A Comparison of the Kalman Filter and the Unbiased FIR Filter for Network Systems with Multiples Output Delays and Lost Data

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Abstract: In this article, a comparison of the UFIR and Kalman filter to estimate a tracking vehicle system variables is developed considering two possible observation output models. The time stamp approach and the predictive compensation are used to analyze the problem from multiple perturbations, which produces random delayed data and losses during transmissions. For the estimation, a transformation model and a decorrelation covariance matrices are developed with the aim of assure optimal conditions and minimizing the estimation error. Finally, several real situations, miss modeling, uncertain noise covariances, and uncertain probabilities are proposed to demonstrate the effectiveness and robustness of the filter proposed.

Keywords: Delayed data, missing data, unbiased FIR filter

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1. Introduction

Network systems have become one of the most popular approaches to engineering in recent years. Consequently, ensuring accurate tracking in an uncertain environment is transformed into a recent problem. [1], [2] in a network environment usually the measurement taken by the sensors suffer from random perturbations or limited communications which let random phenomena or packet errors such as delays, packet dropouts, and lost data, among others [3], [4]. In conventional estimation strategies, deteriorated performance may be produced if the measurement vector which arrives at the processor is adopted. In this sense, new strategies should be designed to improve the accuracy of estimations in unreliable data.

Generally, estimation methodologies depict in an accurate sense the current behavior of the measurements is hard work. In recent decades, many methods have been investigated to characterize this random success. the Bernoulli distribution is the regular model used to describe these phenomenons [5]. One-step random delays and multiple time delays have been studied in which the Kalman filter, H_∞, distributed filter, and others [6]–[8] are been developed by random Bernoulli variables. In [9] the UFIR filter approach was developed using consecutive random variables to depict the real behavior of the measurement; notable robustness was determined compared with other filters.

In relation to transmission losses, The most general compensation framework consists of the zero input and the Hold-input mechanisms [10], [11]. Nothing or the last successfully transmitted measurement is used to compensate when nothing arrives at the processor. However, notables damages are obtained in network congestion. in order to avoid this, new methodologies are used to improve it, such as processing the multiple packet data [12] , predictive compensation, or using the time stamp information [13]–[15].

Motivated by the above discussion and avoiding real mistakes with system knowledge, this paper considers the unbiased FIR filter in network systems with uncertain measurement [16]–[18]. The predictive compensation and time stamp methodology are compared and analyzed by the UFIR filter to improve the accuracy and robustness of state estimation, also expose crucial advantages bounding input bounded output stability, higher robustness than KF, and ignoring zero-mean noise and initial values. Experimental testing based on the Global Positioning System (GPS) is also provided.

2. Model Formulation with Packet Dropouts

The network system and the sensor measurement of the process may be expressed by the following linear model in discrete time:

$$x_n = Fx_{n-1} + w_n \,, \quad (1)$$
$$y_n = Hx_n + v_n \,, \quad (2)$$

where $F \in \mathbb{R}^{K\times K}$, $H \in \mathbb{R}^{K\times M}$ are known matrices with appropriate size; $x_n \in \mathbb{R}^K$ and $y_n \in \mathbb{R}^M$ are the state vector and the measurement vector at time n respectively. $w_n \sim \mathcal{N}(0, Q) \in \mathbb{R}^K$ and $v_n \sim \mathcal{N}(0, R) \in \mathbb{R}^M$ are measurement and model noise vectors with zero mean and non-correlation between other vectors. The covariance matrices satisfy $Q = E\{w_n w_n^T\} \in \mathbb{R}^{K\times K}$ and $R = E\{v_n v_n^T\} \in \mathbb{R}^{M\times M}$.

In order to update the filter with the sensor measurement transmitted at most one time, two approaches are handled to reflect the stochastic and unpredictable delays and multiple packet dropouts during the communication in accordance with the time stamp.

Attempting to express the real behavior during the signal transmission is defiance, the estimation from measurements that are assumed valid is a common failure at estimating accurately. When a measurement is not accurate, compensating it with the latest measurement transmitted successfully, is an efficient compensation strategy. However, in the case that it can not fall, another compensation strategy is needed.

