On the Structure of a Quasi-Optimal Algorithm for Circuit Designing

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Abstract: - The formulation of the process of analog system design has been done on the basis of the control theory application as the problem of a special functional minimization. This approach generalizes the design process and produces the different design trajectories inside the same optimization procedure. Numerical examples show that the potential computer time gain of the optimal design strategy with respect to the traditional design strategy increases when the size and the complexity of the system increase. Some properties of an additional acceleration effect of the design process were analyzed. The special selection of the optimization process start point provides the acceleration effect with a great probability. The positions of the optimal switch points of the control vector were found on the basis of the analysis of the special Lyapunov function of the design process. The combination of the acceleration effect with the start point preliminary selection and the optimal switch points of the control vector serve as the principal ideas to the time-optimal design algorithm construction.

Key-Words: - General design algorithm, control theory application, acceleration effect, start point selection, control vector, Lyapunov function.

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1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. This problem has a special significance for the VLSI electronic circuit design. The reduction of the necessary time for the circuit analysis and improvement of the optimization algorithms are two main sources for the reduction of the total design time. There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[3]. Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The well-known ideas for partitioning of a circuit matrix into bordered-block diagonal form were described in original works using the branches tearing [4] or nodes tearing [5] and jointly with direct solution algorithms give the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macro model representation [6]. The optimization technique is developed both for the unconstrained and for the constrained optimization and can be improved in future. However there is another way to reduce the total computer time for the analog system design. The reformulation of the optimization process on heuristic level was proposed decades ago [7]-[8]. This process was named as generalized optimization and it consists of the Kirchhoff law ignoring for some parts of the optimization process. The special cost function is minimized instead of the circuit equation solution. In practical aspects this idea was developed for the microwave circuit optimization [9] and for the synthesis of high-performance analog circuits [10]-[11]. Nevertheless, we need to generalize all these ideas.

Another way to reformulate the design problem can be proposed. This approach consists of generalization of the total design problem to obtain a set of different design strategies inside the same optimization procedure [12]-[13]. In this case the time-optimal design strategy can be formulated and this strategy can be proposed as a theoretical basis for the time-optimal design algorithm searching. Some preliminary promising results of quasi optimal strategies were obtained in [13]. On the other hand an additional acceleration effect of the design process has been discovered by the analysis of various design strategies with the different initial points [14]. This effect can be proposed as one of the principal elements of the time-optimal algorithm construction. The main step of this construction is the definition of the control vector optimal switch points during the design process. This problem is discussed in this paper by examining the properties of the special Lyapunov function of the design process.

2 Problem Formulation

The design process for any analog system design has been generalized on the basis of the control theory approach as shown in [12]. In this case the design process is defined by means of the optimization procedure (1) and by the analysis of the system (2):

$$X^{s+1} = X^{s} + t_s \cdot H^{s} \qquad (1)$$

$$\left(1-u_j\right) g_j(X) = 0, \quad j = 1,2,\ldots, M \qquad (2)$$

where M is the number of dependent parameters and H is the vector of directional movement. In this case the vector H depends not only on the optimization procedure and objective function structure but from the vector of special control functions $U = \left(u_1, u_2, \ldots, u_m\right)$ that control the design process, where $u_j \in \Omega$; $\Omega = \{0;1\}$. In this case a new generalized objective function is needed to define as $F(X,U) = C(X) + \psi(X)$ with a special additional penalty function $\psi(X,U) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2(X)$. All control variables u_j are the functions of the current point of the design process. The total number of the different design trajectories produced inside the same optimization procedure is practically infinite. The problem of the optimal design strategy searching is formulated now as the typical problem for the functional minimization of the optimal control theory.

The numerical results for the different electronic circuits shown that the optimal control vector U_{opt} can be found and can reduce the total computer time significantly [13]. The optimal trajectory differs from the traditional design strategy ($u_j = 0,\ \forall_{j=1,2,..M}$) and differs from the modified traditional design strategy ($u_j = 1,\ \forall_{j=1,2,..M}$) as shown in [15]. Comparing this result with the results of [9]-[11], we can conclude that the idea implemented in [9]-[11] is not optimal from the computing time point of view.

