systems over time and hence reducing the cascade of chaos within power systems, [32].

The following Simulink model shown in Figure 1 was used to analyse the swing equation for this study.

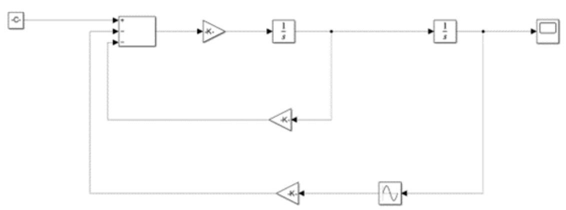

Fig. 1: MATLAB Simulink model used to represent the Swing Equation.

2.1.2 Integrity Diagrams

Integrity diagrams are of crucial significance in nonlinear dynamics since they allow for the evaluation of the dynamic integrity of systems. These diagrams are crucial for measuring the safe basin and erosion profiles, which are fundamental tools for analysing dynamic integrity, [38]. The notion of dynamical integrity has become a key consideration in the design of structures, with significant research dedicated to the management of basin erosion processes, [39]. The concept of global safety, a new approach to evaluating systems, has had a considerable influence on the analysis, control, and design of mechanical and structural systems. The integrity diagrams play a vital part in maintaining the stability and the performance of the nonlinear system, [40]. This is exemplified by analysing vibrational systems, both with and without discontinuities, [41]. Nonlinear Robust Control strategies need integrity diagrams to show solutions when different variables are affected by some external disturbances, [42].

These diagrams also utilise surrogate models to decrease simulation time, uphold correctness, and enable incorporation into circuit simulators for thorough setup analysis during the design phase. Adjustable dead bands are investigated in networked control systems to minimise network traffic. The primary emphasis is on stability analysis utilising robust stability theory, [43]. In addition, nonlinear robust control methods that rely on integrity are employed to address non-modelled dynamics and uncertainties in multivariable systems, hence guaranteeing both robustness and feasibility, [44].

2.2 Results from the Simulink Model

2.2.1 Primary Resonance

The results for primary resonance was obtained for the Simulink model. Figure 2, Figure 3, Figure 4, Figure 5 and Figure 6 show time series, phase portraits and Poincaré maps that were plotted and compared to the analytical results obtained from the previous research work, [2]. The produced figures from MATLAB Simulink show similar behaviour to the analytical work hence providing a strong validation to this study.

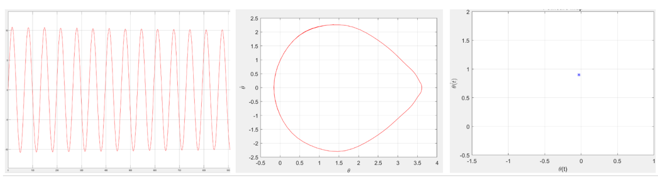

Fig. 2: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 8.61$ rads^{-1}.

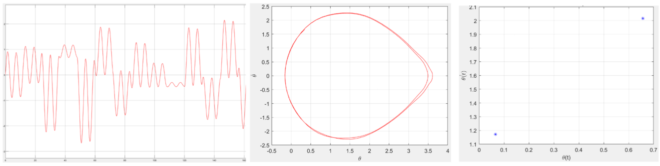

Fig. 3: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 8.43$ rads^{-1}.

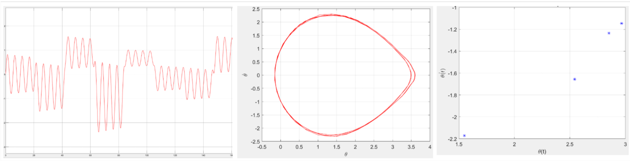

Fig. 4: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 8.282$ rads^{-1}.

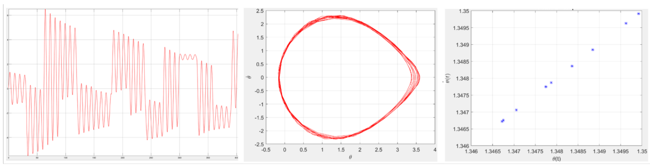

Fig. 5: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 8.275$ rads^{-1}.

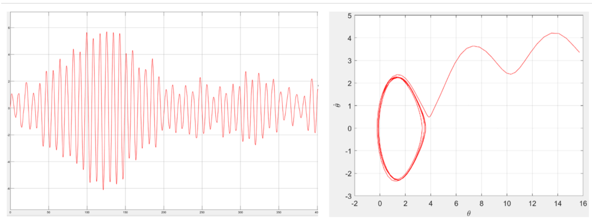

Fig. 6: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 8.2601$ rads^{-1}.