Employing all the information at the packet sensor, as the time stamp, is a physical strategy to make a balance at the signal receiving. Based on [13], when the sensor data is time-stamped the filter perceives if no data or only noise is taken. In accordance, a flag triggers a predictive algorithm. The packet dropout compensation consists of replacing each lost observation with a predictive algorithm based on the last estimation. The following observation model is proposed to address the estimation problem.

$$\begin{cases} z_n = (\alpha_n y_n + (1-\alpha_n)\, y_{n-1}) & if \beta_n = 1 \\ z_n = \hat{y}_n & if \beta_n = 0 \end{cases} \tag{3}$$

where y_n is the measurement vector, $\hat{y}_n$ is the predictive vector, and $z_n \in \mathbb{R}^M$ is the transmitted measurement vector. α_n is a binary random variable with known probabilities $\mathcal{P}\{\alpha_n = 1\} = \overline{\alpha}_n$ and $\mathcal{P}\{\alpha_n = 0\} = 1 - \overline{\alpha}_n$ and β_n is a scalar loss data factor. the predictive observation is defined as:

$$\hat{y}_n = HF\hat{x}_{n-1}\,, \tag{4}$$

In (3), β_n select which information may arrive at each time n. We assume that β_n is independent of α_n. Under $\beta_n = 1$ denote the received measurement output, a currently packet data $z_n = y_n$ if $\alpha_n = 1$ with probability $\bar{\alpha}$, a delay-step data $z_n = y_{n-1}$ if $\alpha_n = 0$ with probability $(1 - \bar{\alpha})$.

In contrast to the time step approach, According to [14] when the processor does not have the knowledge of delays and lost data at the arrival signal, but the probability of this occurring is distinguished, The packet dropout compensation scheduling incorporates the probabilistic information by Bernoulli variables to express the unreliable effects of delays and lost data.

$$\begin{aligned} z_n \;&=\; \alpha_{0,n} y_n + (1-\alpha_{0,n})\left\{ (1-\alpha_{0,n-1})\,\alpha_{1,n} y_{n-1} \right. \\ &\left. + (1-(1-\alpha_{0,n-1})\,\alpha_{1,n})\,\hat{y}_n \right\}, \end{aligned} \tag{5}$$

where $\alpha_{0,n}$ and $\alpha_{1,n}$ are binary random variables with known probabilities $\mathcal{P}\{\alpha_{0,n} = 1\} = \overline{\alpha}_{0,n}$ and $\mathcal{P}\{\alpha_{0,n} = 0\} = 1 - \overline{\alpha}_{0,n}$ and $\mathcal{P}\{\alpha_{1,n} = 1\} = \overline{\alpha}_{1,n}$ and $\mathcal{P}\{\alpha_{1,n} = 0\} = 1 - \overline{\alpha}_{1,n}$.

Following (5) the measurement is received at time n with probability $\bar{\alpha}_0$ when $\alpha_{0,n} = 1$ in contrast delay step data or nothing is received; following the inaccurately data variable $\alpha_{1,n}$, one-step delay data with probability $(1-\bar{\alpha}_0)^2\bar{\alpha}_1$ is obtained if $\alpha_{0,n-1} = 0$ and $\alpha_{1,n} = 1$ in other wise, a predictive compensation is used with probability $(1-\bar{\alpha}_0) - (1-\bar{\alpha}_0)^2\bar{\alpha}_1$.

Our aim is to design the UFIR filter $\hat{x}_n$ based on the optimum variance sense to analyze and compare the significant influence of time stamps to generate an accurate estimation of lost information appears. It is achieved using the observation models (3) and (5).

3. Unbiased Estimator Problem

In the unbiased estimation problem as in many other linear estimators, the observation model of the signal to be estimated should not depend on the previous states; thus, based on the stochastic equations (3) and (5), a system transformation and a FIR filter in sens of the unbiased condition will be derived.

