Comparison between Brazilian Exchange-Traded Funds and Mutual Funds Performance: A multiscale approach

BRUNO MILANI[1], PAULO SERGIO CERETTA
Department of Administrative Sciences
Federal University of Santa Maria
Avenue Roraima 1000, 74C Building, University City, Santa Maria - RS
BRAZIL
[1]brunoprofess@gmail.com, [2]ceretta10@gmail.com

Abstract: The relationship between returns and risk factors are likely to vary depending on the investor's time horizon, but CAPM supposes homogenous expectations. A relatively new approach known as wavelet analysis takes may help to reduce that problem, incorporating different time scales. Taking advantage of that, this paper aims to verify the differences in performance of Brazilian ETFs and mutual funds, according to benchmark, management style and time scale. We have wavelet decomposed share returns of Brazilian ETFs, returns of the five main Brazilian mutual funds categories and the returns of the Brazilian Market proxy into 7 time scales. Then, we estimate an extended-CAPM in each time scale. We found that there are considerable performance/pricing differences between fund/ETFs categories, which are linked to the time horizon assumed.

Keywords: Mutual Funds, ETFs, Multiscale analysis, Brazilian Market

1 Introduction

Mutual funds are considered one of the most important institutions of financial markets and their performance has been the subject of extensive academic studies, especially since Jensen (1967). More recently, Exchanged Traded Funds (ETFs) have become a wide-spread investment vehicle, with unique characteristics that have not been sufficiently studied, especially when it comes to emerging markets ETFs. Also, consolidated asset pricing models are often not enough to analyze the dynamics of a kind of fund that adds a different dimension in relation to conventional investment funds: the variation of share prices.

The traditional CAPM model, developed by Sharpe (1964), Lintner (1965) and Mossin (1966) was based on the relationship between risk and return, outlined by Markowitz (1952). Jensen (1967) applied the CAPM model to the mutual fund performance evaluation, calculating how much a mutual fund variation depends on the systematic (market) variation (Beta), how much is due to manager ability (Alpha) and how much is due to idiosyncratic risk (residual). But ETFs present considerable differences from traditional mutual funds, like traded shares, so investors face the fact that its share price is different from its net asset value (NAV), an unadvised feature of this investment kind.

There are not enough studies regarding the relationship between ETF share price and NAV and their relationship with the market. A concise review of the recent developments is provided by Charupat and Miu (2012), who identify three main literature strands: (a) the ETFs pricing efficiency (how close ETFs prices are from their NAVs); (b) the ETFs performance (how successfully are they achieving their objectives, measuring the difference between NAV returns and underlying index returns); (c) the effects of ETF trading on their underlying securities.

ETFs and mutual funds of emerging markets have received even less academic attention, although they have become increasingly important for investors, due to their fast growing economies. In addition, the dynamics of the relationship between returns and risk factors are likely to vary depending on the investor's time horizon, but CAPM supposes homogenous expectations. So, the need of incorporating different time scales arises. A relatively new approach known as wavelet analysis takes may help to reduce these problems.

This paper focuses in the study of Brazilian mutual funds and ETFs performance, which will be analyzed by an extended version of the CAPM, comparing the ETFs share return and mutual funds return to the return of their benchmark. Moreover, we will adjust our analysis to time scale differences using wavelet decomposition. Therefore, this paper aims to verify the differences in performance of Brazilian ETFs and mutual funds, according to benchmark, management style and time scale.

To comply with that, we have wavelet decomposed share returns of Brazilian ETFs, returns of the five main Brazilian mutual funds categories and the returns of the Brazilian Market proxy into 7 time scales. Then, we estimate the extended-CAPM to the returns in each time scale, besides the extended-CAPM of the original return series, for comparison purposes.

Section 2 will explore theoretical issues, which includes contextualization of ETFs and mutual funds in Brazil, previous studies about funds' performance and previous studies using wavelet decomposition. Section 3 will explain our data and methodological procedures. Section 4 will explore and discuss the results and Section 5 will present concluding remarks.

2 Theoretical Issues

2.1 Exchange Traded Funds, mutual funds and the Brazilian context

Brazilian Mutual Funds are regulated by *Comissão de Valores Mobiliários* (CVM), a governmental institution. In 18/08/2004, CVM issued the Instruction nº 409, which defines the standards that mutual funds should currently obey. There are several types of mutual funds and they are generally classified by the investments they are allowed to make.

In this paper, we will explore mutual funds that invest in stocks. According to CVM, this type of fund should invest at least 67% of its NAV in stocks (spot market).In Brazil, the return of their quotes is updated daily, according to their NAVs. It is important to say that these funds quotes are not traded in stock market, but only directly in financial institutions, mostly banks.

A large part of Brazilian Mutual Funds follow a benchmark and have a specific management style. The main mutual funds benchmarks are Bovespa Index (Ibovespa) and Brasil Index (IBrX). Ibovespa aims to represent the mean return of most negotiated stocks in Bovespa (the main Brazilian stock market) according to the Negotiability Index (NI). The Ibovespa portfolio is rebalanced in each three months, and the criteria to list a stock includes being among the 85% most liquid stocks in the last three rebalancing periods, not to be a classified as a "penny stock" (a stock with value inferior to R$ 1,00) and to represent at least 0,1% of the total financial volume. The IbrX index lists the mean return of exactly of 100 more liquid stocks which are not classified as a "penny stock".

The *Associação Brasileira das Entidades dos Mercados Financeiro e de Capitais* (ANBIMA) is an auto-regulation organization, which defines categories of mutual funds, separated by benchmark and management style. Similarly to other markets, mutual funds in Brazil generally polarize their investments strategies between active and passive. The former tries to beat the market, searching for mispriced securities, and the latter tries to mimic the benchmark and reduce portfolio expenses. Differences between active and passive management in mutual funds were widely studied by academics. The categories of mutual funds analyzed in this study will follow the ANBIMA classification. We choose six main categories, although there are many others ANBIMA.

Recently, a new kind of fund has become increasingly popular in a relatively short period of time: The Exchange-Traded Funds (ETFs), which were originally created as passive investment funds with traded shares. Their main difference in relation to conventional index funds is that, similarly to individual stocks, ETFs shares can be bought and sold throughout the trading day in an exchange market. Over the past twenty years, the number of ETFs has grown from zero to over 2000 in the United States, holding assets of more than US$ 1.000 billion under management (Blackrock, 2010). Studies that have examined the performance of ETFs that mimic U.S. equity indexes conclude that ETF performance is predictable to a high degree of accuracy, generally managing to stay close to their benchmark indexes with low levels of tracking error.

Brazilian ETFs were created in January 2002 by the instruction nº 359 of (CVM). As international ETFs, they should track a reference index, commonly the Ibovespa Index, which represents Brazilian market. But differently than the other countries ETFs, they don't pay dividends to shareholders, reinvesting the stocks dividends in their portfolios.

The instruction nº 359 of CVM determines that at least 95% of an ETF equity should be invested in assets traded in the stock market or other assets authorized by the CVM, in the proportion they integrate the fund reference index, or invested in index futures. This way, the ETF is assured to reflect its reference index variation. The remaining 5% of the fund equity can be invested in government bonds, fixed income bank investments, fixed income mutual funds, commitment transactions and derivatives (exclusively for risk management of the fund portfolio).

In the Brazilian market, the ETFs are one of the few investment fund kinds that can trade shares at a stock market, unlike more developed markets where this possibility is available to many kinds of investment funds. Funds with traded shares puzzle investors in the sense that their total share prices may represent a different value of their underlying fundamentals, i.e., their NAVs. The difference between share prices and their NAVs is called discount and some studies such as in Berk and Stanton (2007) point out to the discount persistence. But the discount and its persistence are not very well explained by current literature and this kind of funds challenge conventional models of asset pricing. Section 3 presents a brief review of the late studies on this subject.