2.2.2 Subharmonic Resonance

Similarly, Figure 7, Figure 8, Figure 9, Figure 10 and Figure 11 were obtained for subharmonic resonance from the Simulink model. Results obtained for subharmonic resonance were compared to the analytical findings from previous research, [6]. The graphs show similar behaviour to the analytical work, hence providing strong confirmation for this study,[6], [10].

Subharmonic resonance is when the excitation frequency is twice the natural frequency of the system. This results in the occurrence of low-frequency oscillations and the possibility of equipment damage, [6], [10]. Studies have demonstrated that by employing Melnikov methods, chaos in the pendulum equation may be mitigated during ultra-subharmonic resonance. This allows for the manipulation of chaos patterns, enabling them to be regulated into period-n orbits by making precise adjustments to certain parameters, [45].

The Ω was reduced and it was observed that the system was losing its stability and entering into chaos.

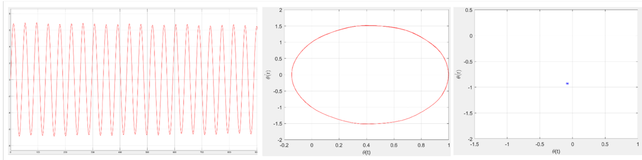

Fig. 7: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 26.01$ rads^{-1}.

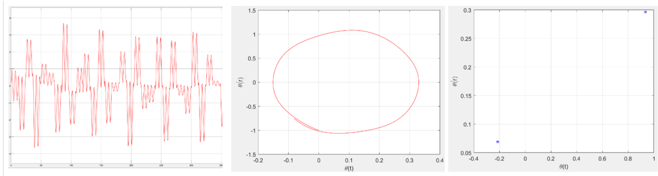

Fig. 8: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 21.042$ rads^{-1}.

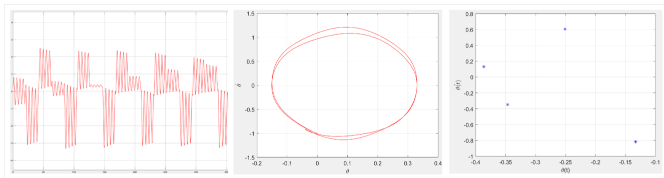

Fig. 9: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 19.4162$ rads^{-1}.

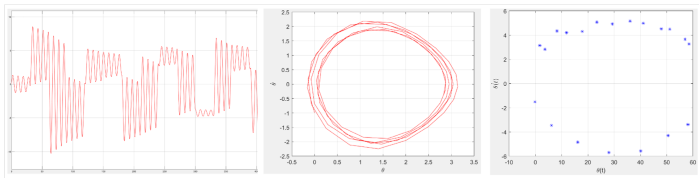

Fig. 10: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 19.375$ rads^{-1}.

2.2.3 Quasiperiodicity

Considering quasiperiodicity where the Ω value is considered to be irrational values, figures similar to the analytical work done previously, [9], were

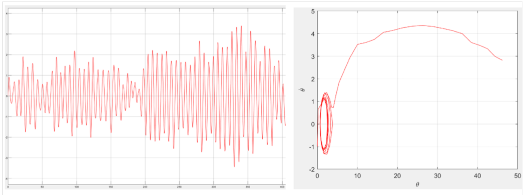

Fig. 11: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 19.37251$ rads^{-1}.

produced and compared. Figure 12, Figure 13, Figure 14, Figure 15 and Figure 16 show the behaviour of the nonlinear system as Ω is reduced exemplifying the significance of the Simulink model.

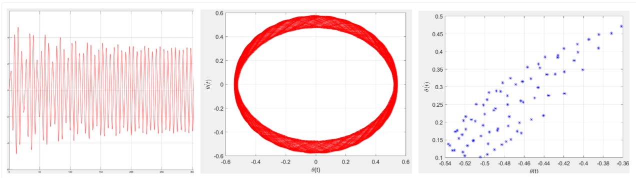

Fig. 12: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 2\pi$ rads^{-1}.

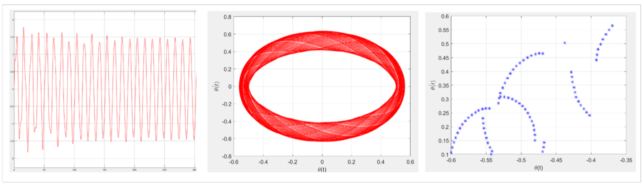

Fig. 13: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = \pi$ rads^{-1}.

