How the Variance-to-Mean Ratio Behaves in Relation to the Basic Principles for Inequality Measures

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Abstract: - The phenomenon of inequality occurs when resources, opportunities, or other attributes are distributed unequally among the elements of a set. Although the literature on inequality focuses heavily on income inequality, inequality encompasses economic, social, and spatial dimensions, being relevant in different fields of society. When addressing inequality, the measure that immediately appears as a candidate for evaluating this inequality is the Gini index, however, there are several circumstances in which other measures of inequality are more appropriate, or in which the information provided by the Gini index is insufficient to adequately characterize or compare inequality. Considering that, in certain situations, the Variance-to-Mean Ratio and the Gini Index appear as viable alternatives to measure inequality, we are interested in analyzing the Variance-to-Mean Ratio regarding its compliance with the four basic criteria for inequality measures, adopting a formal and axiomatic approach. We conclude that the Variance-to-Mean Ratio does not meet all these requirements.

Key-Words: - Dispersion, Gini index, Index of dispersion, Inequality, Inequality measures properties, Variation.

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1 Introduction

Inequality refers to the phenomenon that occurs when resources (or opportunities) among the members of a given society are unevenly and/or unfairly distributed, [1]. Inequality analysis aims to compare the distribution of an underlying resource – as it changes over time, across countries or groups, or both – or to provide a measure of the inequality of a distribution, interpreting inequality as the distance from the concept, not always explicit, of "equality", [2]. The relevance of having a complete picture of the manifestations of inequality has triggered extensive research and the introduction of diverse methodologies to measure, classify, and compare different distributions. Although the literature on inequality places great emphasis on income inequality, it is a relevant theme in various fields of society, and its complexity is highlighted in what is mentioned in [1]: “inequality encompasses distinct yet overlapping economic, social, and spatial dimensions."

The most common approaches to measuring inequality are based on the Lorenz curve, generalized entropy, and the social welfare function. In addition to these three categories of inequality measures, which illustrate inequality in the strict sense, [3] (apud [4]) also considers a category of inequality measures that illustrate division, which includes share and division measures and positional inequality measures.

Since there is no measure of inequality that can encapsulate all the different dimensions, nor allow an objective comparison between them, [1], inequality measures have different degrees of complexity, and each has its own merits and shortcomings. In [5], it is argued that it may be advantageous to use them in a complementary way to provide the most complete picture. Referring to the Gini index, [6] advises that this index should be combined with other measures appropriate to the topic studied.

Due to the long-standing dominance of the economic approach to the study of inequality, the Gini Index (or rather, the numerous versions of this index) has stood out in popularity among the methodologies commonly used to measure inequality. Nevertheless, there are other measures that are suitable for this purpose, such as the Theil's index, the Decile dispersion ratio, the Coefficient of Variation, or the Variance-To-Mean Ratio (VMR), among many others. In [7], [5] or [8], a detailed description of widely used measures of income inequality can be found.

In certain circumstances, alternative measures to the Gini index prove to be even more appropriate than this index. A situation in which the use of alternative measures to the Gini index is of great importance arises when different distributions have the same value of the Gini index, and it is necessary to complement the information provided by this index with other information that allows for proper comparison of the distributions, [9]. In studies on inequality in frequency distributions of non-economic variables, or those not related to income, dispersion measures such as variance and standard deviation are often preferred, see, e.g., [10]. There are, therefore, several situations in which different measures constitute alternatives for assessing inequality. For example, in [11], the Gini index, the coefficient of variation, the Hoover index, and the Variance-to-Mean Ratio (VMR) are considered to evaluate parasite distributions. In [12] the similarity between the square of the coefficient of variation and the Gini index is analyzed and exemplified. In [13] the scatter index was introduced as an alternative to the coefficient of variation.

Settling the statistical analysis of financial contexts on traditional measures, such as standard deviation, variance, and the Gini index, often does not allow for fully leveraging the available data. The integration of the VMR into financial engineering practices can enhance analytical capabilities, leading to more informed decisions, see, e.g., [14]. The advantages of VMR over standard deviation and variance become evident in all circumstances where measuring relative dispersion is important. In terms of comparing the potential of VMR as an alternative to the Gini index, we can refer to cases where the variability of returns in investment portfolios is evaluated, particularly in scenarios where the consistency of returns is crucial. In these cases, it is essential to identify strategies that not only seek high returns but also minimize the uncertainty associated with those returns. In this context, VMR provides a clearer picture of the risk associated with different investment strategies by focusing on the dispersion of returns (the volatility of returns relative to expected return) rather than solely on return inequality, which is the primary concern of the Gini index. The robustness of any statistical data analysis, as well as the validity and rigor of the conclusions drawn, requires the use of a set of measures that allow for a comprehensive understanding of the data under study.