3.1 System Transformation

To provide an unify general model, assume that for each n the state evolution in terms of the previous time as $x_{n-k_n} = F^{-k_n}\left(x_n - \sum_{j=0}^{k_n-1} w_{n-j}\right)$, Hence for a one-step delay $k_n = 1$, $k_n = 1$ $x_{n-1} = F^{-1}(x_n - w_n)$.

for (1)-(3), We can obtain the transformed system $z_n = \bar{H}_n x_n + \bar{v}_n$, with the parameter matrices

$$\bar{H}_n = \alpha_n H + (1-\alpha_n) H F^{-k_n}\,, \tag{6}$$

$$\bar{v}_n = \alpha_n v_n + (1-\alpha_n) v_{n-1} - (1-\alpha_n) H w_n\,, \tag{7}$$

and the covariance measurement noise matrix is

$$\bar{R}_n = \bar{\alpha}_n R_n + (1-\bar{\alpha}_n) R_{n-1} - (1-\bar{\alpha}_n) H Q_{n-1} H^T\,, \tag{8}$$

Now, for (1)-(5), the one-step predictor is designed assuming that the estimation has minimum mean estimated error $\mathbb{E}[e_n] = 0$ to guaranty an unbiased estimation [19]. Note that, when the expectation $\mathbb{E}[\hat{x}_n]$ satisfied as the mean state signal $\mathbb{E}[x_n]$, the predictive algorithm for stationary signals can be expressed as $\hat{y}_n = Hx_n$; Hence, the observation equation provides a unified context to delay a one-step delay and lost packets without delays as follows:

$$\begin{aligned} y_n \;&=\; \alpha_{0,n}(Hx_n + v_n) + (1-\alpha_{0,n})(1-\alpha_{0,n-1})\,\alpha_{1,n} \\ &\quad (Hx_{n-1} + v_n) + (1-\alpha_{0,n})(1-(1-\alpha_{0,n-1})\,\alpha_{1,n}) \\ &\quad Hx_n\,, \end{aligned} \tag{9}$$

without loss of observation, the parameter matrices can be accessed in a compact form as:

$$\begin{aligned} \bar{H}_n \;&=\; \alpha_{0,n} H + (1-\alpha_{0,n})(1-\alpha_{0,n-1})\,\alpha_{1,n} H F^{-k_n} \\ &+ (1-\alpha_{0,n})(1-(1-\alpha_{0,n-1})\,\alpha_{1,n})\,, \end{aligned} \tag{10}$$

$$\begin{aligned} \bar{v}_n \;&=\; \alpha_{0,n} v_n + (1-\alpha_{0,n})(1-\alpha_{0,n-1})\,\alpha_{1,n} v_{n-1} \\ &- (1-\alpha_{0,n})(1-(1-\alpha_{0,n-1})\,\alpha_{1,n})\, H F^{-k_n} w_n\,, \end{aligned} \tag{11}$$

and the covariance $R = E\left\{\bar{v}_n \bar{v}_n^T\right\}$ of noise $\bar{v}_n$ is given by

$$\begin{aligned}\bar{R}_n \;=\;& \alpha_{0,n} R_n + (1-\alpha_{0,n})(1-\alpha_{0,n-1})\,\alpha_{1,n} R_{n-1} \\ & - (1-\alpha_{0,n})(1-(1-\alpha_{0,n-1})\,\alpha_{1,n}) \\ & HF^{-k_n} Q_n H^T F^{-k_n} T\,, \end{aligned} \tag{12}$$

By setting the new measurement noise as $\bar{v}_n$, we can noting at the both models, a correlation between the new measurement noise and the modeling noise as $E\{\bar{v}_n w_n^T\} = -(1-\alpha_n)H$ and $E\{\bar{v}_n w_n^T\} = -(1-\alpha_{0,n})(1-(1-\alpha_{0,n-1})\,\alpha_{1,n})\,HF^{-k_n}$

In the estimation method, the processor produces a linear estimation based on the consideration of withe and uncorrelated noise, for this reason, the derivation of the filter with correlated matrices will be deduced.