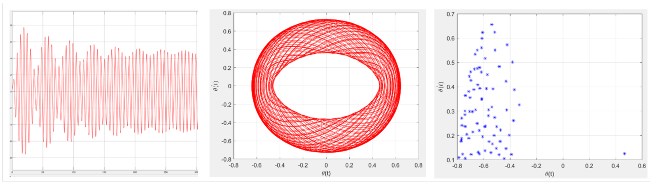

Fig. 14: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 2\pi/3$ rads^{-1}.

2.2.4 Comparing Analytical Method with the Simulink model

The following Figure 17, Figure 18 and Figure 19 were produced to compare the analytical method

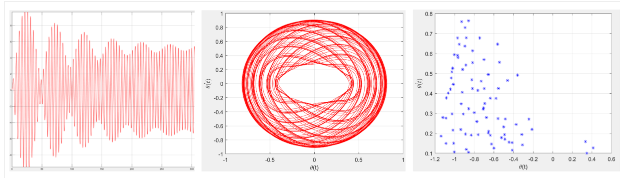

Fig. 15: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = \pi/2$ rads^{-1}.

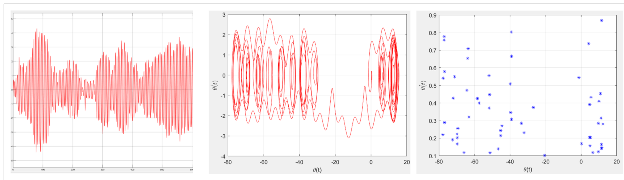

Fig. 16: Time series, Phase portrait and Poincaré map from Simulink when $\Omega = 2\pi/8$ rads^{-1}.

with the Simulink model to validate the results further.

For primary resonance shown in Figure 17, an Ω value 8.2601 rads^{-1} approximately closer to the natural frequency of the system is considered and the Poincaré maps obtained from the analytical work and the simulink model respectively. Both clearly showing similar patterns for the system's behaviour.

Figure 18 depicts subharmonic resonance where the $\Omega = 19.37251$ rads^{-1}. The Poincaré maps obtained from both the analytical and the Simulated model also shows close to similar behaviours validating the Simulink model.

Figure 19 was produced for the case of quasiperiodicity where the Ω is now $\pi/8.5$ rads^{-1}. This also provides strong comparison between the analytically obtained Poincaré map to the Simulink modelled Poincaré map.

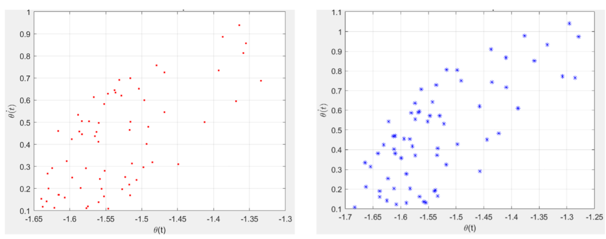

Fig. 17: Poincaré maps from analytical method and Simulink model respectively for Primary Resonance when $\Omega = 8.2601$ rads^{-1}.

2.2.5 Integrity Diagrams

Bifurcation diagrams are essential tools for comprehending nonlinear dynamical systems as

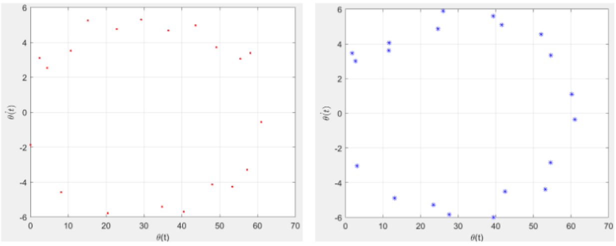

Fig. 18: Poincaré maps from analytical method and Simulink model respectively for Subharmonic Resonance when $\Omega = 19.37251$ rads^{-1}.