In the application of dispersion measures within risk assessment, the adoption of VMR should be considered either as an alternative to traditional measures or as a complement to them in specific contexts where the characteristics of the data align well with its strengths. For this purpose, it is important to have a detailed understanding of the characteristics of VMR.

The present study, focused on the properties of the VMR, was triggered by our interest in evaluating measures of inequality that may constitute alternatives to the Gini index or complement the information provided by this index. In the extensive literature we consulted on the topic of inequality measures, there are few studies that formally address the evaluation of measures in terms of the principles that establish their validity as inequality measures. Usually, there is only a superficial reference to whether or not the criteria are met by the measures addressed in the studies, without the respective proof (e.g., [15], [16], [17], [18], [19]). An interesting survey on inequality measures, with some formalism in the approach to the criteria for inequality measures, can be found in [20]. Two more recent works address the Gini index [21] and the coefficient of variation [22], formally analyzing the basic criteria for inequality measures.

With this work, we hope to contribute to the enrichment of the literature on inequality measures and to a better understanding of the VMR.

Following common practice in the literature, we will consider the terms "dispersion measures" and "variability measures" as synonyms. We will adopt the term "inequality measures" when referring to measures that fulfil the four basic criteria for inequality measures.

The paper is organized as follows. The next section presents the axioms for the measurement of inequality. In Section 3, we address the variance-to-mean ratio and analyze its compliance with the principles for inequality measures. Finally, Section 4 concludes.

2 Axioms for the Measurement of Inequality

The search for adapting inequality measures to the particularities of the variables and data sets to which they are applied has triggered the development of a wide range of measures, as well as versions and corrections to the measures introduced by other authors.

Given the diversity of measures that can be used to assess inequality in a dataset, and the possibility that, ultimately, different measures may lead to different conclusions about the magnitude and direction of inequality, [23], the decision to adopt the most appropriate measure for the situation is critical. The reasons that motivate the choice vary between decisions, sometimes being based on somewhat vague criteria, such as convenience or familiarity, [15], and the use of "borrowed" statistical measures, [24].

According to [15], the choice of a measure of inequality is, in essence, more a decision about the definition of inequality that one intends to study than about the way to measure this inequality. However, as argued by [25], the axioms established to determine what defines the measures of inequality are, themselves, the bases for giving meaning to the concept of inequality. In any case, theoretical and methodological implications are associated with it.

As can be seen in the literature on inequality measures (e.g., [17], [18], [19], [26]), these measures are often presented in a non-formal way, with their properties being addressed from a practical perspective based on examples and illustrations. Perhaps this is proof of the subjectivity that [25] claims is associated with the choice of inequality measures. In practice, the result is a scarcity of studies that formally present the properties of inequality measures and aggregate information scattered across several studies. Sharing the conviction that the comparative assessment of available inequality measures requires a deeper knowledge of the capacity that each measure demonstrates to provide a complete picture, depending on the characteristics of these measures and their strengths and weaknesses, [21], [27], we adopted a formal/theoretical approach to inequality measures, based on the axiomatics that translate the desirable properties of an inequality measure.

Since the beginning of the 20th century, research into measuring (income) inequality has been gaining prominence. Notable contributions to the early stages of this field include the pioneering and influential works of [28], [29], [30], and [31]. Alongside the development of new inequality measures, researchers have been concerned with defining the properties that qualify a measure as a "good" measure of inequality. Based on the work of [31], [32], [33], and [34], it was established that inequality measures must adhere to certain essential properties. Before addressing these properties, it is important to present two preliminary conditions established for candidates to measure inequality, [15].

A measure of inequality:

- is zero for distributions in which all variable values are equal.
- has a positive value for distributions in which the variable takes on at least two different values.

It is typically assumed that there are four essential properties that inequality measures must respect. These four properties are the basic criteria for inequality measures, [15]. According to [35], these four principles constitute an admissible approach to considering an inequality measure as "good".

In what follows, the variable of interest could be income or any other variable for which we intend to measure inequality.

Let us consider a sample (or population) consisting of n individuals, with $n \geq 2$, a distribution $x = \{x_1, x_2, \dots, x_n\}$, with x_i the value of the variable X for the i-th individual, and $I(x_1, x_2, \dots, x_n)$ a function (measure) of inequality.

The desirable properties of an inequality measure are:

1) Anonymity principle (Symmetry)
This principle establishes that the degree of inequality remains the same if there is a change in the income values of pairs of individuals.