2.2 Previous studies about mutual funds and ETFs Performance

Traditional mutual funds have been widely studied by academics, especially regarding its performance and pricing estimated by the CAPM and its extensions. The ETFs, however, still require more attention.

Blitz, Huij and Swinkles (2012) studied the European index funds and exchange-traded funds in the period of January 2003 to December 2008. Their results show that most funds underperform their benchmarks, but there are considerable differences in performance between funds. Also, the expense ratios are an important determinant of relative fund performance, although there is other important factor that could explain this underperformance: the dividend yield.

Shin and Soydemir (2010) test the performance of ETF using Jensen (1967) CAPM. Besides, they test the dependence of discounts on their historical price movements by employing the serial correlation test and runs test and by observing how each market reacts to discrepancy. Their findings pointed out that there are significant tracking errors between ETF performance and their benchmark. It also finds that Asian ETFs appear to be noisier and more prone to momentum trading, meaning that an active management would be more appropriate for Asian markets than for the U.S. markets.

The CAPM is also used by Garg e Singh (2013) to compare Indian index mutual funds and ETFs, arguing that they are competing products. They analyzed five pairs of comparable ETFs and index funds in the period from June 2006 to December 2009. ETFs presented better performance, which can be due to its lower tracking errors (based on NAV) and effectiveness in long-term. Although the ETFs presented higher tracking errors (based on price) than the index funds in a daily basis, they scored well over the index funds in long-term performance.

Blitz and Huij (2012) evaluate the performance of several global emerging markets (GEM) equity exchange traded funds with a sample period from 2003 to 2010. GEM funds exhibit higher levels of tracking errors than ETFs from developed markets. Specially, ETFs that rely on statistical replication techniques are prone to high levels of tracking error, and particularly during periods of high return dispersion. The GEM ETFs underperformance in relation to their benchmark is similar to that of developed markets ETFs and can be explained mainly by expense ratios and the impact of withholding taxes on dividends.

Hughen and Matthew (2009) compared the price transmission dynamics between closed-end country funds and Exchange-traded funds using a sample of funds that invest in foreign securities. With a sample period of March 31, 2000 to March 31, 2001, a vector autoregression model (VAR) is estimated. The analysis shows that ETFs returns are more closely related to their portfolio returns than CEFs are. Innovations in the NAV explain 78% of the 5-day-ahead forecast error variance for ETF share prices but only 54% of the forecast error variance for CEFs.

Huang and Lin (2011) verified if ETFs provide international diversification, creating different regional optimal portfolios containing ETFs, in order to compare direct and indirect investments. They analyzed 19 ETFs traded in NYSE Arca, which covers European, American, Asian and African markets, in the period from 2 June 2003 to 31 March 2009. Three performance measures were used: the Sharpe Index, VAR and Mean-Value at Risk (Mean-VAR). They found no performance differences between direct and indirect investments, what means that investors who invest in foreign markets in ETFs will have no performance difference from those who invest in a more direct way. They also found that diversification benefits are the same before and after the subprime crisis.

Kuok and Chu (2010) examine the short and long term price level linkages between the equity funds under the Hong Kong Mandatory Provident Fund scheme and the benchmark indexes designed by the Hong Kong investment fund association over the period 2001-2008. They use a cointegration test to identify if there is a long run relationship between the price levels and the stock market index and the Granger causality test to analyze short run. According to their findings, 56.43% of the equity funds have their price cointegrated with a stock market index. Granger test pointed that the price level of some funds have both long and short run comovements with the stock market, but other funds that have short run comovements don't present this feature in the long run. This may indicate that some fund managers are designing their portfolios trying to win the market.

Jiang et al. (2010) analyze the first Chinese ETF, the SSE 50, showing that the fund price and NAV are cointegrated, and there is unidirectional causality from price to NAV. The fund is priced closely to its NAV with occasional short excursions away, particularly during the second semester of 2007, when the Chinese market experienced substantial volatilities, reflecting sudden increased market risks as a potential opportunity for arbitrage during financial instability.

The Brazilian Exchange Traded-Funds were studied by Milani and Ceretta (2014), which estimate the Dynamic Conditional Correlation (DCC) between share returns and the Ibovespa, besides the DCC between NAVs returns and the Ibovespa. Their results pointed that the first correlation was higher than the second, which implies that an investor who buys an ETF share is more exposed to systematic risk than the own ETF NAV is. Notwithstanding that the ETF portfolio seek to mimic the market index, their shares may work at a different dynamics than its portfolio. Also, they are allowed to operate derivatives, what contributes to detach NAV variation of a spot market index.

2.3 Previous studies exploring multiscale analysis in financial time series

Gençay et al. (2005) proposed the multiscale measurement of systematic risk, decomposing the traditional Beta into wavelets. The excess log-return of US, UK and Germany markets were individually analyzed with a different range of time for each one, but all of them with daily data. Their results showed that the higher the scale, the stronger the relationship between portfolio return and its beta, which means that the beta was higher at low frequencies (64-128 days dynamics).

Fernandez (2006) formulates a time-scale decomposition of an international version of CAPM that accounts for both market and exchange-rate risk, considering stock indexes of seven emerging countries of Latin America and Asia, for the sample period of 1990-2004. With daily data of the MSCI world index and the MSCI emerging markets index, two approaches are analyzed: the first consists in decomposing each index and recomposing its crystals by DWT and then estimate an OLS regression. The second approach is based on wavelet-variance analysis, which determines estimates for the slopes and the goodness of fit of the model (R^2) by the MODWT variance and covariance formulas. Both methods were used to estimate Beta. The results depended on which world index was used, although the emerging markets appear to depend more on the other emerging markets than the developed ones.

Cifter e Özün (2007) decomposed the variance and returns of 10 stocks of ISE-30 by the MODWT method, and then estimated a CAPM model to six scales. Their results showed that the return-risk maximization of the portfolio with these 10 stocks may be achieved at the scale of 32 days and the risk will be higher in the portfolios established at the scales different than 32 days. Rhaeim et al. (2007) estimated the systematic risk at different scales in the French stock market, with a sample composed of twenty-six actively traded stocks over 2002-2005 periods. Individual stocks and market returns were decomposed into 6 scales. Thus, Beta was estimated by OLS regression. The relationship between excess return and market portfolio becomes stronger at higher scales because beta increases as the scale increases.

Rua and Nunes (2012) illustrated the use of wavelets method assessing the risk of an investor in emerging markets over the last twenty years, using the monthly percentage returns of Morgan Stanley Capital International (MSCI), all country world index and the MSCI emerging markets index, expressed in US Dollars. Using the variance as a measure of total risk, the wavelet spectrum analysis shows that the volatility of monthly stock returns is concentrated at high frequencies, which means that short-term fluctuations dictate the variance of the series. In fact, frequencies associated with movements longer than one year are almost negligible in terms of contributions to total variance. They identify changes in variance across different time-scales in each country, which are clearly linked to well-documented crisis, although there is no evidence of an upward or downward trend in the volatility of emerging countries.

The overall beta of emerging countries is 1.17, seeming to be more stable over time at low frequencies and more time-varying at high frequencies. At high frequencies, one can identify regions in the time-frequency space where the beta is near 3. Given that, their conclusions oppose others like Gençay et al. (2005), Fernandez (2006) and Rhaeim et al. (2007). However, the periods where the beta is high include several crises, which mean that if the crises effects were controlled, these results could not hold.

