Underwater Bearing-only Tracking based on SR-UKFS

BAO-BAO WANG
Jiangsu Automation Research Institute
No.18, Sheng Hu Road, Lianyungang 222006
PEOPLE'S REPUBLIC OF CHINA
wangbaobao_zdh@126.com

PAN-LONG WU
Nanjing University of Science and Technology
No.200,Xiao Ling Wei Street, Nanjing 210094
PEOPLE'S REPUBLIC OF CHINA
wupanlong_zdh@126.com

Abstract: - In order to improve the tracking precision of underwater bearing-only target, a novel filtering-smoothing algorithm based on Square-Root Unscented Kalman Filter (SR-UKFS) is proposed to track underwater target. In the SR-UKFS algorithm, Square-Root Unscented Kalman Filter (SR-UKF) is used as forward-filtering algorithm to provide current location results, Rauch-Tung-Striebel (RTS) algorithm smoothes the previous state vector and covariance matrix using the current location results, therefore an initial value with higher precision is obtained to get more precise locating results. The simulation results show that the SR-UKFS is an effective underwater bearing-only target tracking method, and it performs better than general Unscented Kalman Filter (UKF) and SR-UKF in tracking precision and stability.

Key-Words: Target tracking, Bearing-only, SR-UKFS, Forward-filtering, Backward-smoothing

1 Introduction

Using single observer to track targets avoids complex time synchronization. It has a big practical value with strong independence and good motility. When the observer and target have relative movements and satisfy observability, the observer can track targets [1]. Underwater target motion analysis is a technique for estimation of target motion parameters, which is based on a series of measured data sequence (including azimuth angle and pitch angle) from hydrophone array of sonar platform [2]. The core of this technique is filtering algorithm, which is utilized to locate and track. Because of severe nonlinearity of system, underwater target tracking must face the linearization problems of state equation and measurement equation. Due to the nonlinearity of state equation or measurement equation, related literature adopt many modified Kalman filtering algorithms, like Extend Kalman Filter (EKF), UKF, SR-UKF and so on [3].

EKF property depends on partial nonlinear strength. During the linearization process of EKF, Jacobian matrix is needed, which sometimes has difficulty in realization [4,5]. UKF approximates to-be-estimated parameters by constructing a group of certain weighted sample points, which avoids the linearization modeling of nonlinear objects and calculation of Jacobian matrix as a filtering algorithm which can be directly utilized in nonlinearity system for mode estimation [6]. But in practical utilization, because the data round-off error may make covariance matrix negative definite, which fails the UKF algorithm in calculating the matrix square root. Compared with EKF, on the condition that UKF does not add calculated amount, estimated accuracy and rate of convergence are apparently enhanced. SR-UKF algorithm estimates square-rooting matrix of error to do the recursive calculation, which solves the problem of negative definite covariance matrix and filtering divergence due to calculation error and noise signal in standard UKF algorithm, and enhances the accuracy and stability of filtering [7].

RTS smoothing algorithm is a fixed interval optimal smoothing technique, it greatly simplifies the process of calculation, and is often used to smooth the data from the filtering operation [8]. Smoothing can build a coherent connection at series of estimations, which makes the estimations more resistant to disturbance [9].

Based on SR-UKF, SR-UKFS adds backward smoothing for better accuracy of target state estimation in last moment, and enhances target state estimation accuracy of this moment [10]. According to the relative location of target and observation platform, the paper is based on the information of noise measurement of azimuth angle and pitch angle from passive sonar, tracks the course of underwater target with the utilization of SR-UKFS, and makes the simulation comparisons of standard UKF algorithm and SR-UKF algorithm.

2 Tracking principle and system model

The basic function of passive sonar is to find direction, and a measurement of azimuth information (including azimuth angle and angular altitude) formed by wave beam. Azimuth angle includes the location information of target horizontal direction. Angular altitude includes the information of target depth. Among the research about orientation and tracking problem of underwater target, the shape and size of observer and target can be ignored as particle of space, and the acceleration of target can be seen as a result of noise excitation and a Gaussian random process [11]. The relations of relative movement between target and observing platform can be seen in Fig. 1.