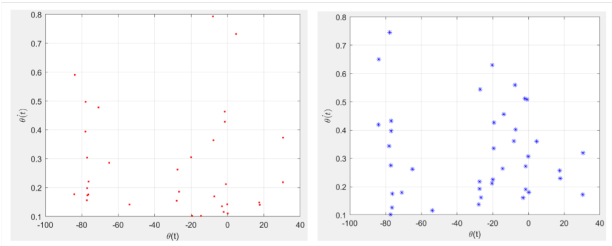

Fig. 19: Poincaré maps from analytical method and Simulink model respectively for Quaisperiodicity when $\Omega = \pi/8.5$ rads^{-1}.

they visually depict the system's behaviour as a parameter Ω is systematically modified. These diagrams identify the specific moments at which the system's solutions undergo qualitative changes, shifting from stable fixed points to either periodic or chaotic behaviour. It can be used to identify these critical points to derive significant parameter values and accompanying state variables. These inputs are crucial for integrity diagrams that illustrate the dynamic behaviours and resilience of the system in the parameter space. This approach also gives a more efficient computing method for analysing system behaviour, [39], [41]. Integrity diagrams also show the safe areas for the systems and it is crucial to stay below the integrity curve to avoid operating at r values exceeding that of the cliff face.

Integrity diagrams are obtained by examining the stability of various behaviours when system parameters change, based on bifurcation diagrams. Illustrating the period-doubling precursor to chaos, the initial step involves creating a bifurcation diagram by manipulating a control parameter Ω, and documenting the system's stable or periodic solutions. The identification of critical locations where period-doubling bifurcations occur, resulting in the emergence of chaos, has been accomplished.

The areas denoting stable equilibrium locations, periodic trajectories, and chaotic dynamics are indicated. An analysis is conducted to determine how the boundaries between these behaviours change in response to perturbations. Prior to and following perturbations, the integrity zones, which denote the parameter ranges in which the system maintains a specific stable state, are computed. The reduction percentage is then derived by comparing the area of these regions before and after the disturbance.

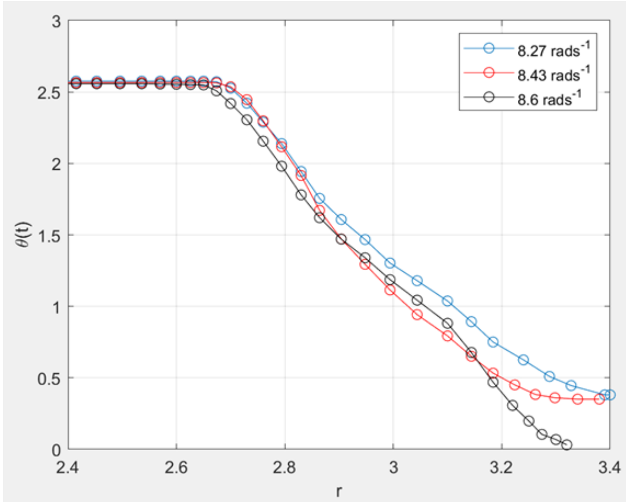

Fig. 20: Integrity Diagrams for Primary Resonance when $\Omega = 8.27$ rads^{-1}, 8.43 rads^{-1} and 8.61 rads^{-1}.

Figure 20 depicts the changes happening within the primary resonance. As the Ω is increased the erosion takes place quicker.

Figure 21 and Figure 22 show the integrity diagrams for the subharmonic resonance and quasiperiodicity respectively. As the r value is increased when the Ω is increased the system's behaviour alters.

The reduction percentage is calculated for the integrity diagrams for primary resonance and analysed. When $\Omega = 8.27$ rads^{-1} the reduction percentage is 24.56% but as Ω is increased to the reduction percentage also increases to 38.17%. 8.6 rads^{-1} exemplifying that the stable region reduces as the parameter is changed.

Therefore, it has been confirmed that the system's integrity is significantly compromised as the parameter is increased. For subharmonic resonance, when the Ω is equal to 19.4162 rads^{-1}, results in approximately a 44.13% decrease in stable behaviour. At $\Omega = 26.01$ rads^{-1}, the rise in bias results in a reduction of around 51.28%, indicating that the consistent performance of the system is greatly reduced.

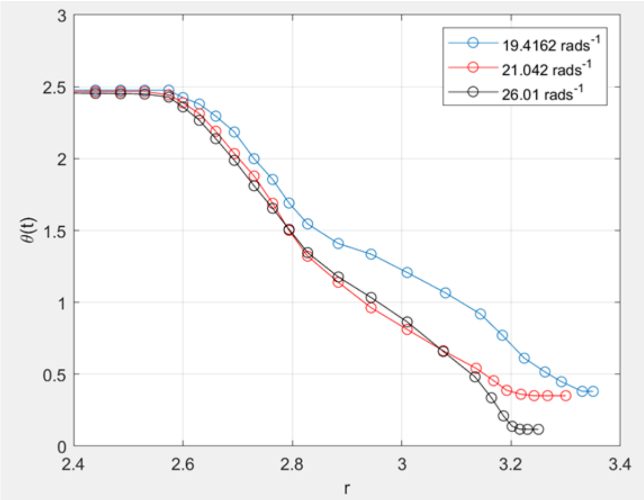

Fig. 21: Integrity Diagrams for Subharmonic Resonance when $\Omega = 19.4162$ rads^{-1}, 21.042 rads^{-1} and 26.01 rads^{-1}.