3.2 Correlated Matrices at the Filter

Referring to linear estimators as the Kalman filter which is dependent on the noise covariance a new gain may be defined in accord with the mean square error and the correlated noise.

The estimation error at the step-time n is defined as $e_n = x_n - \hat{x}_n$ where $\hat{x}_n$ is the estimation. Taking in to account that a recursive estimation has the unify structure $\hat{x}_n = F\hat{x}_{n-1} + K_n(z_n - \bar{H}_n F\hat{x}_{n-1})$, the estimation covariance may be expressed as $P_n = E\{e_n e_n^T\}$ as follow:

$$\begin{aligned} P_n \;=\;& E\{(x_n - F\hat{x}_{n-1} - K_n(z_n - \bar{H}_n F\hat{x}_{n-1})) \\ & (x_n - F\hat{x}_{n-1} - K_n(z_n - \bar{H}_n F\hat{x}_{n-1}))^T\} \\ =\;& \left(I_n - K_n\bar{H}_n\right) P_n^- \left(I_n - K_n\bar{H}_n\right)^T + K_n\bar{R}_n K_n^T \\ & - \left(I_n - K_n\bar{H}_n\right)\phi_n k_n^T - k_n\phi_n\left(I_n - K_n\bar{H}_n\right)^T \end{aligned} \tag{13}$$

where $\phi_n = E\{w_n \bar{v}_n^T\}$ and $P_n^- = FP_{n-1}F^T + Q$

The innovation gain filter should be obtained according to the robustness criteria as an optimal estimation. Then, the gain filter computing by minimizing the mean-squares estimation error is given by

$$\begin{aligned} \frac{\partial tr P_n}{\partial K_n} \;=\;& -2\left(P_n^- \bar{H}_n^T + \phi_n\right) + 2K_n(\bar{H}_n P_n^- \bar{H}_n^T \\ & + \bar{R}_n + \bar{H}_n\phi_n + \phi_n^T \bar{H}_n^T)\,. \end{aligned} \tag{14}$$

and

$$K_n = \left(P_n^- \bar{H}_n^T + \phi_n\right)(\Gamma_n)^{-1}\,. \tag{15}$$

Finally, The covariance matrix at (13) is modified to the cross-covariance as

$$P_n^- = P_n^- - K_n\left(\bar{H}_n P_n^- + \phi_n^T\right)\,. \tag{16}$$

where $\Gamma_n = \bar{H}_n P_n^- \bar{H}_n^T + \bar{R}_n + \bar{H}_n\phi_n + \phi_n^T \bar{H}_n^T$

In a conclusion, we can observe the significance of the noise covariance matrices at the filter derivation. In contrast, we will be shown next as the UFIR filter will be computed to avoid the use of the noise covariance matrices.

3.3 UFIR Filter Algorithm

As has been already indicated, our aim is to design an unbiased estimation based on a measurement vector with uncertain failures, considering the different model approaches.

Let x_n at the horizon $[m, n]$, the state vector, the observation vector and the matrices are calculated by an extended system equation on the horizon as $X_{m,n} = A_{m,n}X_m + B_{m,n}W_{m,n}$ and $Y_{m,n} = C_{m,n}x_m + D_{m,n}w_{m,n} + v_{m,n}$ where $X_{m,n} = \left[x_m^T\, x_{m+1}^T \ldots x_n^T\right]^T$ and $Y_{m,n} = \left[y_m^T\, y_{m+1}^T \ldots y_n^T\right]^T$

$$A_N = \left[I\; F^T \ldots F^{N-1^T}\right]^T, \tag{17}$$

$$B_N = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ F & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F^{N-2} & F^{N-3} & \cdots & I & 0 \\ F^{N-1} & F^{N-2} & \cdots & F & I \end{bmatrix}. \tag{18}$$

$$C_{m,n} = \begin{bmatrix} \bar{H}_m \\ \bar{H}_{m+1}F \\ \bar{H}_{m+1}F^2 \\ \vdots \\ \bar{H}_n F^{n-1} \end{bmatrix}, \tag{19}$$

$$D_{m,n} = \begin{bmatrix} \bar{H}_m & 0 & 0 & \ldots & 0 \\ \bar{H}_{m+1}F & \bar{H}_{m+1} & 0 & \ldots & 0 \\ \bar{H}_{m+2}F^2 & \bar{H}_{m+2}F & \bar{H}_{m+2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{H}_n F^{N-1} & \bar{H}_n F^{N-2} & \bar{H}_n F^{N-3} & \ldots & \bar{H}_n \end{bmatrix} \tag{20}$$