Counterpointing results are also found by Masih et al. (2010), who estimates beta at different time scales in the context of the emerging Gulf Cooperation Council (GCC) equity markets by applying wavelet analysis, finding a multiscale tendency. They analyzed companies of the Saudi stock market (88), Muscat Securities Market (114), Kuwait stock exchange (189), Bahrain stock exchange (43), Doha securities market (38), Abu Dhabi securities market (61) and Dubai financial market (46), in different time ranges, comprising February 2007 to April 2008, with daily data. Each return series is separated into its components multiresolution (multihorizon) constituents using orthogonal Haar wavelet transformation. Then, an OLS estimation is ran to each stock and for each frequency, generating several multiscale Betas. They found that Beta and its variability increase between lowest and highest scale, which makes long-term investors more exposed to systematic risk than short-term investors. Also, R^2 decreases when moving to higher scales (longer interval), which means that market return is more able to explain individual stock return at higher frequencies, similarly as the study of Rua and Nunes (2012).

Additionally, Rua and Nunes (2012) also computed the wavelet of R^2 as a multiplication of the country's conditional Beta by the wavelet of market return divided by the country return, analogously to the traditional R^2. This is due to the importance of the systematic risk in explaining total risk, since the overall value of R^2 was near 0.5, but changing considerably over time and frequencies. In low frequencies, 80% of total variance is explained by the systematic risk, but in high frequencies, only 30%.

Deo and Shah (2012) applied the multiscale Beta estimation approach based on wavelet analysis to all stocks comprising BSE-Sensex, using the wavelet decomposition from the maximal overlap discrete wavelet transform (MODWT). With daily data from the BSE-30 (a representative index of the thirty biggest companies of the Indian stock market) from 5 January 2010 to 31st march 2012 (562 observations), they separate out each return series into its constituent multi-resolution (multi-horizon) components. The MODWT was chosen because giving up orthogonality, they gain attributes that are more desirable in economic applications, as the possibility to handle data of every length, not just powers of two; it is translation invariant – that is, a shift in the time series results in an equivalent shift in the transform; it has increased resolution at lower scales since it oversamples data; the choice of a particular wavelet filter is not so crucial; it is slightly affected by the arrival of new information. To each scale of stock return series, two equations are estimated by the OLS method, one with the conventional Beta and other with two coefficients analogous to Beta, one associated to a short periodicity series and the other to a long-periodicity series of market returns.The market index is also decomposed and the Beta coefficient estimated in each level. Beta coefficients were significant in all cases but, they observed that the R^2 is higher at lower scales, implying that major part of market portfolio influence on individual stocks is between medium to higher frequencies. If market risk is concentrated at the medium and higher frequencies, the model predictions would be more relevant at medium to long-run horizons as compared to short time horizons.

Conlon et al. (2008) explored multiscale analysis for Hedge Funds, due to their wide acceptance by institutional investors because their seemingly low correlation with traditional investments and attractive returns. The Hedge Funds correlation and market risk scaling properties are analyzed by the MODWT, with monthly data from April 1994 to October 2006, tracking over 4500 funds holding at least US$ 50 million under management. They found that both correlation and market risk level with respect to S&P500 varies greatly according to the strategy and time scale examined. The correlation between Convertible Arbitrage, Fixed Income and Multi-strategy, besides the S&P500 and the Hedge Fund Composite Index was found to increase as the time scale increases. But the correlation between Dedicated Shorted Bias, Equity Market Neutral, Global Macro and Managed Futures strategies correlation with S&P500 and the Hedge Fund Composite Index was found to decrease as the time scale increases. Also, the market risk level held by different Hedge Funds strategies varies according to the time horizon studied. The level of market risk of convertible Arbitrage, Emerging Markets, Event-Driven and Long/Short Equity was found to increase as the time scale increased. The market risk of Dedicated Short Bias, Global Macro and

Managed Futures was found to decrease as the time scale increased.

Milani and Ceretta (2014b) used wavelet decomposition to verify the differences in scale of the risk pricing in emerging markets, based on international CAPM model. They verified a Beta tendency to increase at lower frequencies, as well as the model goodness-of-fit (R^2). Their results were consistent with Rua and Nunes (2012) in the sense that the emerging market dependency to the world market is higher at large scales.

Thus, in general, there is certain consensus among the studies, in the sense that betas are higher at low frequencies (large scales), pointing that an asset (or a fund) dependency on the market is stronger and easily verified in the long-run analysis. R^2 are also higher at low frequencies, showing that the market return is more able to explain a stock return in the long run, which may be due to a high degree of speculative behavior at the short-run.

2.4 About Wavelets

Stock Market participants are a diverse group, which operate in different time scales, associated with different time horizons. However, most previous studies focus on only two scales: short-run and long-run. This has happened mainly because of the lack of an empirical tool. Recently, wavelet analysis has attracted attention as a mean to fill this gap (In and Kim, 2014).

Wavelets are small “waves” that grow and decay in a limited time period. The wavelets transforms decompose a time series in terms of some elementary functions, called the daughter wavelets or, simply, the wavelets ($\psi_{\tau,s}(t)$). These wavelets are new time series resulting from a mother wavelet $\psi(t)$ that can be expressed as a function of the time position τ (translation parameter) and the scale s (dilatation parameter), which is related to the frequency.

Wavelets are similar to sine and cosine functions because they oscillate around zero, but differ because they are localized both in the time and frequency domains. In contrast to Fourier analysis, wavelets are compactly supported, because all projections of a signal onto the wavelet space are essentially local, not global, and thus it doesn’t need to be homogeneous over time. In fact, wavelet analysis can be seen as a refinement of Fourier analysis.

Wavelets are flexible in handling a variety of non-stationary signals, considering the non-stationarity as an intrinsic property of the data rather than a problem to be solved. Basic wavelets are characterized into father and mother wavelets. A father wavelet (scaling function) represents the smooth baseline trend, while the mother wavelets (wavelet function) are used to describe all deviations from trends. Formulations (1) and (2), respectively represents the father and mother wavelets.

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}\phi(2^j x - k). \quad (1)$$

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k). \quad (2)$$

Where $j, k \in \mathbb{Z}$, for some coarse scale j_0, that will be taken as zero. j=1, in a j-level decomposition. The father wavelet integrates to one and reconstructs the trend component (longest time scale component) of the series. The mother wavelets integrate to zero and describe all deviations from the trend. In order to compute the decomposition, wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets need to be calculated first. They are $D_{j,k}$ (smooth; mother wavelet) and $S_{j,k}$ (detailed; father wavelet), as expressed by the formulation (3), that generates an orthonormal system. For any function f that belongs to this system we may write, uniquely:

$$f(x) = \sum_k S_{0,k}\,\phi_{0,k}(x) + \sum_{j\geq 0}\sum_k D_{j,k}\psi_{j,k}(x). \quad (3)$$

In (3), $S_{0,k} = \int f(x)\phi_{0,k}dx$ and $D_{j,k} = \int f(x)\psi_{j,k}\,dx$ are the Smooth and Detail component wavelet coefficients. We could also understand that $f(x)$ is reconstructed, containing the separate components of the original series at each frequency j. After we decompose the function $f(x)$ into j crystals, the crystals d_j are recomposed into a time domain. Formulation (3), thus, represents the entire function $f(x)$, where $\sum_k D_{j,k}\psi_{j,k}(x)$ is the recomposed series in the time domain from the crystal d_j and $\sum_k S_{0,k}\,\phi_{0,k}(x)$ is the recomposition of the residue. In this sense, $\sum_k D_{j,k}\psi_{j,k}(x)$ represents the contribution of frequency j to the original series.