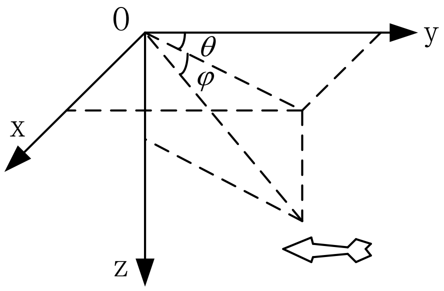

Fig.1. Location relationship between target and observation platform

In passive sonar tracking system, the information of sonar measurement can be obtained from a spherical coordinate [12]. And target dynamic model is usually constructed in rectangular coordinate system. Like this, the target tracking problem of sonar becomes a nonlinear estimation problem. The main methods to solve this problem are EKF, UKF and SR-UKF. This paper adopts SR-UKFS algorithm to track the target, that is to say, firstly uses SR-UKF algorithm to estimate target state, and then uses RTS smoothing algorithm to obtain the target state estimation of last moment, and finally adopts SR-UKF algorithm to estimate the target state of this moment.

In rectangular coordinate, a linear dynamic model and a nonlinear observing model can be used to establish target motion model. The state variable is $\boldsymbol{X}(k)=[x(k),y(k),z(k),v_x(k),v_y(k),v_z(k)]^T$, and x , y , z mean the relative location of X , Y , Z directions; v_x , v_y , v_z mean the relative speed of X , Y , Z directions. The discrete state equation of system is

$$\boldsymbol{X}(k+1)=\boldsymbol{\Phi}(k+1,k)\boldsymbol{X}(k)+\boldsymbol{\Gamma}(k)\boldsymbol{U}(k) \tag{1}$$

where

$$\boldsymbol{\Phi}(k+1,k)=\begin{bmatrix}1&0&0&T&0&0\\0&1&0&0&T&0\\0&0&1&0&0&T\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

$$\boldsymbol{\Gamma}(k)=\begin{bmatrix}T^2/2&0&0\\0&T^2/2&0\\0&0&T^2/2\\T&0&0\\0&T&0\\0&0&T\end{bmatrix}$$

$\boldsymbol{\Phi}(k+1,k)$ means state transition matrix; $\boldsymbol{\Gamma}(k)$ is the matrix of system noise ; $\boldsymbol{U}(k)=\left[u_x(k),u_y(k),u_z(k)\right]^T$ is the systematic process noise caused by target acceleration, process noise is zero-mean Gaussian white noise, its variance matrix is $\boldsymbol{Q}=diag([\sigma_{u_x},\sigma_{u_y},\sigma_{u_z}])$. T is the sampling period of system.

The observed quantity of system includes azimuth angle $\theta(k)$ and pitch angle $\varphi(k)$, which can be seen in Fig. 1. In rectangular coordinate system, the formula of azimuth angle and pitch angle can be seen in Formula (2):

$$\begin{bmatrix}\theta(k)\\\varphi(k)\end{bmatrix}=\begin{bmatrix}\tan^{-1}(x(k)/y(k))\\\tan^{-1}(z(k)/\sqrt{x(k)^2+y(k)^2})\end{bmatrix} \tag{2}$$

When the target depth $z(k)$ is a fixed value, the observed quantity of azimuth angle and pitch angle can be transferred to the position quantity of target in direction X , Y , and the transition measurement equation of system can be seen in Formula (3):

$$\boldsymbol{Z}(k)=H(\boldsymbol{X}(k))=\begin{bmatrix}x(k)\\y(k)\end{bmatrix}$$

$$=\begin{bmatrix}z(k)/\left(\tan(\varphi(k))\sqrt{1+\tan^2(\theta(k))}\right)\\z(k)\tan(\theta(k))/\left(\tan(\varphi(k))\sqrt{1+\tan^2(\theta(k))}\right)\end{bmatrix}+\boldsymbol{V}(k) \tag{3}$$

The measuring error of azimuth angle and pitch angle is relatively independent zero-mean Gaussian white noise , $\boldsymbol{V}(k)=\left[v_\theta(k),v_\varphi(k)\right]^T$ is the measurement noise caused by azimuth angle and pitch angle , its variance matrix is $\boldsymbol{R}$, $\boldsymbol{R}=diag([\sigma_\theta,\sigma_\varphi])$.