In many practical applications, knowing the progressive system parameters depend on many conditions and may become in a laborious assignment if initial qualities or noise statistics are undistinguished. In the iterative unbiased FIR filter in contrast to many other filters, the initial states are not an essential requirement for the estimation. The UFIR filter is designed in two stages; at first, a previous estimation is obtained by a batch algorithm based on a convolution theory. Then, this estimation is computed as the initial parameters of an iterative algorithm where the last estimation Is updated based on the measurements and known probabilities.

The UFIR filter has the feature to neglect the zero mean noise, in this sense the uncorrelated matrices are not required and the filter can be derived in a direct form. Let $\hat{X}_{m,n} = \left[\hat{x}_m^T\, \hat{x}_{m+1}^T \ldots \hat{x}_n^T\right]^T$ as the estimation vector,and the unbiased condition $E\{x_n\} = E\{\hat{x}_n\}$ a computational procedure of the UFIR filter considering the two possibles scenarios with a observation equations such as 3 and 5 can be summarised as follow [16]. The estimation is summarized in the following steps: Note that, only if the time step approach is implemented, in the case of the observation equation (3), the predictive step is applied.

- Predictive value: if $\beta_n = 0$ which indicate that some data is lost, the observation vector is replaced by the predictive value obtained by (4)

- Matrices definition: The algorithm convert the original data y_n to two signals z_n given by (3) or (5) respectively, and the observation matrix can be calculated according to the system model information, from (3) and (5) in (6) and (10).

  Then the filter operates as follows:
- Filter algorithm
  - The batch algorithm at the time-step n can be computed by

$$\hat{x}_s = (H_{m,s}^T H_{m,s})^{-1} H_{m,s}^T Y_{m,s}\,, \qquad (21)$$

where the matrix gain G_s is calculated as $G_s = (H_{m,s}^T H_{m,s})^{-1}$, it is known as the *generalized noise power gain* (GNPG). The matrix $H_{m,s}$ is obtained as

$$H_{m,s} = \begin{bmatrix} \bar{H}F^{-N+1-k_m} \\ \vdots \\ \bar{H}F^{-1-k_{s-1}} \\ \bar{H}F^{-k_s} \end{bmatrix}. \qquad (22)$$

  - Then a previous batch estimation, a iterative form of the UFIR filter is applied to reduce the computational effort and complexity,

$$\hat{x}_l = F\hat{x}_{l-1} + K_l^{\mathrm{UF}}(y_l - \bar{H}F\tilde{x}_{l-1})\,, \qquad (23)$$

where the gain matrix is $G_l = [\bar{H}^T\bar{H} + (FG_{l-1}F^T)^{-1}]^{-1}$ and $K_l^{\mathrm{UF}} = G_l\bar{H}^T$;
Note that $\hat{x}_{l-1} = \hat{x}_s$

- Optimal horizon: The tuning parameter N is an essential criterion to achieve a minimum estimated error. N_{opt} is calculated by an optimal condition, minimizing the square error according to N, $N_{\mathrm{opt}} = \arg\min_N[\mathrm{tr}P_n(N)]$ The error covariance matrix of the filter is defined by $P_n = E\{(x_n - \hat{x}_n)(x_n - \hat{x}_n)^T\}$. For the batch algorithm, a previous error covariance is given by

