# Application of the Simplex Method in the Production of Communication Modules

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*Abstract:* - This paper aims to highlight the important role of operations research, particularly the simplex method, in solving everyday problems. To this end, a specific linear programming problem is considered in the paper. The problem will be solved using the simplex method. After solving the problem, some useful conclusions will be drawn. The following two sections provide an overview of interesting ideas for future research in linear programming and the simplex method. The contribution of this paper is then compared with the contributions of some similar studies. Finally, the study concludes with its conclusion.

*Key-Words:* - operations research, linear programming, simplex method, optimal basic solution, optimal solution, applications, future research.

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## 1 Introduction

Operations research, as is well known, represents an entire scientific discipline. This discipline involves the study and improvement of analytical methods that can aid in making better decisions. One of the key tools in operations research is linear programming. The linear programming problem is an optimization problem for a given linear function with given linear constraints, [1], [2], [3], [4], [5] The simplex method is one of the most well-known and efficient methods for solving linear programming problems, [6], [7]. Therefore, the simplex method is applied in various industries such as logistics, production, finance, and telecommunications. As mentioned in the abstract, the following section will present a linear programming problem that will then be solved using the simplex method. The problem involves profit maximization, demonstrating the practical application of this method in solving real-world optimization problems. The simplex procedure will improve the basic solutions of the problem step by step, i.e., from table to table, until the optimal basic solution is reached. The optimality of the obtained basic solution will be examined in each table in its last row by calculating the differences $z_j - c_j = \sum_{i=1}^{m} c_i t_{ij} - c_j$ , where $i$ is the index of the basic vector, $c_j$ is the coefficient of the variable $x_j$ in the objective function, and $t_{ij}$ is the number in the $i$ – th row and $j$ – th column of the table. The value of the objective function for the basic solution shown in the same table is calculated in the last field of that row (from left to right). The optimal basic solution is given by the table in which all differences $z_j - c_j$ are non-negative.

## 2 Application of the Simplex Method to Solve a Specific LP Problem

A company specializing in the production of electronic equipment manufactures two types of communication modules daily: wireless modules and wired modules. Each module goes through three sets of machines $S_1$, $S_2$ and $S_3$, each specialized for a certain stage of the production of each module. To produce one wireless module, it takes two hours on machine $S_1$, three hours on machine $S_2$, and two hours on machine $S_3$. To produce one wired module, it takes one hour each on machines $S_1$ and $S_3$ and eight hours on machine $S_2$. The capacities of the machines are 10, 22, and 8 hours per day, respectively. The revenue from selling one wireless module is 140, and from selling one wired module is 100 monetary units. The goal is to determine the optimal production plan that maximizes revenue.

### 2.1 Solution

The provided data are shown in Table 1.

Table 1. The setting of the task to be solved

| Machine | Module Type | | Daily Machine Capacity (hours) |
|---|---|---|---|
| | Wireless | Wired | |
| $S_1$ | 2 | 1 | 10 |
| $S_2$ | 3 | 8 | 22 |
| $S_3$ | 2 | 1 | 8 |
| Revenue per unit sold | 140 | 100 | |

## 2.2 Mathematical Model

The linear programming model, [8], [9] for this problem is formulated as follows:

Maximize: $140x_1 + 100x_2$

Subject to:

$$2x_1 + x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 22$$
$$2x_1 + x_2 \leq 8$$
$$x_1, x_2 \geq 0,$$

Where $x_1$ is the number of wireless modules produced, and $x_2$ is the number of wired modules produced.

## 2.3 Conversion to Standard Form

To solve using the simplex method, the inequalities must be converted into equalities by introducing slack variables $x_3$, $x_4$, and $x_5$.

Maximize: $140x_1 + 100x_2 + 0x_3 + 0x_4 + 0x_5$

Subject to:

$$2x_1 + x_2 + x_3 = 10$$
$$3x_1 + 8x_2 + x_4 = 22$$
$$2x_1 + x_2 + x_5 = 8$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

This is a maximum linear programming problem.

The following section will find its optimal solution using the simplex method.

## 2.4 Initial Simplex Tableau

The initial simplex tableau is constructed as shown in Table 2.

Table 2. Initial simplex tableau

| | $c_j$ | 140 | 100 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| $c_i$ | ***Base*** | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | **RHS** |
| **0** | $A_3$ | 2 | 1 | 1 | 0 | 0 | 10 |
| **0** | $A_4$ | 3 | 8 | 0 | 1 | 0 | 22 |
| **0** | $A_5$ | 2 | 1 | 0 | 0 | 1 | 8 |
| $z_j - c_j$ | | **−140** | **−100** | **0** | **0** | **0** | **0** |

## 2.5 Simplex Method Iterations

The simplex method proceeds iteratively to improve the solution by pivoting.