3 Design Trajectory Subsets

The idea of the system design problem definition as the problem of the control theory does not have dependency from the optimization method (the function H form) and can be embedded into any optimization procedure. The numerical results for the different electronic circuits show [12] that the optimal control vector U_{opt} and the optimal trajectory X_{opt} exist and allow reducing the total computer time significantly. The main problem is to construct the optimal algorithm, which permits to realize all advantage of the optimal strategy. The analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal design strategy relatively the traditional strategy increases when the size and complexity of the system increase.

On the basis of the described methodology, by means of the start point of the vector X variation, an additional acceleration effect of the design process was discovered [13]. This effect appears for all analyzed circuits when at least one coordinate of the start point is negative and gives the possibility to reduce the total computer time additionally. This effect can serve as a basis for constructing an optimal algorithm if a sequence of control function switching points is established. So, the main problem of constructing an optimal algorithm is the task of finding the optimal switching point for control functions in the design process.

The analysis of some examples gives the possibility to conclude that all the trajectories that appear for the different control vector U can be separated in two subsets. In Fig. 1 there is a three-node circuit that has four admittances y_1, y_2, y_3, y_4 (K=4) and three nodal voltages V_1, V_2, V_3 (M=3). The nonlinear elements of the circuit have been defined by the following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot \left(V_1 - V_2\right)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot \left(V_2 - V_3\right)^2$.

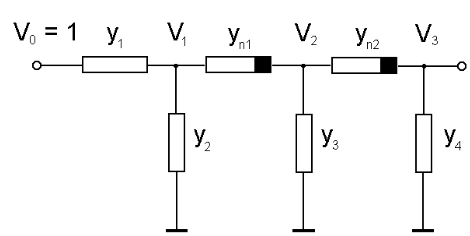

Fig. 1 Circuit with four independent (K=4) and three dependent (M=3) variables

The mathematical model (2) of this circuit includes now three equations and the optimization procedure (1) includes four equations. The one plane trajectory projections of the different design strategies, which correspond to the different control vector *U* are shown in Fig. 2. These projections correspond to the plane $y_4 - V_3$ and the points *S* and *F* correspond to the start and the final points of the design process. The complete basis of the different design strategies includes eight strategies.

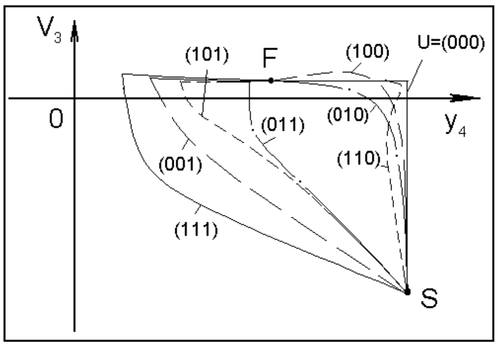

Fig. 2 $y_4 - V_3$ plane trajectory projections for different control vector *U*

We can define the two subsets of the trajectories: 1) the trajectory projection, which corresponds to the traditional strategy *U*=(000) and the like traditional strategy projections (010), (100), (110) and 2) the trajectory projection, which corresponds to the modified traditional strategy (111) and the like modified traditional strategy projections (001), (011), (101). The main differences between two these groups are the different curve behavior and the different approach to the final point. The curves from two these groups draw to the finish point from the opposite directions. The time-optimal algorithm has includes one or some switching points where the switching is realized from the like modified traditional strategy to like traditional strategy with an additional adjusting. At least one negative component of the start value of the vector *X* is needed to realize the acceleration effect. In this case the optimal trajectory can be constructed.

The similar behavior of the complete basis of various design strategies with a constant control vector is observed for all the studied circuits. We can separate all the trajectories to two subsets. The first subset includes the traditional and like traditional design trajectories and the other one includes the modified and like modified traditional trajectories. It can be concluded that the trajectories of the second group can serve as the first part of the trajectory of the optimal algorithm, and the trajectories of the first group serve as a continuation. The next principal problem of the time-optimal algorithm construction is the unknown optimal position of the control function switching points that provide the minimal computer time.