$$P_n^- = (B_n^{(N)} - \mathcal{H}_{m,n}Y_{m,n})\bar{Q}_n(B_n^{(N)} - \mathcal{H}_{m,n}Y_{m,n})^T + \mathcal{H}_{m,n}R_n\mathcal{H}_{m,n}^T\,, \qquad (24)$$

and for the iterative algorithm, the covariance matrix is given by

$$P_n = (I - \mathcal{G}_n\bar{H}_n^T\bar{H}_n)(FP_{n-1}F^T - B_nQ_nB_n^T)(I - \mathcal{G}_n\bar{H}_n^T\bar{H}_n)^T + \mathcal{G}_n\bar{H}_n^T\bar{R}_n\mathcal{G}_n^T\bar{H}_n\,, \qquad (25)$$

4. Experimental Example

In this section, a simulation example is illustrated to show the implication of the current signal model in the estimation process by two different kinds of models and inspect the advantages of the unbiased FIR filter between other algorithms as the Kalman filter when it is influenced by uncertain measurements. The observation signal consists on GPS coordinates of a Beijing's vehicle transmitted via wireless communication to a central station [21]. Consider the vehicle dynamics such as distance and velocity in the north and east directions as the state vector $x_n = \begin{bmatrix} x_{1n} & x_{2n} & x_{3n} & x_{4n} \end{bmatrix}^T$, where $x_{1n} = x_n$, $x_{2n} = \dot{x}_n$, $x_{3n} = y_n$ and $x_{4n} = \dot{y}_n$.

The vehicle trajectory in the north-east direction in coordinates x and y is shown in Fig.1.

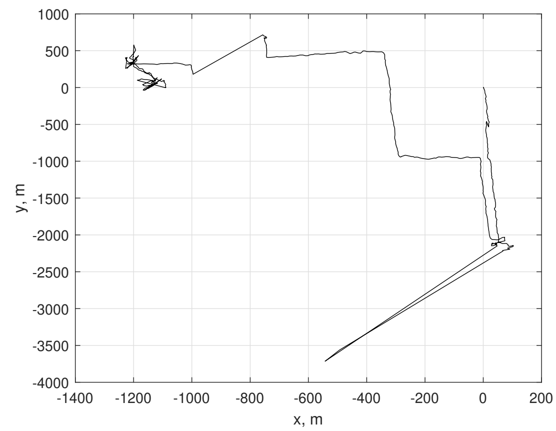

Fig. 1. GPS-measured vehicle trajectory in the north y and east x coordinates.

We assume the following discrete systems

$$x_n = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{n-1} + w_n\,,$$

$$z_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_n + v_n$$

where the measurement output arrived at the processor with known probabilities, delays, and losses. The data can be delayed by no more than one step and only the first data packet can be processed. Multiple lost packet data may appear as a consequence of the delays or sensor issues.

To unify these examples, the additive noises are defined as a zero-mean white process; due to the severe assignment to have the appropriate noise statistics development, we do it based on general knowledge. A vehicle in the residential district moves with an average speed of $11\,\mathrm{m/s}$. Based upon, we have found that the optimal filter mode will be obtained with the standard deviation in the acceleration noise of $\sigma_{3w} = 0.2\,\mathrm{m/s}$ by neglecting noise in the first and second states, $\sigma_{1w} = \sigma_{2w} = 0\,\mathrm{m/s}$. The GPS navigation service produces an error of about 15 meters with a probability of 95%. Accordingly, we assign $\sigma_v = 3.75\,\mathrm{m}$ and form the noise covariance matrices as

$$Q = \sigma_{w2}^2 \begin{bmatrix} \frac{\tau^2}{4} & \frac{\tau}{2} & 0 & 0 \\ \frac{\tau^2}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{\tau^2}{4} & \frac{\tau}{2} \\ 0 & 0 & \frac{\tau^2}{2} & 1 \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