3 SR-UKFS algorithm

UKF is adopted by Julier[13], and widely used in the field of nonlinear estimation. But in practice, due to the round-off error in numerical calculation, sometimes we may get negative definite covariance matrix, which leads to the stoppage of UKF filter[14]. In order to avoid failure, SR-UKF uses covariance square root to replace the covariance to take part in recursive calculation, which ensures the half positive definitiveness of covariance in basic state has a better numerical characteristics [15].

In the process of UKF filtering, the calculation of new Sigma point in every update requires a considerable number of calculation. Everytime we must calculate the square root of state covariance matrix $\boldsymbol{P}$, and assumes $\boldsymbol{S}\boldsymbol{S}^T = \boldsymbol{P}$. In the process of SR-UKF filtering, $\boldsymbol{S}$ will be recorded to avoid heavy decomposition calculation in every resampling, which enhances the operation speed of UKF. QR disintegration and Cholesky disintegration update are two important concepts in SR-UKF.

QR disintegration: for matrix $\boldsymbol{A} \in \boldsymbol{R}^{L \times N}$ $(N \geq L)$, finding an orthogonal matrix $\boldsymbol{Q} \in \boldsymbol{R}^{N \times N}$ and an upper triangular matrix $\boldsymbol{R} \in \boldsymbol{R}^{N \times L}$ to make $\boldsymbol{A}^T = \boldsymbol{Q}\boldsymbol{R}$, which is doing a QR disintegration for matrix A. $qr\{\cdot\}$ can be used as QR disintegration with R as returned value. According to the analysis knowledge of matrix, $\boldsymbol{R}^T = \boldsymbol{S} = chol(\boldsymbol{P})$, that is Ris also the transposition of Cholesky coefficient S in matrix $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^T$

Cholesky disintegration update: if S is the Cholesky disintegration of matrix $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^T$, and $\boldsymbol{S} = chol(\boldsymbol{P})$, so the Cholesky disintegration update of matrix $\boldsymbol{P} \pm \sqrt{\boldsymbol{v}}\boldsymbol{u}\boldsymbol{u}^T$ can be marked as $\boldsymbol{S} = cholupdate\{\boldsymbol{S}, \boldsymbol{u}, \pm\boldsymbol{v}\}$. $\boldsymbol{u}$ usually is a column of vectors, but if $\boldsymbol{u}$ is a matrix which includes factors in M column, and uses vectors in M column to M times first order Cholesky update successively. In filtering process, S displaces P to participate in recursive calculation which can ensure the nonnegative definitiveness of covariance matrix for effective filtering.

Both UKF algorithm and SR-UKF algorithm, target state estimation of this moment is related to both current measured value, and the state estimation of last moment. Thus, the state estimation accuracy enhancement of last moment can enhance the tracking accuracy. SR-UKFS uses RTS algorithm as the forward filter algorithm to backward smooth the received target state estimation, to obtain a precise target state estimation of last moment, and uses SR-UKF for secondary filtering. Fig. 2 shows the the process of SR-UKFS algorithm.

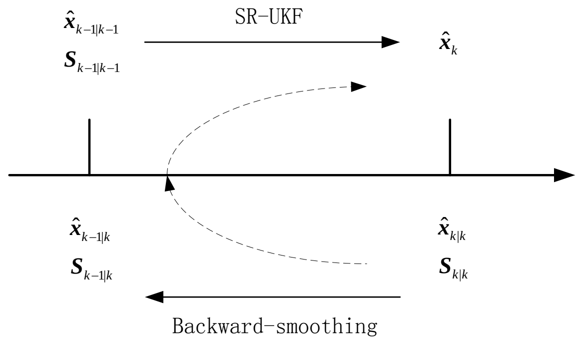

Fig.2. The process of SR-UKFS

The calculation step of SR-UKFS is given as:

Step 1: Parameters initialize

$$\hat{\boldsymbol{x}}_0 = E\left[\boldsymbol{x}_0\right] \tag{4}$$

$$\boldsymbol{P}_0 = E\left[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \boldsymbol{x}_0)^T\right] \tag{5}$$

$$\boldsymbol{S}_0 = chol(\boldsymbol{P}_0) \tag{6}$$

Step 2: Selection of sigma points

$$\boldsymbol{\chi}_{k-1} = \left[\hat{\boldsymbol{x}}_{k-1} \;\; \boldsymbol{x}_{k-1} + \sqrt{n+\kappa}\boldsymbol{S}_k \;\; \boldsymbol{x}_{k-1} - \sqrt{n+\kappa}\boldsymbol{S}_k\right] \tag{7}$$

Step 3: Time update equations

$$\boldsymbol{\chi}^*_{k|k-1} = \boldsymbol{\Phi}_{k|k-1}\boldsymbol{\chi}_{k-1} \tag{8}$$

$$\hat{\boldsymbol{x}}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \boldsymbol{\chi}^*_{i,k|k-1} \tag{9}$$

$$\hat{\boldsymbol{S}}_k^- = qr\left(\left[\sqrt{W_1^c}\left(\boldsymbol{\chi}^*_{1:2n,k|k-1} - \hat{\boldsymbol{x}}_k^-\right)\sqrt{\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{\Gamma}^T}\right]\right) \tag{10}$$

$$\boldsymbol{S}_k^- = cholupdate\left(\hat{\boldsymbol{S}}_k^-, \boldsymbol{\chi}^*_{0,k} - \hat{\boldsymbol{x}}_k^-, W_0^{(c)}\right) \tag{11}$$

$$\boldsymbol{\chi}_{k|k-1} = \left[\hat{\boldsymbol{x}}_k^- \;\; \boldsymbol{x}_k^- + \sqrt{n+\kappa}\boldsymbol{S}_k^- \;\; \boldsymbol{x}_k^- - \sqrt{n+\kappa}\boldsymbol{S}_k^-\right] \tag{12}$$

$$\boldsymbol{Z}_{k|k-1} = H(\boldsymbol{\chi}_{k|k-1}) \tag{13}$$

$$\hat{\boldsymbol{Z}}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \boldsymbol{Z}_{i,k|k-1} \tag{14}$$

Step 4: Measurement update equations

$$\hat{\boldsymbol{S}}_{\bar{Z}_k} = qr\left(\left[\sqrt{W_1^{(c)}}\left[\boldsymbol{Z}_{1:2n,k|k-1} - \hat{\boldsymbol{Z}}_k^-\right]\sqrt{\boldsymbol{R}}\right]\right) \tag{15}$$

$$\boldsymbol{S}_{\bar{Z}_k} = cholupdate\left(\hat{\boldsymbol{S}}_{\bar{Z}_k}, \boldsymbol{Z}_{0,k} - \hat{\boldsymbol{Z}}_k^-, W_0^{(c)}\right) \tag{16}$$

$$\boldsymbol{P}_{x_k Z_k} = \sum_{i=0}^{2n} W_i^{(c)} \left[\boldsymbol{\chi}_{i,k|k-1} - \hat{\boldsymbol{x}}_k^-\right]\left[\boldsymbol{Z}_{i,k|k-1} - \hat{\boldsymbol{Z}}_k^-\right]^T \tag{17}$$

$$\boldsymbol{K}_k = (\boldsymbol{P}_{x_k Z_k} / \boldsymbol{S}^T_{\bar{Z}_k}) / \boldsymbol{S}_{\bar{Z}_k} \tag{18}$$

$$\hat{\boldsymbol{x}}_k = \boldsymbol{x}_k^- + \boldsymbol{K}_k(\boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_k^-) \tag{19}$$

$$\boldsymbol{U} = \boldsymbol{K}_k \boldsymbol{S}_{\bar{Z}_k} \tag{20}$$

$$\boldsymbol{S}_k = cholupdate(\boldsymbol{S}_k^-, \boldsymbol{U}, -1) \tag{21}$$