Considering a time series $f(t)$ that we want to decompose into various wavelet scales. Given the father wavelet, such that its dilates and translates constitute an orthonormal basis for all subspaces that are scaled versions of the initial subspace, we can form a Multiresolution Analysis for $f(t)$. The wavelet function in formulation (3) depends on two parameters, scale and time: the scale or dilation factor j controls the length of the wavelet, while the translation or location parameter k refers to the location and indicates the non-zero portion of each wavelet basis vector.

The Discrete Wavelet Transform (DWT) is the usual approach for this multiresolution analysis, but it is restricted to sample sizes to a power of 2, i.e., for j levels we must have a sample of size 2^j. In order to overcome this difficulties, in this study we adopt the Overlap Discrete Wavelet Transform (MODWT), which can handle data of any length, not just powers of two; it is translation invariant, i.e., a shift in the time series results in an equivalent shift in the transform; it has also increased resolution at lower scales since it oversamples the data; the choice of a particular filter is not so crucial if MODWT is used and it isn't affected by the arrival of new information, except for the last few coefficients.

Differently from DWT, MODWT is a highly redundant linear filter that transforms a series into coefficients related to variations over a set of scales (Gençay et al. 2001). This way, giving up of orthogonality, MODWT gains attributes that are more desirable in economic applications.

2.5 Higher moments

We intend to use and extended version of CAPM, which incorporates co-skewness and co-kurtosis. If we cannot expect a perfectly normal distribution, the effect of skewness and kurtosis should be considered. Many authors worked in the construction of this model, as Kraus and Litzenberger (1976), Ang and Chua (1979) which included the co-skewness; and Fang and Lai (1997) and Chunhachinda et al. (1997) which included the co-kurtosis. The extended CAPM, which includes co-skewness and co-kurtosis, can be described by Equation (4):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + \gamma_i(r_{M,t} - r_{f,t})^2 + \delta_i(r_{M,t} - r_{f,t})^3 + \varepsilon_{i,t}. \quad (4)$$

Where $r_{i,t}$ is the return of portfolio i; $r_{f,t}$ is the risk-free asset return; $r_{M,t}$ is the market Proxy return; α_i is the linear coefficient; β_i is the co-variance coefficient; γ_i is the co-skewness coefficient; δ_i is the co-kurtosis coefficient; $\varepsilon_{i,t}$ is the error term.

3 Data and Methodology

We used daily share returns of Brazilian ETFs and daily returns of the five main categories of Brazilian mutual funds. Our sample period is from 03/11/2011 to 30/10/2013, with 713 observations. We did not use data previous to 03/11/2011 because there were few ETFs in Brazil during that period and we did not use data posterior to 30/10/2013 because we did not have access to it. So, the sample period was chosen by the availability criteria and the data concerning ETFs and mutual funds was obtained with ANBIMA. The Ibovespa was used as a market proxy and it was obtained from the website of BOVESPA, the main Brazilian Stock Market.

To reduce the amount of analyzed data, we worked with the mean return of each fund category, weighted by its NAV. In an attempt to ease the heteroscedasticity problems, we used the squared market return as an independent variable in our models. In addition, we also use an autoregressive term in each regression, to control for the problems related to autocorrelation. This way, our paper uses the series presented in Table 1.

Table 1. Analyzed return series, representing different categories of mutual funds

Name	Description
Active Ibovespa (IA)	Weighted mean return of actively managed funds whose benchmark is Ibovespa.
Passive Ibovespa (IP)	Weighted mean return of passively managed funds whose benchmark is Ibovespa.
Active IBrX (XA)	Weighted mean return of actively managed funds whose benchmark is IBrX.
Passive IBrX (XP)	Weighted mean return of passively managed funds whose benchmark is IBrX.
Free Funds (F)	Weighted mean return of funds without a defined benchmark or kind of management.
ETFs (E)	Weighted mean return of Exchange-Traded Funds. Not divided into categories of management and benchmark, due to its reduced number.
IBovespa (M)	Return of the Bovespa index, which represents the Brazilian

	Market, i.e., our market proxy.

We will begin our analysis with the summary statistics of the series presented in Table 1. Considering that the summary statistics already calculates the series mean and standard deviation, we will present the Sharpe Ratio (1966) with them. After, we will estimate the CAPM model with higher moments, via OLS regressions, as can be shown by Equation (5).

$$r_{i,t} = \alpha_i + \beta_1 r_{m,t} + \beta_2 r_{m,t}^2 + \beta_3 r_{m,t}^3 + \varepsilon_{i,t}. \quad (5)$$

Where $r_{i,t}$ is the weighted average return of each Fund Category at time *t*; $r_{m,t}$ is the market return at time *t*; $\varepsilon_{i,t}$ is the error term of the regression of each Fund Category at time *t*; α_i is the linear coefficient for each Fund Category. β_1, β_2 and β_3 are parameters.

The estimation of Equation (5) is important for comparison purposes, since we will estimate a similar equation hereafter to the series divided into time scales.

After the estimation of Equation (5) for each return categories specified in Table 1, as well as the Ibovespa volatility series (squared Ibovespa return) was decomposed into wavelets by the MODWT transform, generating a multistage decomposition of the series at seven different scale crystals (*j*) as follows:D1 (2-4 days); D2 (4-8 days); D3 (8-16 days); D4 (16-32 days); D5 (32-64 days); D6 (64-128 days); D7 (128-256 days).

After the series decomposition, we estimate other OLS regressions explaining Funds returns by Market returns for each time scale, analogously to the CAPM and to Equation (4). So, Equation (5) was estimated for each scale *j* of each Fund Category *i*.

$$r_{i,t}(\tau_j) = \alpha_i(\tau_j) + \beta_1 r_{m,t}(\tau_j) + \beta_2 r_{m,t}^2(\tau_j) + \beta_3 r_{m,t}^3(\tau_J) + \varepsilon_{i,t}(\tau_j). \quad (6)$$

Where $r_{i,t}(\tau_j)$ is the weighted average return of each Fund Category wavelet *j* at time *t*; $r_{m,t}(\tau_j)$is the market return wavelet *j* at time *t*; $\varepsilon_{i,t}(\tau_j)$ is the error term of the regression of each Fund Category at time *t* in each time scale *j*; $\alpha_i(\tau_j)$ is the linear coefficient for each Fund Category in each time scale; β_1, β_2 and $\beta_3$are parameters.

Considering the large data quantity and the deriving difficulty to analyze it, we chose to disregard the intermediates D2, D4 and D6 crystals, allowing us to focus on D1, D3, D5 and D7 crystals. This way, we were able to verify the scale differences on a larger frequency range, reducing the outputs and making it easier to interpret them.

To comply with our objective, we will discuss the differences between estimated coefficients of each funds category of each time scale. Also, considering that the tracking error is an important feature of performance analysis, we will analyze the adjusted R^2 coefficient, which is calculated based on the regression error. Section 4 will present and discuss the results obtained.

4 Results

Before the CAPM estimations, we analyzed the summary statistics of each series defined previously in Table 1, as presented in Table 2. We have also calculated the Sharpe Index as the ratio between the mean and the standard deviation.