4.1 State Estimation

According to the observation models used at the theoretical definitions and assuming that the values of the packet arrival probabilities are $\bar{\alpha}_0 = 0.7$ and $\bar{\alpha}_1 = 0.9$, and the lost event variable is the independent random variable β, the accuracy of the proposed estimation algorithm by the model (3) and (5) is showed in Figure 2 and 3 respectively. Figure 2 displays the trajectory signal in the north direction, the UFIR estimation, and the Kalman estimation by the first scenario with the output equation (3). A satisfactory estimation performance has been obtained. The UFIR filter archive a large overshoot but a short transient compared with the KF development. Note that when a loss appears the predictive compensation is activated and a successful estimation is obtained. In Figure 3 the trajectory signal in the north direction, the UFIR estimation, and the Kalman estimation by the second scenario with the output equation (5) are shown. opposite to the first scenario development, the filters lost the capability to track the trajectory when lost data appear, largest overshoot and transitory are obtained.

The error signal development of the proposed filter and Kalman filter are compared for each observation model. In Figure 5 the estimated error using the first observation model with the time-stamp approach (3) is depicted. Big error values can be observed in contrast to the estimation error obtained in Figure 4. The losses increase the variations and uncertain estimations.

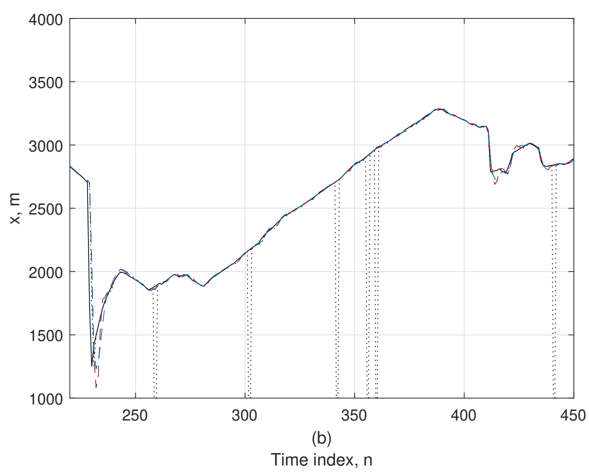

Fig. 2. GPS-based vehicle tracking estimation in the y, m direction by the UFIR filter and KF, using the observation model (3).

In order to show the influence of the missing measurements and uncertain noise parameters at the veracity estimations, an error factor is introduced in the algorithms. The actual matrices Q and R are substituted in the algorithms with $\alpha^2 Q$ and $\beta^2 R$, where $\alpha = \frac{1}{\beta}$ and β indicates an error in the noise standard deviation. In Figure 6 and Figure 7 the effect of errors in the noise covariance is shown, α and β is variate from 0.1 to 10. From these figures, we can observe that better performance of the filters is obtained when the covariance matrices have a minimum error. The Kalman filter is influenced by the error variables but the UFIR filter is not indeed influenced by unreliable noise.

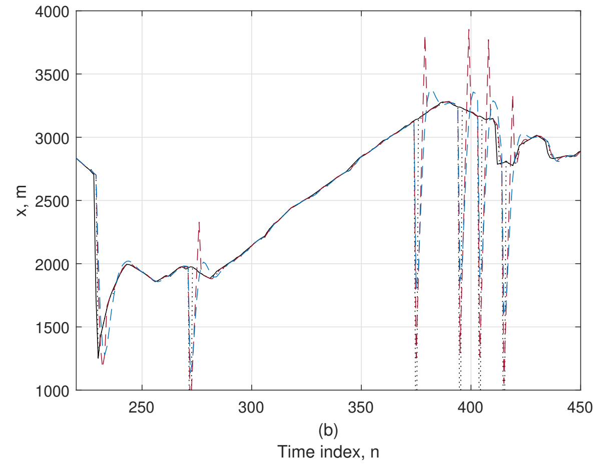

Fig. 3. GPS-based vehicle tracking estimation in the y, m direction by the UFIR filter and KF, using the observation model (5).