Step 5: Smoothing process

$$\hat{\boldsymbol{x}}_{k-1|k} = \boldsymbol{x}_{k-1} + D(\boldsymbol{x}_{k-1} - \boldsymbol{\chi}^*_{k|k-1}) \tag{22}$$

$$\boldsymbol{S}_{k-1|k} = \boldsymbol{S}_{k-1} + D(\boldsymbol{S}_{k-1} - \boldsymbol{S}_{k|k-1})D^T \tag{23}$$

$$D = \boldsymbol{S}_{k-1}\boldsymbol{\Phi}_k(\boldsymbol{S}_{k|k-1})^{-1} \tag{24}$$

Step 6: Second forward SR-UKF filtering

Replacing $\hat{\boldsymbol{x}}_{k-1}$, $\boldsymbol{S}_{k-1}$ by $\hat{\boldsymbol{x}}_{k-1|k}$, $\boldsymbol{S}_{k-1|k}$, and repeating step 2 to step 4.

where

$$W_0^{(m)} = \lambda / (n+\lambda)$$

$$W_0^{(c)} = \lambda / (n+\lambda) + (1-\alpha^2+\beta)$$

$$W_i^{(m)} = W_i^{(c)} = 1/\left[2(n+\lambda)\right] \quad (i = 1, 2 \cdots 2n)$$

$\lambda = \alpha^2(n+\kappa) - n$ is scale parameter， n is the dimension of state vector of system， α decides the degree of dispersion of Sigma and usually sets as a small positive number， and $0.001 < \alpha \le 1$. κ is another scale adjustment parameter， which is usually set as 0. β includes the prior knowledge of x probability distribution. For random variables which follow the Gaussian distribution， $\beta = 2$ is the best situation.

4 Simulation results and analysis

Within the observation time, sonar observation platform is motionless. The sonar observation platform is considered as coordinate reference point, underwater target aircraft straightly motions at a constant speed 500 meters away from observation platform in a stable state, and the speed is 8m/s. The initial azimuth angle is 300 with respect to observation platform, the depth is 45 meters. The relations of relative location constitute the three-dimensional situation with known depth. And $z = 45$ m， $v_z = u_z = 0$， the target state vector can be simplified as $\boldsymbol{X}(k) = [x(k), y(k), v_x(k), v_y(k)]^T$. The simulation condition and related parameters: sampling period $T = 0.1s$， noise variance matrix of system $\boldsymbol{Q} = diag([1,1])$; the measured noise standard deviation of azimuth angle and pitch angle in passive sonar $\sigma_\theta = 0.1$ rad， $\sigma_\varphi = 0.1$ rad. The initial state vector of target $\boldsymbol{X}(0|0) = (250, 433, -4, -6.9)^T$, the covariance matrix of original state error $\boldsymbol{P} = diag([50, 50, 5, 5])$， the simulation time is 46s. The filtering parameter of SR-UKFS is set as $\alpha = 0.002$， $\beta = 2$， $\kappa = 0$.

Under above-mentioned conditions, the author evaluates the performances of UKF, SR-UKF and SR-UKFS, which are used for underwater pure orientation targets. The target trajectory is shown in Fig.3. Fig.4 and Fig. 5 are the estimation variance of location of x and y direction, Fig. 6 and Fig. 7 are estimation variance of speed of x and y direction. Tab.1 is the comparison of location and speed mean square after the filters of UKF, SR-UKF and SR-UKFS. The simulation results clearly show that UKF has the worst tracking accuracy, the stability of SR-UKF and SR-UKFS are better, and the tracking accuracy of SR-UKFS is better than SR-UKF.