Table 2. Summary Statistics and Sharpe Index for the six categories of Brazilian Funds and Ibovespa Index

	Active Ibovespa	Passive Ibovespa	Active IBrX	Passive IBrX	Free	ETFs	Ibovespa index
Mean	-0.0060	-0.0220	-0.0080	-0.0060	0.0140	-0.0210	-0.0340
Median	0.0200	-0.0450	0.0220	0.0150	0.0360	-0.0110	-0.0460
Minimum	-7.8600	-7.9800	-7.7090	-7.8740	-6.3870	-8.2150	-8.4310
Maximum	4.1170	4.8780	4.5910	4.6860	3.4950	4.8920	4.9750
Stdev	1.0460	1.3930	1.1670	1.1810	0.8680	1.2910	1.4270
Skewness	-0.7030	-0.2040	-0.4300	-0.4190	-0.5830	-0.3410	-0.2610
Kurtosis	5.5460	1.9900	3.5060	3.5730	5.6910	2.8670	2.1510
Sharpe	-0.0057	-0.0158	-0.0069	-0.0051	0.0161	-0.0163	-0.0238

Table 2 shows that ETFs presented the worst Sharpe Index of the funds sample, followed by passively managed funds whose benchmark is Ibovespa. Only the free funds presented positive mean and consequently positive Sharpe Index, even though all types of funds presented better results than the market proxy. Besides the fact that Free Funds presented the higher average return, they also presented the smallest standard deviation, letting no doubt that they were the best investments in the period, according to the Sharpe Index analysis. ETFs presented the second higher standard deviation, after passively managed funds whose benchmark is Ibovespa, but the market standard deviation was higher than all the funds standard deviation.

The maximum and minimum returns pointed that the amplitude of Free Funds is the smallest, while the amplitude of ETFs returns is the largest, followed by the amplitude of passively managed funds whose benchmark is Ibovespa. The amplitude, i.e., distance from the maximum and minimum point is an indication of risk. This confirms the analysis that the best performance, in this sample, is due to the Free Funds and the worst is due to ETFs, although they still had better performance than the market proxy.

All types of funds presented negative skewness, what means that there is a large probability of extreme negative values than extreme positive values. All funds presented high kurtosis coefficients, a natural feature of financial time series that indicates that values distant from the average are very common. The Free funds presented the second smallest skewness coefficient but the largest kurtosis coefficient, showing that they are exposed to risks related to the fourth moment, which does not increase the probability of extreme negative values.

After the summary statistics presentation, we continue the analysis estimating Equation (5) for each fund type. The results can be shown in Table 3.

Table 3. Equation (5) coefficients

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
Active Ibovespa	α	0.0326	2.9499	**0.0033**	0.9442
	$\beta_{1,AI}$	0.6740	81.4610	**0.0000**	
	$\beta_{2,AI}$	-0.0063	-2.1731	**0.0301**	
	$\beta_{3,AI}$	0.0031	5.8646	**0.0000**	
Passive Ibovespa	α	0.0056	1.6249	0.1046	0.9969
	$\beta_{1,PI}$	0.9768	375.3504	**0.0000**	
	$\beta_{2,PI}$	0.0027	2.9349	**0.0034**	
	$\beta_{3,PI}$	-0.0001	-0.3321	0.7399	
Active IBrX	α	0.0265	1.9774	**0.0484**	0.9336
	$\beta_{1,AX}$	0.7670	76.2172	**0.0000**	
	$\beta_{2,AX}$	-0.0034	-0.9603	0.3372	
	$\beta_{3,AX}$	0.0020	3.0734	**0.0022**	
Passive IBrX	α	0.0258	1.9056	0.0571	0.9341
	$\beta_{1,PX}$	0.7752	76.3491	**0.0000**	
	$\beta_{2,PX}$	-0.0019	-0.5348	0.5929	
	$\beta_{3,PX}$	0.0022	3.3800	**0.0008**	
Free	α	0.0445	3.6122	**0.0003**	0.8992
	$\beta_{1,F}$	0.5417	58.7131	**0.0000**	

	$\beta_{2,F}$	-0.0046	-1.4143	0.1577	
	$\beta_{3,F}$	0.0029	4.8960	**0.0000**	
ETFs	α	0.0138	1.2516	0.2112	0.9636
	$\beta_{1,E}$	0.8767	106.2721	**0.0000**	
	$\beta_{2,E}$	-0.0018	-0.6066	0.5443	
	$\beta_{3,IP}$	0.0010	1.9510	0.0515	

As it is expected, β_2 is larger for passive funds than for active funds. The coefficient of β_2 is analogous to the CAPM Beta, so it is normal that passive funds present larger Beta than active funds, since the first deliberately tries to replicate a benchmark. However, if that is the purpose of passive funds, $\beta_{2,PX}$ presents a small coefficient (0,7752). One possible explanation is due to the fact that we use the Ibovespa index as our market proxy, while this category uses IBrX as a benchmark. Nevertheless, this argument loses part of its consistency when we analyze the actively managed funds, which β_2 coefficients are larger for those whose benchmark is IBrX than for those whose benchmark is Ibovespa.

We can perceive that the free funds have the smallest Beta coefficient among all fund categories, another feature that differentiates them from the others. ETFs have high Beta coefficient, what can be explained by the fact that many ETFs were created with the promise of representing a portion of a market index, although this passive behavior is currently not verified in all ETFs. The fact that ETFs have traded shares also contributes to make their returns more dependent of market returns. We can perceive that the two categories of funds with higher Beta are exactly the same two categories with smallest Sharpe Index (1966). This can be, at least partially, explained by the effect of Euro Zone Debt Crises in the sample, what made the more exposed to systematic risk funds exactly the ones with worse performance.

ETFs and passive funds whose benchmark is Ibovespa also present higher adjusted R^2 coefficients, confirming that their returns are better explained by market returns than the other funds. Free funds present the smallest adjusted R^2 coefficient, another feature indicating that their returns are not so well explained by market returns.

Active and Free funds have significant positive linear coefficients, what means that the fund managers can generate part of the fund return without systematic risk exposition, according to Jensen (1967) interpretation. So, if Jensen's Alfa is considered a performance measure, Free funds have the best performance, followed by active funds whose benchmark is Ibovespa.

Funds whose benchmark is Ibovespa presented significant β_2 coefficients, although it is negative for actively and positive for passively managed funds. All funds, except Passive funds whose benchmark is Ibovespa and ETFs presented significant positive β_3 coefficient, evidencing that bad performance may be associated with the non-significance of co-kurtosis coefficient. The co-kurtosis is often associated with the volatility concept, but we can see that the fund categories with no significant co-kurtosis coefficient presented the higher standard deviations. All regressions present significant F-test, showing that they are globally significant.

After we explore the coefficients obtained from Equation (5), based on the original time series, we proceeded the performance analysis in different time scales, as specified by Equation (6). Table 4 presents the estimated coefficient of the Estimation of Equation (6) to each time scale of returns from active funds whose benchmark is Ibovespa.

Table 4 – Estimated coefficients of Equation (6) for active funds whose benchmark is Ibovespa, in different time scales

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
D1 (2-4 days)					

Active Ibovespa (τ_1)	$\alpha(\tau_1)$	0.0000	0.0000	1.0000	0.9520
	$\beta_{1,AI}(\tau_1)$	0.6540	89.3950	**0.0000**	
	$\beta_{2,AI}(\tau_1)$	-0.0098	-0.9819	0.3265	
	$\beta_{3,AI}(\tau_1)$	0.0035	9.5950	**0.0000**	
D3 (8-16 days)					
Active Ibovespa (τ_3)	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9422
	$\beta_{1,AI}(\tau_3)$	0.7084	79.2122	**0.0000**	
	$\beta_{2,AI}(\tau_3)$	-0.0306	-1.9214	0.0551	
	$\beta_{3,AI}(\tau_3)$	0.0049	10.2723	**0.0000**	
D5 (32-64 days)					
Active Ibovespa (τ_5)	$\alpha(\tau_5)$	0.0000	-0.0000	1.0000	0.9458
	$\beta_{1,AI}(\tau_5)$	0.6863	78.1228	**0.0000**	
	$\beta_{2,AI}(\tau_5)$	-0.0153	-1.3135	0.1895	
	$\beta_{3,AI}(\tau_5)$	0.0075	16.8325	**0.0000**	
D7 (128-256 days)					
Active Ibovespa (τ_7)	$\alpha(\tau_7)$	0.0000	-0.0000	1.0000	0.9728
	$\beta_{1,AI}(\tau_7)$	0.6801	96.4210	**0.0000**	
	$\beta_{2,AI}(\tau_7)$	-0.0433	-8.3443	**0.0000**	
	$\beta_{3,AI}(\tau_7)$	0.0009	2.8058	**0.0052**	

In Table 4 we can see that the β_2 coefficient, analogous to the CAPM Beta, is not much different among the scales, although it is a little higher in the D3 crystal. The β_3 coefficient is significant only in the largest scale (D7 crystal), negatively. The β_4 Coefficient significant in all scales, but higher in the middle ones (D3 and D5 crystals).