The discovery of the additional acceleration effect of the design process [13] allows defining the essential features of the optimal algorithm. The optimal algorithm consists of one or several trajectory jumps from quasi modified traditional strategy to quasi traditional strategy. Acceleration effect leads to large computer time gain. For instance the time gain for two transistor cells amplifier is near 100 times and for three transistor cell amplifier more than 400 times as shown in papers [14]-[15]. On the other hand the construction of the optimal algorithm on basis of the acceleration effect turns on unknown switch point positions for the control functions u_j.

4 Initial Point Optimal Selection

4.1 Two-dimensional problem

The problem of the initial point selection for the design process is one of the main problems of the time-optimal algorithm construction. Analysis of the acceleration effect for the simplest electronic circuit in Fig. 3 is shown below.

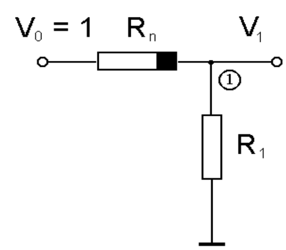

Fig. 3 Topology of a simplest electronic circuit

The nonlinear element has the next dependency: $R_n = r_0 + bV_1^2$. The vector *X* of the state variables has two components $X=(x_1,x_2)$, where x_1 is the independent parameter ($x_1 \equiv r_2$) and $x_2 \equiv V_1$. The objective function is determined by the formula $C(X) = (x_2 - k_V)^2$, where k_V has a fixed value. The optimization procedure in accordance with the new design methodology can be defined by the equations:

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U), \quad i = 1,2 \qquad (3)$$

where the right hand side $f_i(X,U)$ for the gradient method can be defined as:

$$f_1(X,U) = -\frac{\delta}{\delta x_1} F(X,U),$$
$$f_2(X,U) = -u_1 \frac{\delta}{\delta x_2} F(X,U) + \frac{(1-u_1)}{t_s}\left[-x_2^s + \eta_2(X)\right] \qquad (4)$$

The vector of the control variables U consists on one coordinate u_1 only. The equation (2) is transformed now to the next form:

$$(1-u_1)g_1(X) = 0 \qquad (5)$$

$F(X,U)$ is the generalized objective function, $\eta_2(X)$ is the implicit function $\left(x_2^{s+1} = \eta_2(X)\right)$ and it gives the value of the parameter x_2 from the equation (5). The vector of the control variables U consists on one coordinate u_1 only for this example.

The idea of the system design problem definition as the problem of functional minimization of the control theory does not have dependency from the optimization method and can be embedded into any optimization procedure as shown in [12].

We can select the initial point of the design process with the negative coordinate x_2 as shown in [13]. In this case the acceleration process is realized. We analyze the characteristics of the acceleration effect to decide what value of the coordinate x_2 is better. The family of the design curves for the circuit in Fig. 3, which corresponds to the modified traditional design strategy (u_1 = 1) and the negative initial value of the second coordinate (x_2<0) of the vector X is shown in Fig. 4 for the 2-D phase space. These curves have different start points but the same final point F. The start points were selected on the circle arc and have the different initial coordinates. The special curve *S-F*, which is marked by thick line, is the separating curve. This curve separates the trajectories that are the candidates for the acceleration effect achievement (all curves that lie under the curve S-F), and the trajectories that can not produce the acceleration effect (curves that lie over the curve *S-F*). It is clear that the projections of the final point F to all curves of the first group define the switch point of the optimal trajectory, which produces the acceleration effect.