For a symbolic and more formal presentation of this principle, let us consider the incomes of n individuals, in ascending order,

$$x_1 \leq \dots \leq x_i \leq x_j \leq \dots \leq x_n$$

If we rearrange the observations, for example, considering that two of the individuals who previously had incomes of x_i and x_j, respectively, now have incomes $x'_i = x_j$, $x'_j = x_i$ after the rearrangement the incomes in ascending order are:

$$x_1 \leq \dots \leq x'_j \leq x'_i \leq \dots \leq x_n$$

The function I verifies the Anonymity principle if:

$$I(x_1, x_2, \dots, x_i, x_j, \dots, x_n) = I(x_1, x_2, \dots, x'_i, x'_j, \dots, x_n), \forall\, x_i \neq x_j, i, j = 1, \dots, n.$$

2) Pigou-Dalton transfer principle
This principle establishes that a mean-preserving progressive [regressive] transfer of a positive amount of income, this is, a transfer

from a richer [poorer] individual to a poorer [richer] individual without reversing the ranking between both, must lead to a lower [higher] value of the inequality measure.

For a symbolic and more formal presentation of this principle, let us consider the incomes of n individuals, in ascending order,

$$x_1 \leq \cdots \leq x_i < x_j \leq \cdots \leq x_n.$$

With a transfer of a quantity $\delta > 0$, from the individual x_j to the individual x_i, such that $x_i + \delta < x_j - \delta$, the function I verifies the Pigou-Dalton Principle if:

$$I\left(x_1, x_2, \ldots, x_i, x_j, \ldots, x_n\right) > I(x_1, x_2, \ldots, x_i + \delta, x_j - \delta, \ldots, x_n).$$

3) Scale invariance (Relative income principle)
This principle establishes that if there is a recalling of the income values, that is if everyone's income changes by the same proportion, the value of the measure of inequality remains the same. So, the measure cannot be affected by the absolute values of the income, only their relative values matter.

For a symbolic and more formal presentation of this principle, let us consider two distributions variables $x = \{x_1, x_2, \ldots, x_n\}$ and $y = kx = \{kx_1, kx_2, \ldots, kx_n\}$, with k a positive constant. The function I is scale invariant if:

$$I(x_1, x_2, \ldots, x_n) = I(kx_1, kx_2, \ldots, kx_n).$$

4) Principle of Population
This principle establishes that the inequality value remains unchanged if the population is replicated one or more times, that is, when a population with n individuals is combined with other similar populations, resulting in a population of kn individuals and the same proportion of the population receiving any income.

For a symbolic and more formal presentation of this principle, let us consider a distribution $x = \{x_1, x_2, \ldots, x_n\}$ and a replication of this distribution by order k, for $k > 2$,

$$\{x_1, \ldots, x_1, x_2, \ldots, x_2, \ldots, x_n, \ldots, x_n\}$$

where each x_i, $i = 1, \ldots, n$, is repeated k times.
The function I verifies the Principle of Population if:

$$I(x_1, x_2, \ldots, x_n) = I(x_1, \ldots, x_1, x_2, \ldots, x_2, \ldots, x_n, \ldots, x_n).$$

In cases where the inequality measure is used to compare distributions with the same mean, it is considered sufficient to satisfy only two properties: (1) Anonymity principle (Symmetry) and (2) Pigou-Dalton transfer principle. However, when used to compare two distributions with different means it is necessary to add a mean-invariance property, usualy the property of scale invariance, [36].

3 Variance-to-Mean Ratio Properties

Let us consider a sample (population) of n individuals, with $n \geq 2$, and x_i the value of the variable X for the i-th individual. The dispersion measure known as the variance-to-mean ratio (VMR), also known as index of dispersion, is defined as:

$$VMR = \frac{s^2}{\bar{x}},$$

where

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j,$$

$\bar{x} \neq 0$, and

$$s^2 = \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2$$

are the mean value and the variance of X, respectively.

Note: We will only consider the case where n is very large, since similar results would be obtained when this is not the case. Therefore, the variance will not be subject to Bessel's correction, which would take the form $s^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2$.

The VMR is commonly used to quantify the degree of dispersion (or randomness) of a set of observed occurrences of a given phenomenon by comparison with a standard statistical model.

Generally, the VMR is used only for variables that take positive values, such as count data or time between events, and the VMR is only defined when the mean value is non-zero. The VMR is widely used in applications to biology, ecology, physics, and engineering (see e.g., [11], [37], [38], [39], [40], [41], [42]) to measure the heterogeneity/randomness in certain phenomena.