**Iteration 1:**

1. **Pivot Selection:** Identify the most negative $z_j - c_j$. Here, $z_1 - c_1 = -140$ is the most negative, indicating $A_1$ enters the basis.
2. **Determine Leaving Variable:** Calculate the ratio of RHS to the pivot column for each row. The minimum ratio determines the leaving variable.
   Row 1: $10/2 = 5$
   Row 2: $22/3 \approx 7.3$
   Row 3: $8/2 = 4$ (minimum)
   Thus, $A_5$ leaves the basis.
3. **Pivot Operation:** Perform row operations to make the pivot element 1 and other elements in the pivot column 0.

   Thus, we derive Table 3.

Table 3. Second simplex tableau

| | $c_j$ | 140 | 100 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| $c_i$ | ***Base*** | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | **RHS** |
| **0** | $A_3$ | 0 | 0 | 1 | 0 | −1 | 2 |
| **0** | $A_4$ | 0 | $\frac{13}{2}$ | 0 | 1 | $-\frac{3}{2}$ | 10 |
| **140** | $A_1$ | 1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 4 |
| $z_j - c_j$ | | **0** | **−30** | **0** | **0** | **70** | **560** |

**Iteration 2:**

1. **Pivot Selection:** Identify the most negative $z_j - c_j$. Here, $z_2 - c_2 = -30$ is the most negative, indicating $A_2$ enters the basis.
2. **Determine Leaving Variable:** Calculate the ratio of RHS to the pivot column for each row. The minimum ratio determines the leaving variable.
   Row 1: Not applicable (coefficient is 0)

Row 2: $10/\frac{13}{2} = \frac{20}{13} \approx 1.5$ (minimum)

Row 3: $4/\frac{1}{2} = 8$

Thus, $A_4$ leaves the basis.

3. **Pivot Operation:** Perform row operations to make the pivot element 1 and other elements in the pivot column 0.

As a result, Table 4 is obtained.

Table 4. Third simplex tableau

| | $c_j$ | **140** | **100** | **0** | **0** | **0** | |
|---|---|---|---|---|---|---|---|
| $c_i$ | ***Base*** | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | **RHS** |
| **0** | $A_3$ | 0 | 0 | 1 | 0 | $-1$ | 2 |
| **100** | $A_2$ | 0 | 1 | 0 | $\frac{2}{13}$ | $-\frac{3}{13}$ | $\frac{20}{13}$ |
| **140** | $A_1$ | 1 | 0 | 0 | $-\frac{1}{13}$ | $\frac{8}{13}$ | $\frac{42}{13}$ |
| $z_j - c_j$ | | **0** | **0** | **0** | $\mathbf{\frac{60}{13}}$ | $\mathbf{\frac{820}{13}}$ | $\mathbf{\frac{7\,880}{13}}$ |

Since $z_j - c_j = 0$ only under the basic vectors, and all other differences $z_j - c_j$ are positive, the third table provides the only optimal basic solution. Therefore, the given problem has exactly one optimal solution.

The optimal solution is:

$$x_1^* = \frac{42}{13} \approx 3.23, \quad x_2^* = \frac{20}{13} \approx 1.54,$$

$$z^* = 140x_1^* + 100x_2^* = \frac{7\,880}{13} \approx 606.15$$

with additional variables:

$$x_3^* = 2, \quad x_4^* = x_5^* = 0.$$

# 3 Conclusions from the Solution

The optimal production plan is to produce approximately 3.23 wireless modules and 1.54 wired modules daily, resulting in a maximum revenue of approximately 606.15 monetary units. Machines $S_2$ and $S_3$ operate at full capacity (because $x_4^* = x_5^* = 0$), while machine $S_1$ is underutilized by 8 hours (since $x_3^* = 2$).