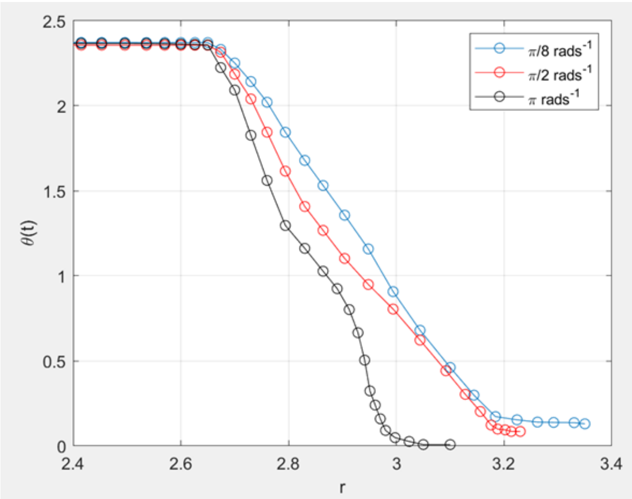

Fig. 22: Integrity Diagrams for Quasiperiodicity $\Omega = \pi/8$ rads^{-1}, $\pi/2$ rads^{-1} and π rads^{-1}.

Considering the case of quasiperiodicity, as Ω is varied from $\pi/8$ rads^{-1}, $\pi/2$ rads^{-1} and π rads^{-1}, the stable region is diminished. The stable region is reduced to 39.01% when $\Omega = \pi/8$ rads^{-1}, but rapidly increases in value to 67.23% as Ω is increased to π rads^{-1}.

3 Discussion

The main objective of this study is to thoroughly analyse the dynamical behaviour of the swing equation when altering parameters within the system. This paper also compares the analytical methods and numerical simulations to provide strong results for the research. This work also aims to comprehensively comprehend the dynamics of the swing equation and its influence on power system stability by using the Simulink tool.

Analytical tools are essential for evaluating the resonances in the swing equation. Through the utilisation of mathematical modelling and computations, these methods offer precise insights that are based on minimal assumptions. The simulated model provides graphical representations of the behaviour of the swing equation for primary resonance, subharmonic resonance and quasiperiodicity when variables are changed. These graphs provide authentication to the analytical results obtained. This method also provides engineers to make sound decisions when considering the stability of electric grids.

Load variations are a common occurrence in power systems within nonlinear dynamics. The swing equation implemented in Matlab offers insight into the analytical and numerical findings acquired earlier. The data acquired from these scenarios is vital for the administration of the electrical grid, aiding in preserving system stability and reliability. Obtaining a better understanding of the dynamics in the swing equation can be helpful in reducing power outages within electric systems and aid in avoiding unavoidable circumstances.

4 Conclusion

This comprehensive analysis on the swing equation and its MATLAB Simulink model compare and validate the results obtained from the research done by the same authors. The time series, phase portraits and Poincaré maps all obtained from the simulink model show the dynamical behaviour of the power system when Ω is altered. Bifurcation diagrams were studied in detail to obtain the integrity diagrams and improving the understanding of the swing equation and its behaviour. The results derived from the Simulink model for primary resonance, subharmonic resonance, and quasiperiodicity exhibit analogous behavioural patterns to the prior analytical work conducted by the same authors.

This study builds upon the recent research studies carried out by the same researchers, advancing their prior discoveries. Its objective is to improve current methods by providing a deeper comprehension of the fundamental mathematics, rather than substituting them. This research enhances the understanding of fundamental principles and system stability in power systems, specifically focussing on the Simulink model. It adds to improving control techniques and preventive measures for power systems. Its objective is to reduce the disruptive consequences resulting from the actions advantageous to power system engineers and researchers.

The findings also show substantial information regarding dynamical behaviour of the swing equation. This can help to improve power systems which are complex with minute intricate detailing within the electronics sector.

In the future, scholars may explore methods to enhance these conditions within the context of swing equations in power grids. This could yield significant novel insights into the enduring sustainability and adaptability of power networks. They can also deepen our knowledge of complex nonlinear systems and generate enhancements that improve resilience.

Declaration During the preparation of this work the authors used Grammarly for language editing. After using this service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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