Considering the nature of the variables to which it is typically applied, VMR evaluates the degree of dispersion based on the Poisson distribution, [11]. The relevance of using the Poisson distribution as a reference is based on the equality between the mean value and the variance of this distribution, thereby giving a unitary value to the VMR.
Using the Poisson distribution as a reference, that is, by comparison with the value $VMR = 1$, a distribution is classified as:

- under-dispersed when $0 < VMR < 1$;

- over-dispersed when $VMR > 1$.

A possible use of VMR is its application to evaluate whether observed data can be modeled through a Poisson process. This procedure, called the VMR test (or Poisson test), is applied in several areas (see, e.g., [43], [44], [45], [46]).

Since in this approach we intend to evaluate the VMR as a measure of inequality and as a candidate to complement the information provided by the Gini Index, we exclusively consider the case of variables that assume only positive values. This guarantees the applicability of the VMR and sets aside the situations in which the Gini Index behaves poorly, falling outside the range [0,1].

As mentioned previously, [15] established that candidates for measures of inequality should be zero for distributions in which all variable values are equal, and should have a positive value for distributions in which the variable assumes at least two different values.

Let us begin by showing that the VMR is zero when all observations are equal.

Proposition 1: When $x_i = a$, $i = 1, \dots, n$, with a constant, $VMR = 0$

Proof: When $x_i = a$, $i = 1, \dots, n$, the mean value of X will be $\bar{x} = a$ and the variance will be $s^2 = 0$, so $VMR = \frac{s^2}{\bar{x}} = \frac{0}{a} = 0$

Let us now consider the case in which a variable assumes only two different values, more specifically when only one of the values is non-zero.

Proposition 2: When $x_k = c$, with $c > 0$, and $x_j = 0$, $j = 1, \dots, n, j \neq k, VMR > 0$.
Proof: When $x_k = c$ $(c > 0)$ and $x_j = 0$, $j = 1, \dots, n, j \neq k$, the mean value of X is

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n} = \frac{(n-1) \times 0 + c}{n} = \frac{c}{n}.$$

and the variance is:

$$s^2 = \frac{1}{n} \sum_{j=1}^{n} \left(x_j - \frac{c}{n}\right)^2$$
$$= \frac{1}{n}\left[(n-1)\left(0 - \frac{c}{n}\right)^2 + \left(c - \frac{c}{n}\right)^2\right] =$$
$$= \frac{1}{n}\left[(n-1)\frac{c^2}{n^2} + \left(\frac{nc}{n} - \frac{c}{n}\right)^2\right] =$$
$$= \frac{1}{n}\left[(n-1)\frac{c^2}{n^2} + \left(\frac{c(n-1)}{n}\right)^2\right]$$
$$= \frac{1}{n}\left[(n-1)\frac{c^2}{n^2} + \frac{c^2(n-1)^2}{n^2}\right]$$
$$= \frac{1}{n}\left[(n-1)\frac{c^2}{n^2} + (n-1)^2\frac{c^2}{n^2}\right]$$
$$= \frac{1}{n}\left[(n - 1 + n^2 + 2n + 1)\frac{c^2}{n^2}\right] =$$
$$= \frac{1}{n}\left[(n^2 + 3n)\frac{c^2}{n^2}\right] =$$
$$= (n+3)\frac{c^2}{n^2}$$

So, the VMR is:

$$VMR = \frac{s^2}{\bar{x}} = \frac{(n+3)\frac{c^2}{n^2}}{\frac{c}{n}} = (n+3)\frac{c}{n} .$$

We have $c > 0$, since the variable X only takes positive values, so $VMR = (n+3)\frac{c}{n} > 0$.

Next, we will analyse the four principles that enable a measure to the measurement of inequality.

Proposition 3: VMR verifies the anonymity principle (Symmetry)

Proof: Let us consider a sample (population) of $n \geq 2$ individuals, and $x_i \geq 0$ the value of the variable X for the i-th individual. Let us consider the incomes of n individuals ($n \geq 2$), with $x_i \geq 0$ representing the income of the i-th individual, in ascending order,

$$x_1 \leq \cdots \leq x_i \leq x_j \leq \cdots \leq x_n.$$

Rearranging the observations so that two individuals who previously had incomes of x_i and x_j, respectively, now have incomes $x'_i = x_j$, $x'_j = x_i$, the arrangement of the observations after this change will be:

$$x_1 \leq \cdots \leq x'_j \leq x'_i \leq \cdots \leq x_n$$

Let us represent by x^* and x^{**} the distributions of income before and after the rearrangement. Since neither the mean value nor the variance are sensitive to the order of observations, the mean values, before and after the rearrangement, will be the same:

$$\overline{x^*} = \overline{x^{**}}$$

and the variances, before and after the rearrangement, will be the same too:

$$s^2(x^*) = s^2(x^{**}).$$

Therefore, there is no change in the value of the VMR,

$$VMR(x^*) = VMR(x^{**}).$$

Proposition 4: VMR verifies the Pigou-Dalton transfer principle.