So, we can perceive that depending on the scale, the main determinant of the fund returns may change. In the small scale (D1), the market return is the main influence on funds returns; in the middle scales (D3 and D5), co-kurtosis becomes also important and the influence of market returns is higher than in the small scale; in the large scale (D7), the co-skewness coefficient increases while co-kurtosis decreases its importance. Differently of what we concluded in the analysis of Table 2, co-skewness affects the fund returns only in the long run, and not homogeneously.

Table 5 brings the estimated coefficient of Equation (6) for passive funds whose benchmark is Ibovespa.

Table 5. Estimated coefficients of Equation (6) for passive funds whose benchmark is IBovespa, in different time scales

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
D1 (2-4 days)					
Passive Ibovespa (τ_1)	$\alpha(\tau_1)$	0.0000	-0.0000	1.0000	0.9967
	$\beta_{1,PI}(\tau_1)$	0.9764	373.6779	**0.0000**	
	$\beta_{2,PI}(\tau_1)$	0.0025	0.7039	0.4817	
	$\beta_{3,PI}(\tau_1)$	-0.0004	-2.9903	**0.0029**	

		D3 (8-16 days)			
Passive Ibovespa (τ_3)	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9972
	$\beta_{1,PI}(\tau_3)$	0.9877	401.8459	**0.0000**	
	$\beta_{2,PI}(\tau_3)$	0.0084	1.9185	0.0555	
	$\beta_{3,PI}(\tau_3)$	-0.0007	-5.0964	**0.0000**	
		D5 (32-64 days)			
Passive Ibovespa (τ_5)	$\alpha(\tau_5)$	0.0000	-0.0000	1.0000	0.9969
	$\beta_{1,PI}(\tau_5)$	0.9927	386.2909	**0.0000**	
	$\beta_{2,PI}(\tau_5)$	0.0053	1.5475	0.1222	
	$\beta_{3,PI}(\tau_5)$	-0.0007	-5.7355	**0.0000**	
		D7 (128-256 days)			
Passive Ibovespa (τ_7)	$\alpha(\tau_7)$	0.0000	0.0000	1.0000	0.9976
	$\beta_{1,PI}(\tau_7)$	0.9973	324.0633	**0.0000**	
	$\beta_{2,PI}(\tau_7)$	0.0120	5.2932	**0.0000**	
	$\beta_{3,PI}(\tau_7)$	-0.0015	-10.1073	**0.0000**	

The β_2 coefficient is very high in all scales of this fund's category, what is natural because passive funds are designed to follow the market. Also, β_2 is higher at large scales. Similarly of the results shown in Table 4, the co-skewness coefficient is significant only in the large scale (D7 crystal), but in this case it is positive. Again, we can perceive that it would be naïve to disregard the scale differences, because Equation (5) was taking us to believe that co-skewness coefficient was significant for all passive funds whose benchmark is Ibovespa, homogeneously.

All β_3 coefficients are significant, negatively. That is another difference of active funds whose benchmark is Ibovespa and may be related with investors profile differences. Actively managed funds take advantage of co-kurtosis, but it has a negative effect in passively managed. Is important to remember that Active Ibovespa Funds present the second best performance, but passive Ibovespa funds presented the second worse, so a positive co-kurtosis coefficient can have a connection with a better performance.

The Table 6 will present the estimated coefficients of Equation (6) for active funds whose benchmark is Ibovespa.

Table 6. Estimated coefficients of Equation (6) for active funds whose benchmark is IBrX, in different time scales

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
		D1 (2-4 days)			
Active IBrX (τ_1)	$\alpha(\tau_1)$	0.0000	-0.0000	1.0000	0.9393
	$\beta_{1,AX}(\tau_1)$	0.7511	80.9471	**0.0000**	
	$\beta_{2,AX}(\tau_1)$	-0.0119	-0.9411	0.3470	
	$\beta_{3,AX}(\tau_1)$	0.0025	5.3355	**0.0000**	
		D3 (8-16 days)			
Active IBrX	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9330

(τ_3)	$\beta_{1,AX}(\tau_3)$	0.8038	76.0201	**0.0000**	
	$\beta_{2,AX}(\tau_3)$	-0.0279	-1.4824	0.1387	
	$\beta_{3,AX}(\tau_3)$	0.0029	5.1430	**0.0000**	
D5 (32-64 days)					
Active IBrX (τ_5)	$\alpha(\tau_5)$	0.0000	0.0000	1.0000	0.9674
	$\beta_{1,AX}(\tau_5)$	0.7964	112.3291	**0.0000**	
	$\beta_{2,AX}(\tau_5)$	0.0336	3.5711	**0.0004**	
	$\beta_{3,AX}(\tau_5)$	0.0019	5.4608	**0.0000**	
D7 (128-256 days)					
Active IBrX (τ_7)	$\alpha(\tau_7)$	0.0000	-0.0000	1.0000	0.9803
	$\beta_{1,AX}(\tau_7)$	-4.0167	-34.8839	**0.0000**	
	$\beta_{2,AX}(\tau_7)$	1.1051	13.0365	**0.0000**	
	$\beta_{3,AX}(\tau_7)$	0.8347	151.8262	**0.0000**	

The $\beta_{1,AX}(\tau_7)$ coefficient shown in Table 6 is, at least, curious. It is negatively significant, i. e., the fund returns decrease when the market returns increase. That may be due to the use of Ibovespa as market proxy. Ibovespa and IBrX are surely heavily correlated, but in the long run funds whose benchmark is IBrX may have a negative relationship with Ibovespa, possibly because they compete for investors.

Another possible interpretation is that the co-kurtosis coefficient is more important to explain long-term returns than the market beta. The $\beta_{3,AX}(\tau_7)$ is considerably higher than the others co-kurtosis coefficients, showing that Active IBrX funds are more prone to market volatility in the long term.

Similarly to Table 4 results, the co-skewness coefficient (β_2) is significant only in the D5 and D7 crystals, but positively in this case. The co-kurtosis coefficient (β_3) is significant in all scales, but very high in the large scale (D7 crystal).

The coefficient of adjusted R^2, again, is higher in large scales. In fact, in the D7 crystal estimation, almost all variance is explained. The F-test p-value shows that the estimation is globally significant. Now, Table 7 presents the estimated coefficients of Equation (6) to passive funds whose benchmark is IBrX.