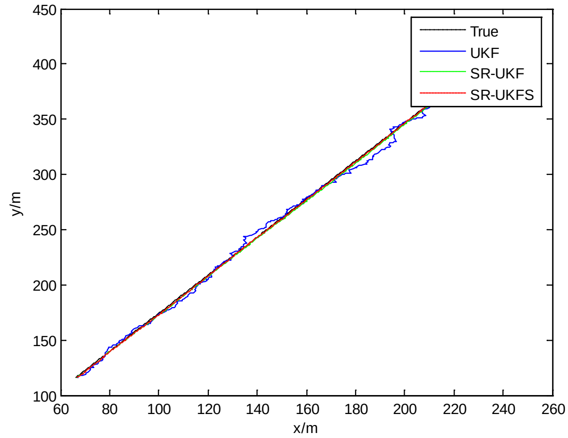

Fig.3. Moving trace of target

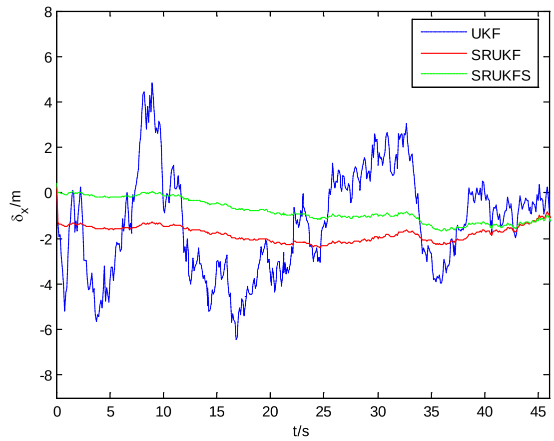

Fig.4. Estimation variance of location in line x

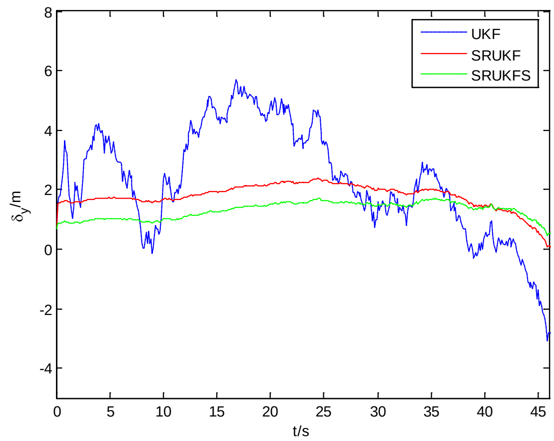

Fig.5. Estimation variance of location in line y

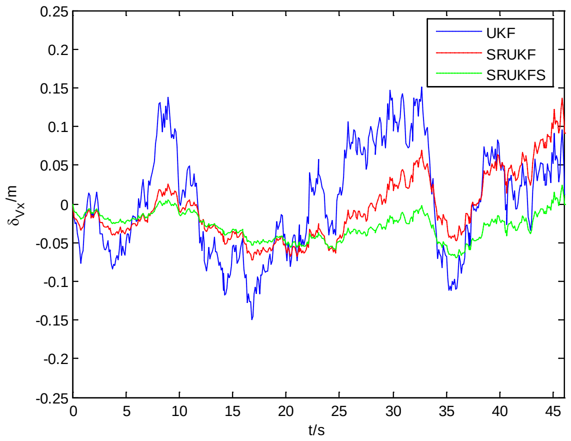

Fig.6. Estimation variance of speed in line x

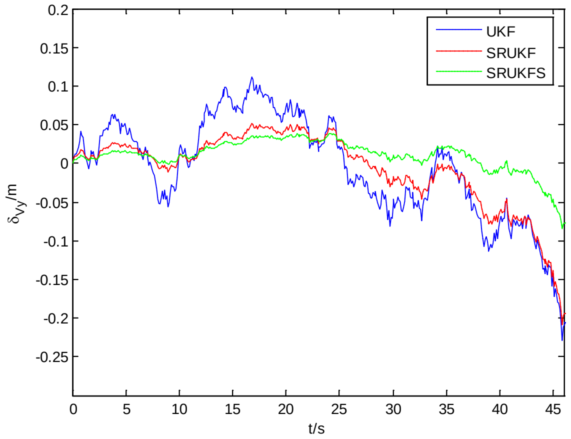

Fig.7. Estimation variance of speed in line y

Tab.1 Comparison between RMS estimation variances of three different conditions

	x(m)	*y(m)*	*Vx(m/s)*	*Vy(m/s)*
UKF	2.1689	2.7573	0.0581	0.0801
SR-UKF	1.6650	1.6993	0.0402	0.0679
SR-UKFS	0.8526	1.1940	0.0280	0.0341