Proof: Let us consider the incomes of n individuals in ascending order, $x_1 \leq \cdots \leq x_i \leq \cdots \leq x_j \leq \cdots \leq x_n$, and a transfer of a quantity $\delta > 0$, from the individual x_j to the individual x_i, such that $x_i + \delta < x_j - \delta$. Let us also consider $p < i$, $i < h < j$ and $m > j$.

The distributions of income before and after the transfer of the quantity δ will be represented by $x^* = (x_1, x_2, \dots, x_i, \dots, x_j, \dots, x_n)$ and $x^\delta = (x_1, x_2, \dots, x_i + \delta, \dots, x_j - \delta, \dots, x_n)$.

The mean value of the income before and after the transfer are, respectively:

$$\overline{x^*} = \frac{x_1 + x_2 + \cdots + x_i + \cdots + x_j + \dots + x_n}{n}$$

and

$$\overline{x^\delta} = \frac{x_1 + x_2 + \cdots + x_i + \delta + \cdots + x_j - \delta + \dots + x_n}{n}$$

so $\overline{x^*} = \overline{x^\delta}$.

The variance of the income before and after the transfer are, respectively:

$$s^2(x^*) = \frac{(x_1 - \overline{x^*})^2 + (x_2 - \overline{x^*})^2 + \cdots + (x_i - \overline{x^*})^2}{n} + \\ + \frac{\cdots + (x_j - \overline{x^*})^2 + \cdots + (x_n - \overline{x^*})^2}{n}$$

and

$$s^2(x^\delta) = \frac{(x_1 - \overline{x^*})^2 + (x_2 - \overline{x^*})^2 + \cdots + (x_i + \delta - \overline{x^*})^2}{n} + \\ + \frac{\cdots + (x_j - \delta - \overline{x^*})^2 + \cdots + (x_n - \overline{x^*})^2}{n}$$

so

$$n^2[s^2(x^\delta) - s^2(x^*)] = 2\delta(x_j - x_i) + 2\delta^2 \Leftrightarrow$$
$$s^2(x^\delta) - s^2(x^*) = \frac{2\delta(x_j - x_i) + 2\delta^2}{n^2}$$

therefore, the VMR verifies the Pigou-Dalton Principle since:

$$VMR(x^*) > VMR(x^\delta)$$

that is, the transfer resulted in a decrease in inequality between individuals.

Proposition 5: VMR is not scale invariant.

Proof: Let us consider two distributions of income $x^* = (x_1, x_2, \dots, x_n)$ and $y^* = kx^* = (kx_1, kx_2, \dots, kx_n)$, with k a positive constant.
We have $\overline{y^*} = k\overline{x^*}$ and $s^2(y^*) = k^2 s^2(x^*)$

so,

$$VMR(y^*) = \frac{k^2 s^2(x^*)}{k\overline{x^*}} = kVMR(x^*)$$

$$VMR(y^*) > VMR(x^*)$$

4 Conclusion

In the study of inequality, measurement through the Gini index has prevailed, however the extension of the scope of the study of inequality to phenomena not related to income or non-economic phenomena has boosted the proposal of other measures of inequality. Considering the economic, social and spatial dimensions of inequality, and the complexity of inequality measures, the choice of the appropriate inequality measure is of utmost importance. Diversity of situations and contexts require different measures of inequality, either due to their better adequacy or due to the need to complement the information provided by other measures, namely by the Gini index. Considering that, in certain situations, the Variance-to-Mean Ratio (VMR) and the Gini Index appear as viable alternatives for measuring inequality, we were interested in analysing the VMR in terms of its compliance with the four basic criteria for inequality measures. It was not our purpose to judge the VMR as better or worse than others, our objective was to formally address the properties of the VMR, and, according to these properties, verify compliance with the four basic criteria for inequality measures.

We proved that the VMR meets the two preliminary conditions established for candidates to measure inequality. Regarding the four basic criteria for inequality measures, VMR complies with the anonymity principle, the Pigou-Dalton transfer principle, and the principle of population, but do not comply with the principle of scale invariance.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work the author used Perplexity.ai for language editing, and references checking. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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