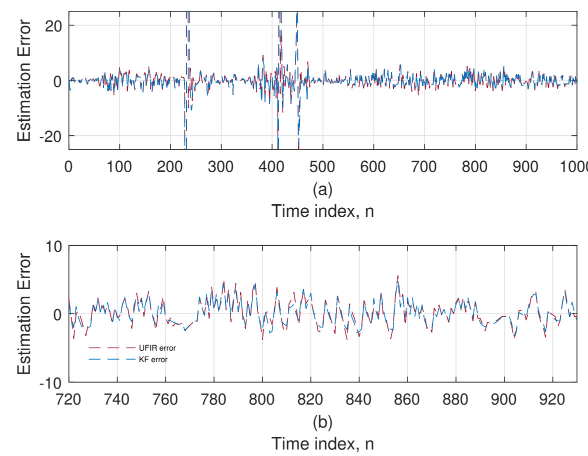

Fig. 4. Estimation error produced by by the UFIR filter and Kalman filter using the observation model (3) in the y direction; (a) full scale and (b) $720 \leq n \leq 920$.

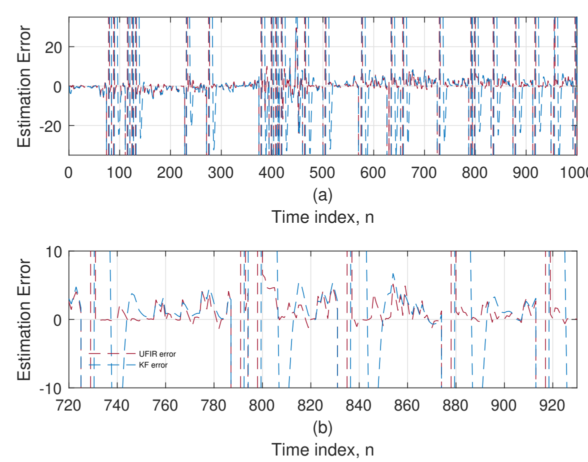

Fig. 5. Estimation error produced by by the UFIR filter and Kalman filter using the observation model (5) in the y direction; (a) full scale and (b) $720 \leq n \leq 920$.

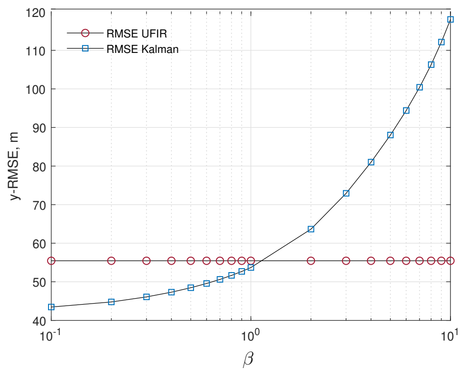

Fig. 6. Effect of the data transmission probability α and covariance matrices on the RMSEs produced by the UFIR filter and Kalman filter using the obser...

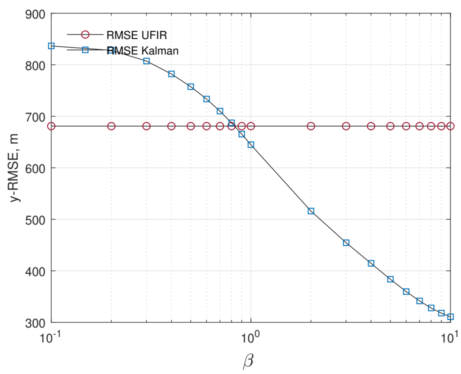

Fig. 7. Effect of the data transmission probability α and covariance matrices on the RMSEs produced by the UFIR filter and Kalman filter using the observation model (5).

5. Conclusions

In this paper, a comparison of the UFIR and Kalman filter development using the most conventional models to describe the uncertain measurement with random delays and losses was presented. The time-stamp approach and the predictive compensation were used. Considering the resulting estimation performance, a considerable influence of the lost data model was observed. An increase in the estimation error was obtained when a Bernoulli distribution model was proposed to describe the multiple lacks of information, in contrast, a better estimation performance was achieved when a predictive algorithm determined the value at the vector which is not valid. To achieve these results, the system state-space model has been reformulated in a way such that the delay factor was removed from the state to matrices. An experimental vehicle tracking was presented to compare the effectiveness of the UFIR filter with the KF.

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