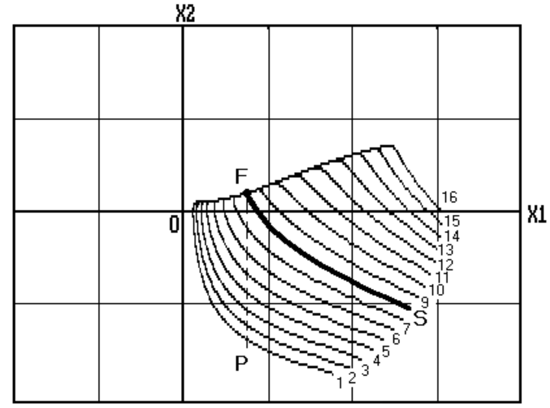

Fig. 4 Trajectories of the modified traditional strategy for the different start points with the negative coordinate x_2

All curves of the first group (1-7) approach to the final point F from the left side, and all curves of the second group (9-16) approach to the final point from the right side. The comparison of the relative computer time for all curves of the Fig. 4 is shown in Fig. 5 as the function of the curve number n. The separating curve *S-F* has the minimal computer time among all of the trajectories.

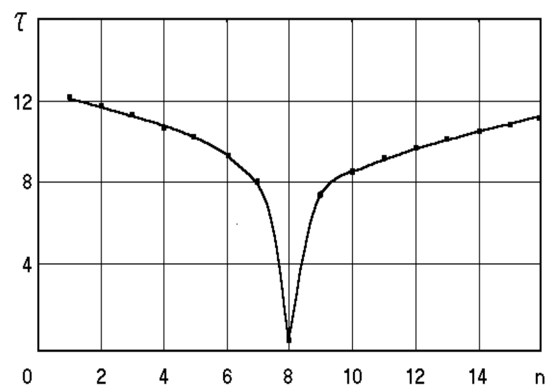

Fig. 5 Relative computer time τ as the function of the curve number n

At the same time this curve cannot be used as the basis for the time-optimal trajectory construction because the projection of the point F to this curve is the same point F, but the movement slows down considerably near this point. Only the curves that lie under the curve *S-F* can be serve as the first part of the time-optimal trajectory with the following jump to the point F. The relative computer time τ of the optimal trajectories with acceleration effect (on the basis of the curves 1-7, Fig. 4) is shown in Fig. 6 as the function of the curve number n. The curves 9-16 can be optimized too but the time reduction about 10-15% only takes place.

Fig. 6 shows that the total computer time increases when the start point approaches to the curve *S-F*, and on the contrary, the acceleration can be obtained if the start point lies far from the curve *S-F* (from curve 7 to curve 1). So, the start point selection with at least one negative initial coordinate of the vector *X* and the value of this coordinate that gives the start point position under the separating line are the sufficient conditions for the acceleration effect appearance.

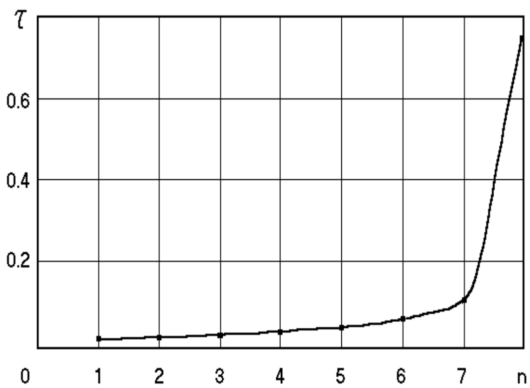

Fig. 6 Relative computer time τ of the optimal trajectories with acceleration effect as the function of the curve number *n*