Table 7. Estimated coefficients of Equation (6) for passive funds whose benchmark is IBrX, in different time scales

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
D1 (2-4 days)					
Passive IBrX (τ_1)	$\alpha(\tau_1)$	0.0000	-0.0000	1.0000	0.9967
	$\beta_{1,PI}(\tau_1)$	0.9764	373.6779	**0.0000**	
	$\beta_{2,PI}(\tau_1)$	0.0025	0.7039	0.4817	
	$\beta_{3,PI}(\tau_1)$	-0.0004	-2.9903	**0.0029**	
D3 (8-16 days)					
Passive IBrX (τ_3)	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9307
	$\beta_{1,PI}(\tau_3)$	0.8041	74.6087	**0.0000**	
	$\beta_{2,PI}(\tau_3)$	-0.0300	-1.5631	0.1185	

	$\beta_{3,PI}(\tau_3)$	0.0029	5.1569	**0.0000**	
		D5 (32-64 days)			
Passive IBrX (τ_5)	$\alpha(\tau_5)$	0.0000	0.0000	1.0000	0.9932
	$\beta_{1,PI}(\tau_5)$	-0.0156	-6.6587	**0.0000**	
	$\beta_{2,PI}(\tau_5)$	0.9816	314.9015	**0.0000**	
	$\beta_{3,PI}(\tau_5)$	0.0005	4.5556	**0.0000**	
		D7 (128-256 days)			
Passive IBrX (τ_7)	$\alpha(\tau_7)$	0.0000	-0.0000	1.0000	0.9706
	$\beta_{1,PI}(\tau_7)$	0.7323	94.7659	**0.0000**	
	$\beta_{2,PI}(\tau_7)$	-0.0609	-10.7013	**0.0000**	
	$\beta_{3,PI}(\tau_7)$	0.0001	0.3906	0.6962	

The results of Table 7 show another case of negative β_1 coefficient, in the D5 crystal. This time, the co-skewness coefficient is considerably higher than the others, differently of the case presented in Table 6. Although the co-kurtosis coefficient is significant in the D5 crystal, it is very small. It seems that the co-variance coefficient was substituted by the co-skewness coefficient, which is more important to explain funds returns here. In the D7 crystal, the co-skewness coefficient is significant again, but the co-kurtosis is not.

Table 8 will present the estimated coefficients of Equation (6) applied to the Free funds.

Table 8. Estimated coefficients of Equation (6) for Free Funds in different time scales

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
		D1 (2-4 days)			
Free (τ_1)	$\alpha(\tau_1)$	0.0000	-0.0000	1.0000	0.9077
	$\beta_{1,F}(\tau_1)$	0.5157	61.2506	**0.0000**	
	$\beta_{2,F}(\tau_1)$	0.0015	0.1269	0.8990	
	$\beta_{3,F}(\tau_1)$	0.0038	9.1471	**0.0000**	
		D3 (8-16 days)			
Free (τ_3)	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9013
	$\beta_{1,F}(\tau_3)$	0.5767	59.9288	**0.0000**	
	$\beta_{2,F}(\tau_3)$	-0.0007	-0.0424	0.9662	
	$\beta_{3,F}(\tau_3)$	0.0034	6.6446	**0.0000**	
		D5 (32-64 days)			
Free (τ_5)	$\alpha(\tau_5)$	0.0000	-0.0000	1.0000	0.8392
	$\beta_{1,F}(\tau_5)$	-0.0363	-4.5544	**0.0000**	
	$\beta_{2,F}(\tau_5)$	0.6311	59.5855	**0.0000**	
	$\beta_{3,F}(\tau_5)$	0.0004	1.0881	0.2769	
		D7 (128-256 days)			
Free (τ_7)	$\alpha(\tau_7)$	0.0000	-0.0000	1.0000	0.9415

$\beta_{1,F}(\tau_7)$	0.5633	63.5722	**0.0000**
$\beta_{2,F}(\tau_7)$	-0.0577	-8.8529	**0.0000**
$\beta_{3,F}(\tau_7)$	0.0024	5.6781	**0.0000**

The first differences we can perceive in these results are the considerably smaller β_1 coefficients, in comparison to the other fund categories. This is in congruence with the previous analysis (Table 2) that shows that this Fund Category was less exposed to systematic risk in the period. In the D5 crystal there is, again, a negative β_1 coefficient, which is compensated by a large co-skewness coefficient. One more time, the co-skewness appeared to be more present is the larger scales (D5 and D7). Table 9 brings the analysis of estimated coefficients of Equation (6) for ETFs.

Table 9. Estimated coefficients of Equation (6) for ETFs

Dependent Variable	Parameter	Coefficients	t-value	p-value	Adjusted R^2
D1 (2-4 days)					
ETFs (τ_1)	$\alpha(\tau_1)$	0.0000	0.0000	1.0000	0.9857
	$\beta_{1,E}(\tau_1)$	-1.1828	-11.6890	**0.0000**	
	$\beta_{2,E}(\tau_1)$	-0.1174	-0.8501	0.3955	
	$\beta_{3,E}(\tau_1)$	0.9271	183.4023	**0.0000**	
D3 (8-16 days)					
ETFs (τ_3)	$\alpha(\tau_3)$	0.0000	-0.0000	1.0000	0.9610
	$\beta_{1,E}(\tau_3)$	0.8992	104.5487	**0.0000**	
	$\beta_{2,E}(\tau_3)$	-0.0645	-4.2062	**0.0000**	
	$\beta_{3,E}(\tau_3)$	0.0006	1.3164	0.1885	
D5 (32-64 days)					
ETFs (τ_5)	$\alpha(\tau_5)$	0.0000	0.0000	1.0000	0.9330
	$\beta_{1,E}(\tau_5)$	-0.0697	-9.5469	**0.0000**	
	$\beta_{2,E}(\tau_5)$	0.9400	96.9677	**0.0000**	
	$\beta_{3,E}(\tau_5)$	0.0021	5.7230	**0.0000**	
D7 (128-256 days)					
ETFs (τ_7)	$\alpha(\tau_7)$	0.0000	0.0000	1.0000	0.9892
	$\beta_{1,E}(\tau_7)$	0.8540	145.7296	**0.0000**	
	$\beta_{2,E}(\tau_7)$	-0.0196	-4.5397	**0.0000**	
	$\beta_{3,E}(\tau_7)$	0.0039	14.1094	**0.0000**	

The analysis of ETFs coefficients shows that there are negative β_1 in the D1 and the D5 crystals, where the co-kurtosis and the co-skewness coefficients, respectively, presented higher coefficients. So, the effect of higher moments in the fund's returns, one more time, appears to be larger than the market-return influence.

The co-skewness coefficient is significant in all scales, except in the smaller one; the co-kurtosis coefficient is significant in all the scales, except in the scale of D3 crystal. These results allowed us to outline some conclusions, presented in Section 5.

5 Final Considerations

The results presented in Section 4 take us to these considerations:

a) The Active Ibovespa, Passive Ibovespa and Active IBrX share a commom standard: The β_1 and the β_3 coefficients explain the fund's returns and there are not other dependent significant variables. Also, the β_1 is always smaller in the small scale.
b) The funds whose benchmark is IBrX and the Free funds present significant co-skewness coefficients, but only in the larger scales, represented by the D5 and D7 crystals.
c) Although we always expect large co-variance coefficients, there are several cases where they are small or even negative. In these cases, the co-skewness or co-kurtosis compensates the equation with higher coefficients.
d) When negative co-variance coefficients happen in the D5 crystal, they are compensated with higher co-skewness coefficients; when they happen in the D1 or D7 crystals, they are compensated with higher co-kurtosis coefficients. There was not any case of negative co-variance coefficient in the D3 crystal.
e) When the co-skewness coefficient compensates a negative co-variance coefficient, its coefficient is positive.
f) Free funds presented the larger linear coefficient. According to the Jensen (1967) CAPM these fund managers aggregate returns better than the others, so the Free funds performance is the best. As presented in Table 1, their Sharpe index was the highest, what also indicates that this fund category have the best performance. Table 8 shows that these funds have the smaller co-variance coefficient, indicating that they were less exposed to the systematic risk.
g) The second better performance can be attributed to actively managed funds whose benchmark is Ibovespa, which also presented small co-variance coefficients. Free Funds and Active Ibovespa Funds presented, respectively, the higher kurtosis coefficient (Table 1). Their co-kurtosis coefficients, presented in Table 2 for the original series and in Table 3 and Table 7 for the waveleted series, are larger than the others in most cases. These characteristics indicate that these funds are able to take advantage of a different risk, represented by co-kurtosis, and that may be linked with the fact that they present better performance.
h) Passive Ibovespa funds and ETFs presented the worse performance, according to the Sharpe Index. Also, these two fund categories were among the three that could not generate significant linear coefficients with the original series, indicating a bad performance according to Jensen (1967) interpretation. It is important to note that these were the same two categories that did not generated significant co-kurtosis coefficients, reinforcing that this feature is linked to its performance.
i) In the wavelet series regression, Passive Ibovespa Funds presented negative co-kurtosis coefficients, what reinforces the conclusions of the “h” item.
j) Active Ibovespa, Passive IBrX and Free Funds presented negative co-skewness coefficient in the larger scale (D7), while Passive Ibovespa and Active IBrX funds presented positive co-skewness in the large scale (D7). Since the first had better performance than the latter, we can infer that negative co-skewness, in the long run, can help to increase the performance.

A possible explanation for the performance of Free Funds is that these funds presented better performance because although our sample included the Eurozone debt crisis period, they were able to reduce their exposure to the market risk. This was possible because their regulation allow them not to restrain to a specific management strategy or benchmark.

References:

[1] Ang, J.S. and Chua, J.H. Composite Measures for the Evaluation of Investment Performance. *Journal of Financial and Quantitative Analysis*, v. 14, 1979, pp. 361-384.

[2] Berk, J. B and R. Stanton Managerial Ability, Compensation, and the Closed-End Fund Discount. *The Journal of Finance* v. 62, 2007, pp. 529-556.

[3] Blitz, David. Huij, Joop. Evaluating the performance of global emerging markets equity exchange-traded funds. *Emerging Markets Review*, v. 13, 2012, pp. 149-158.

[4] Blitz, David. Huij, Joop. Swinkles, Laurens. The Performance of European Index Funds and Exchange-Traded Funds. *European*

Financial Management, v. 18, n. 4, 2012, pp. 649-662.

[5] Charupat, N. and P. Miu. Recent Developments in exchange-traded fund literature: pricing efficiency, tracking ability, and effects on underlying securities. *Managerial Finance* v. 39, 2012, pp. 427-443.

[6] Chunhachinda, Pornchai. Dandapani, Krishnan. Hamid, Shahid. Prakash, Arun J. Portfolio selection and skewness: evidence from international stock markets. *Journal of Banking and Finance*, v. 21, i. 2, 1997 pp. 143-167.

[7] Cifter, A. and Özün, A. Multiscale Systematic Risk: An application on ISE-30. *MPRA Paper No*. 4288, 2007.

[8] Conlon, T., Crane, M. and Ruskin, H. Wavelet Multiscale Analysis for Hedge Funds: Scaling and Strategies. *Physica A: Statistical Mechanics and Its Applications*, v. 387, i. 21, 2008, pp. 5197-5204.

[9] Deo, M. Shah, A. Wavelet-Based Beta Estimation: Applications to Indian Stock Market. *Kuwait Chapter of Arabian Journal of Business and Management Review*, v. 1, n. 11, 2012.

[10] Fang, Hsing. Lai, Tsong-Yue. Co-kurtosis and capital asset pricing. *Financial Review*, v. 32, i. 2, 1997, pp. 293-307.

[11] Fernandez, V. The international CAPM and a wavelet-based decomposition of Value At Risk. *Studies in Non-linear Dynamics & Econometrics*, v. 9, i. 4, 2005.

[12] Gargh, Swati. Singh, Y. P. An empirical comparison of ETFs and Index funds performance in India. *International Journal of Applied Financial Management Perspectives*, v.2, i. 3, 2013.

[13] Gençay, R., Selçuk, F. and Whitcher, B. Multiscale Systematic Risk. *Journal of International Money and Finance*, v. 24, p. 55-70, 2005.

[14] Gençay, Ramazan. Selçuk, Farouk. Whitcher, Brandon J. An introduction to wavelets and other filtering methods in Finance and Economics. *Academic Press*, 2001.

[15] Huang, Mei-Yueh. Lin, Jun-Biao. Do ETFs provide effective international diversification? *Research in International Business and Finance*, v. 25, i. 3, 2011, pp. 335-344.

[16] Hughen, J. Christopher. Mathew, Prem G. The efficiency of international information flow: Evidence from the ETF and CEF prices. *International Review of Financial Analysis*, v. 18, 2009, pp. 40-49.

[17] Jensen, M. C. The performance of mutual funds in the period of 1945-1964. *Journal of Finance*, v. 23, n. 2, 1967, pp. 389-416.

[18] Jiang, Y., F. Guo and T. Lan. Pricing Efficiency of China's Exchange-traded Fund Market. *Chinese Economy* v. 43, 2010, pp. 32-49.

[19] In, Francis. Kim, SangBae. An Introduction to Wavelet Theory in Finance: A wavelet multiscale approach. *World Scientific*. Singapore, 2014.

[20] Kuok, P. and K. Chu. The price linkages between the equity fund price levels and the stock markets: evidences from cointegration approach and causality analysis of Hong Kong Mandatory Provident Fund (MPF). *International Review of Financial Analysis* v. 19, 2010, pp. 281-288.

[21] Kraus, Alan. Litzenberger, Robert H. Skewness preference and the valuation of risk assets. *The Journal of Finance*, v. 31, i. 4, 1976, pp. 1085-1100.

[22] Milani, Bruno. Ceretta, Paulo Sergio. Dynamic Correlation Between Share Returns, NAV Variation and Market Proxy of Brazilian ETFs. *Engineering Economics*, v. 25, i. 1, 2014, pp. 21-30.

[23] Milani, Bruno. Ceretta, Paulo Sergio. A Multiscale Approach to Emerging Market pricing. *Economics Bulletin*, v. 34, i. 2, 2014.

[24] Sharpe, W. F. Mutual Fund Performance. *The Journal of business*, v. 39, n. 1, 1966, pp.119-138.

[25] Shin, Sangheon; Soydemir, Gökçe. Exchange-traded funds, persistence in tracking errors and information dissemination. *Journal of Multinational Financial Management*, v. 20, 2010, pp. 214-234.

[26] Lintner, John. Security Prices, Risk and Maximal Gains from Diversification. *Journal of Finance*, v. 20, 1965, pp. 587-616.

[27] Markowitz, H. M. Portfolio selection. *Journal of Finance*, v. 7, n. 1, 1952, pp. 77-91.

[28] Masih, M., Alzahrani, M. and Al-Titi, O. Systematic Risk and time scales: New evidence from an application of wavelet approach to the emerging Gulf stock markets. *International Review of Financial Analysis*, v. 19, 2010, pp. 10-18.

[29] Mossim, Jan. Equilibrium in a Capital Asset Market. *Econometrica*, v. 34, 1966, pp. 768–783.

[30] Rhaeim, N., Ammou, S. B. and Mabrouk A. B. Wavelet estimation of systematic risk at Different Time Scales: Application to French

stock market. *The international Journal of Applied Economics and Finance*, v. 1, i. 2, 2007, pp. 113-119.

[30] Rua, A. and Nunes, L. C. A wavelet-based assessment of market-risk: The emerging markets case. *The Quarterly Review of Economics and Finance*, v. 52, 2012, pp. 84-92.