5 Conclusion

In this paper, the author utilizes passive sonar to obtain the information of pitch angle and azimuth angle of underwater pure orientation target and combines with the SR-UKFS algorithm to track underwater motion targets. Based on SR-UKFS algorithm, better filtering effect can be achieved to enhance the tracking accuracy. The results of simulation show that SR-UKFS can be used in underwater motion target tracking, and the precision of filtering is significantly better than standard UKF and SR-UKFS algorithm.

References:

[1] Hu You-feng, Jiao-Bing-li. Passive underwater target motion analysis based upon bearing-elevation measurement in three dimensions[J]. Acta simulate ayatematica sinica, Vol.15, No.6, 2003, pp. 776-779.

[2] Wu Pan-long, Wang Bao-bao, Cai Ya-dong, et al. Single observer passive target tracking based on extended H_∞ filter[J]. Journal of Chinese Inertial Technology, Vol.18, No.5, 2010, pp. 591-594.

[3] Wang Bo, Xu De-min, Shen Meng. Underwater passive target motion analysis based on UKF[J]. Journal of Projectiles, Rockets, Missiles and Guidance, Vol.25, No.2, 2005, pp. 423-426.

[4] Pan-long Wu, Bao-bao Wang, Cun-hui Ji. Design and realization of short range defence radar target tracking system based on DSP/FPGA[J]. WSEAS Transactions on System, Vol. 10, No. 11, 2011, pp.376-386.

[5] Bao-bao Wang, Lian-zheng Zhang. Information Fusion of Airborne radar and ESM for maneuvering target tracking system based on IMM-BLUE [J]. WSEAS Transactions on System, Vol. 13, No. 11, 2014, pp.699-707.

[6] S. J. Julier, J. K. Uhlmann, H. F. Durrant-Whyte. A new method for the nonlinear transformation of means and covariances in filters and estimators. IEEE Trans On Automatic Control, 2000, pp. 477-482.

[7] Wu Pan-long, Kong Jian-shou. Under bearing-only target tracking based on Square-root UKF[J]. Journal of Nanjing University of Science and Technology (Natural Science), Vol.33, No.6, 2009, pp. 751-755.

[8] H. E Rauch, F. Tung, C. T. Striebel. Maximum likelihood estimates of linear dynamic system [J]. AIAA J, 1965, pp.1445-1450.

[9] Y. Cao, X. C. Mao. Improved Filtering-Smoothing Algorithm for GPS Positioning[C], in: Beijing, China: Proceedings of the 11th International IEEE Conference on Intelligent Transportation Systems, 2008, pp. 857-861.

[10] Fan ZHANG, Panlong WU, Longmei ZHAO. Improved SR-UKF algorithm for Mobile Robot Tracking[J]. Journal of Computational Information System, Vol.8, No.15, 2012, pp. 6499-6506.

[11] Farina A. Target tracking with bearing-only measurements[J]. Signal Process, Vol.78, No.1, 1999, pp. 61-78.
[12] Yu Chun-lai, Zhan Rong-hui, Wan Jian-wei. Research on robust UKF algorithm for single observer passive target tracking based on polar coordinates[J]. Journal of National University of Defense and Technology, Vol.30, No.5, 2008, pp. 73-79.
[13] Julier S, Uhlmann J, Hugh F. A new method for the nonlinear transformation of means and covariance in filters and estimators[J]. IEEE Transaction on Automatic Control, Vol.45, No.3, 2000, pp. 477-482.
[14] Sadhu S, Modndal S, Srinivasan M. Sigma point Kalman filter for bearing only tracking[J]. Signal Processing, Vol.45, 2006, pp. 3769-3777.
[15] Rudolph van der Merwe, Eric A.Wan. The square root unscented Kalman filter for state and parameter estimation[C]. Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing, New York, 2001, pp. 3461-3464.