4.2 *N*-dimensional problem

All above mentioned conclusions are correct for the *N*-dimensional problem too. We need to analyze the different projections of the *N*-dimensional curve in this case. The solution of the N-dimensional problem is complicated by a large number of different admissible trajectories and a large number of different projections of trajectories [14]-[15]. In this case we need to choose the most perspective trajectories and analyze them [16]. It can be done by some approximate methods of the control theory [17]. In present paper it was done by the careful analysis of all possibilities. The total set of the various trajectories can be divided in two different subsets. The first subset consists of the trajectories that are similar to the traditional design strategy trajectory. The second subset consists of the trajectories that are similar to the modified traditional strategy trajectories. In this case the trajectories of the second group serve as the candidates for the first part of the optimal trajectory and the first group trajectories serve as the candidates to the jump produce. Two of these main steps together with the following different trajectory adjustment make up the essence of the optimal algorithm construction. We can decide from the experience that not all of the feasible projections are important to the acceleration effect obtained. First of all the admittance-voltage two-dimensional projections are more important. Variables that are included in the objective function formula have a greater importance among all of these projections. By this preliminary selection we can reduce the number of the more perspective candidates for the time-optimal algorithm elaboration. This problem final solution will be based on the optimal algorithm intrinsic structure. However, the results obtained here serve as the next step on the way of this problem solution. Now it is clear that the optimal algorithm must include the special conditions for the start point selection to the acceleration effect reach. On the other hand, the problem of determining the optimal position of the switching point of the control vector is discussed in the next section.

5 Switch Point Definition

On the basis of the analysis in previous section we can conclude that the time-optimal algorithm has one or some switch points where the switching realize from like modified traditional strategy to like traditional strategy with an additional adjusting. At least one negative component of the start value of the vector *X* is needed for the optimal trajectory obtained. The main problem of the time-optimal algorithm construction is the unknown sequence of the switch points during the design process. We need to define a special criterion that permits realizing the optimal or quasi-optimal algorithm by means of the optimal switch points searching. In this paper we propose to use a Lyapunov function of the design process for the optimal algorithm structure revelation, in particular for the optimal switch points searching. There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let define the Lyapunov function as:

$$V(Y,U)=\sum_i\left(\frac{\partial F(Y,U)}{\partial x_i}\right)^2 \qquad (6)$$

where *F(Y,U)* is the generalized objective function of the optimization procedure. This form holds all of the necessary characteristics of the standard Lyapunov function definition. It is supposed that the vector *Y* is defined as the difference between two vectors *X* and *A*, where *A* is the stationary point of the design process (the final point). In this context this function is used for the analysis of the design trajectories behavior with the different switching points. We can define now the system design process as a transition process that provides the stationary point during some time. The problem of the time-optimal design algorithm construction is the problem of the transition process searching with the minimal transition time. There is a well-known idea [18]-[19] to minimize the transition process time by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions $f_i(X,U)$. By this conception it is necessary to change the functions $f_i(X,U)$ by means of the control vector U selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum of $-\dot{V} = -dV/dt$) at each point of the process. Unfortunately the direct using of this idea does not serve well for the time-optimal design algorithm construction. It occurs because the change of the design strategy produces not only continuous design trajectories but non-continuous trajectories too. In this case we need to correct the idea to maximize *-dV/dt* at each point of the design process. We define other principle: it is necessary to obtain the maximum speed of the Lyapunov function decreasing for that trajectory part which lies after the switch point. In this case the trajectories with the different switch points are compared to obtain the maximum value of *-dV/dt*. This idea is realized by some probes comparing with the different switch points. The best probe can be selected that provides the maximum value of *-dV/dt* after the switching. Numerical results support this idea. The two-cell transistor amplifier circuit is shown in Fig. 7.

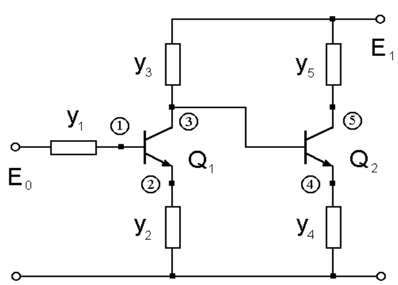

Fig. 7 Two-cell transistor amplifier

As shown in [15] the optimal design strategy for this amplifier has the time gain near 100 comparing with the traditional design strategy. Analysis of the Lyapunov function time derivative gave the next results. The behavior of the function *dV/dt* for this circuit for three neighbor switch points 1, 2 and 3 that correspond to the five consecutive integration steps before (a), (b), in (c) and after (d), (e) the optimal point is shown in Fig. 8.