## 3.1 Dual Problem Analysis

The optimal values of the dual variables are:

$$y_1^* = 0, \quad y_2^* = \frac{60}{13}, \quad y_3^* = \frac{820}{13}.$$

The original problem has an optimal solution, and so does the dual, with equal optimal values of their objective functions. Indeed, for the dual objective function $w = 10y_1 + 22y_2 + 8y_3$, it holds that:

$$w^* = 10y_1^* + 22y_2^* + 8y_3^* = \frac{7\,880}{13}.$$

## 3.2 Impact of Capacity Changes

If the capacity of the third machine (8 hours) is increased by one hour – through overtime or otherwise, the following is obtained:

$$10y_1^* + 22y_2^* + (8+1)y_3^* = \frac{7\,880}{13} + y_3^*.$$

Thus, the value of the objective function will increase by $y_3^*$, meaning an increase in the capacity of the third machine by 1 hour would cause, through another optimal program, an increase in maximum revenue by $\frac{820}{13}$ monetary units. Similarly, if the capacity of the second machine is increased by 1 hour (from 22 to 23 hours), the following is obtained:

$$10y_1^* + (22+1)y_2^* + 8y_3^* = \frac{7\,880}{13} + y_2^*.$$

Therefore, an increase in the capacity of the second machine by one hour would cause an increase in maximum revenue by $\frac{60}{13}$ monetary units. Since $y_1^* = 0$, an increase in the capacity of the first machine by one hour would not lead to an increase in total revenue. This is logical because the capacity of the first machine is not fully utilized in the optimal program. Namely, according to the optimal program, the first machine operates only 8 out of the possible 10 hours. If the capacity of the first machine were increased, for example, from 10 to 11 hours, the revenue would not increase because the highest revenue is obtained when the first machine operates exactly 8 hours.

# 4 Comparison with Related Studies and Ideas for Future Research

Numerous studies and papers have been written on the topic of linear programming, but mostly they all offer only theory and complex mathematical formulas without practical examples from the real world. This paper illustrates the application of the simplex method in solving a specific production problem. It shows a detailed analysis of the capacity utilization of the machine highlighting each iteration and calculation. By applying the simplex method to a concrete problem involving the production of wireless and wired communication models, it is shown how to maximize profit under given constraints. Some studies [10], [11] investigate theoretical progress and improvement of algorithmic efficiency, but also without concrete practical applications in industry. Studies, e.g. [12], [13] discuss resource allocation but do not study how capacity is used in individual industries. Theoretical texts [3], [14] explore the concept of duality in linear programming, but without detailed analysis of real applications. More recent research [15] investigates technological aspects more than the application of the simplex method itself. Works like [16] propose algorithmic improvements and their application, but again without emphasizing their practical application. In contrast to the above, this paper investigates the dual problem and interprets the optimal values of the dual variables, showing their economic importance and influence on making the right decisions.

Further research should go towards a more efficient application of the simplex method, ie it should be investigated how the simplex method can be adapted for efficient computing with large data sets in real time, which requires the development of new algorithms or the improvement of existing ones. Algorithmic improvements include the development of heuristic and metaheuristic approaches, such as genetic algorithms and simulated annealing, to find near-optimal solutions in cases where the simplex method is too computationally demanding, while integration with advanced technologies, such as artificial intelligence (AI) and machine learning, would significantly improve efficiency and solving accuracy.

In the future, we should investigate the possibilities of applying linear programming in solving life problems such as those in healthcare, for example when optimizing the schedule of doctors and nurses in hospitals, in order to reduce costs and improve patient care. Analogously, the application of linear programming should be investigated in process optimization in renewable energy, waste management and other green technologies, minimizing environmental impact while maximizing efficiency and profitability.

In addition to everything known and already mentioned, there are great possibilities of adaptation and application of linear programming for the optimization of key infrastructure elements such as traffic management, smart energy distribution, improving the availability of services and security for citizens.

# 5 Conclusion

This study contributes to the existing body of knowledge by providing a clear, practical example of the simplex method's application in a real-world setting, a detailed analysis of machine capacity utilization, and a comparison with dual problem solutions. It bridges the gap between theoretical research and practical application, offering insights valuable for both academic researchers and industry practitioners. Highlighting future research directions, also paves the way for further exploration of the simplex method in conjunction with modern technological advancements.

The solved example points to the great importance of operations research, i.e., in this specific case, linear programming and the simplex method, in everyday life. Indeed, with another objective function or with different constraints, a similar problem can arise in any field. Thanks to modern technology, modeling and solving linear programming problems with a larger amount of data in a much shorter time have contributed to the significant development of operations research. In recent years, the focus in the field of linear programming has shifted from military and economic problems to general industrial areas, and social, or urban sciences, and their application continues to grow, adapting to modern technologies and societal challenges. Additionally, in the context of contemporary technological innovations, operations research includes the application of artificial intelligence, machine learning, and smart system technologies. These technologies enable process automation and faster decision-making.

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