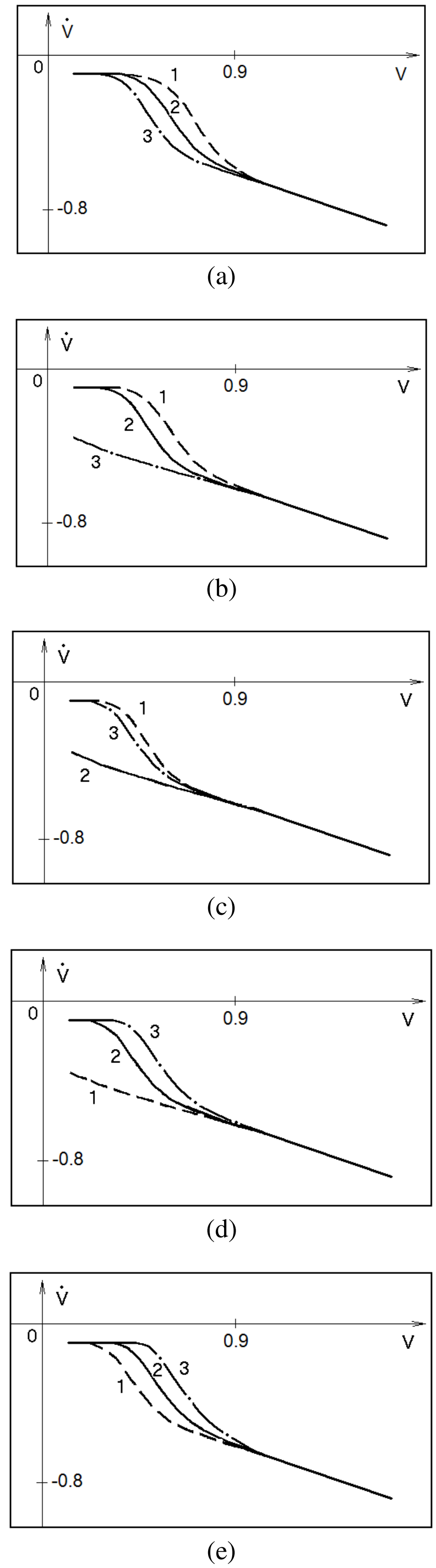

Fig. 8 Time derivative of Lyapunov function for three switch points 1,2,3 of integration steps before (a), (b), in (c) and after (d), (e) the optimal point

The optimal switch point corresponds to the curve 3 of Fig. 8 (b), or curve 2 of Fig. 8 (c), or curve 1 of Fig. 8 (d). It is clear that this point corresponds to the maximum negative value of function *dV/dt* and at the same time corresponds to the minimum value of the total design steps. In this case we suppose that the optimal position of the switch point is found. This switch point or some switch points serve as the basis to the time-optimal design algorithm construction. It is clear that we are forced to lose the computer time to do some probes and to look for the optimal position of the switch points. It means that we can never obtain the time gain, which characterizes the veritable optimal strategy. It was determined from the analyzed examples that the time loses can have the same order as the optimal algorithm own computer time. So, the maximum real time gain is equal to 50 for the circuit in Fig. 7. This result twice worse than the theoretic prediction, but this gain is also significant.

6 Conclusion

The traditional approach to designing analog systems is not time-optimal. The problem of constructing a speed-optimal algorithm can be solved as a problem of optimizing a functional in control theory. Three main ideas can be attributed to the construction of an optimal algorithm: the effect of accelerating the design process, the optimal choice of the starting point of the design algorithm, and determining the optimal switching point of the control vector to implement the best trajectory. The choice of the starting point allows, with a high probability, to obtain an additional effect of accelerating the design process. This effect additionally reduces the total computation time and serves as the basis for constructing an optimal or quasi-optimal algorithm. The optimal position of the required switching points can be obtained by analyzing the Lyapunov function of the design process. Minimization of the time derivative of this function is the main criterion for determining the optimal switching point. Thus, the combination of above three ideas serves to construct the basic elements of a quasi-optimal algorithm.

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The author has no conflict of interest to declare that is relevant to the content of this article.

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