All-to-all broadcast in Augmented Optical Linear Array

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Abstract: - All-to-all broadcast communication refers to a process where each terminal node in a network sends a distinct message to every other terminal node. This intensive exchange of information is a crucial component in ultra high-performance computing and communication frameworks, finding applications in areas such as network control mechanisms and data center operations. In telecommunication networks, it is critical for efficient resource management, as each terminal node must collect information about all others. Optical networks using Wavelength Division Multiplexing (WDM) form the backbone of these systems, where efficient utilization of wavelengths is essential to minimize costs and complexity. The optical linear array topology is commonly employed in Local Area Networks, Wide Area Networks, and Metropolitan Area Networks. To optimize the performance in such networks, modifications of the network topology are necessary to minimize both wavelength requirement and hop count. In this study, a linear array network with T terminal nodes is augmented by directly interconnecting terminal nodes with an index difference of $\left\lfloor \frac{T-1}{2} \right\rfloor$, which can also be termed as linear array with $\left\lfloor \frac{T-1}{2} \right\rfloor$length extension. This augmented linear array is analyzed to identify the number of wavelengths required atmost for establishing all-to-all broadcast by grouping non-overlapping connections on the same wavelength. The results found indicate that the Augmented Linear Array achieves a reduction of approximately 10% to 24% in wavelength requirements compared to a linear array with a two-length extension, also achieving 50% reduction in hop count. However, it exhibits a 10% increase in wavelength usage in comparison to a linear array with a three-length extension, but it offers a hop count reduction more than 20%.

Key-Words: - Augmented Linear Array, WDM, Wavelength allocation, All-optical, All-to-all broadcast

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1 Introduction

WDM optical networks are a leading solution for managing the rapidly growing demand for data traffic. These networks offer extensive bandwidth by utilizing multiple wavelengths over a single optical fiber. A WDM optical network consists of interconnected optical routing terminal nodes, which typically include optical sources and detectors, with wavelength converters as optional components. These terminal nodes are connected over optical fibers, with each fiber supporting a specific number of wavelengths. In most network designs, each link is represented as a pair of fibers: one for the forward connections and the other for reverse connections.

A connection or lightpath, denoted as (x, y), establishes a route from a source terminal node x to a destination terminal node y, facilitating the transfer of data along the path. When wavelength converters are unavailable, a unique wavelength must be assigned to the entire path for a connection. All-to-all broadcast, also known as gossiping, involves the distribution of unique messages from every terminal node to remaining all terminal node available in the network. This form of communication is essential in advanced computing and large-scale communication systems [1-4] such as enterprise data centers, which often consist of hundreds or thousands of interconnected terminal nodes [5-7] utilizing WDM optical networks. Additionally, all-to-all broadcast is critical in interconnected and distributed computing networks [8-12], where each terminal node must exchange information with all others to efficiently manage network resources.

WDM optical networks [13-14] serves as the backbone of emerging communication and computing networks, supporting ever-increasing traffic demands. A key resource in these networks is the set of available wavelengths, and minimizing their usage is vital to reducing network costs and complexity. Lowering the number of wavelengths reduces [16-19] the need for expensive components such as laser diodes, photodetectors, and their associated circuitry. Efficient wavelength allocation prevents any two connections sharing a link from using the same wavelength, ensuring nonblocking performance [20-22].

The linear array network, widely employed in LANs, MANs, WANs, and interconnection networks, is valued for its simplicity and regular structure [23-25]. However, rising traffic demands necessitate modifications to these networks, similar to the enhancements made in ring topologies [26-28]. By introducing additional links between alternate terminal nodes in the linear array, new fibers create alternative routes for connections [29-31]. These changes decrease the number of hops needed for data transmission and minimize wavelength requirements.

However, the increasing complexity of modern applications and traffic demands calls for innovative network topologies with enhanced properties. Many of these new or modified topologies can be decomposed into multiple basic linear arrays or modified linear arrays, as these form the foundational structure of network designs [29-31]. While previous work has demonstrated the advantages of modified ring topologies over simple rings, this study explores a similar approach for linear array networks.

This study focuses on wavelength allocation for all-to-all broadcast communication in an Augmented Linear Array, deriving the number of wavelengths needed atmost for efficient operation. The findings are particularly relevant for analyzing practical long-haul networks and/or interconnection networks such as mesh networks, as these can be broken down into multiple linear arrays or Augmented Linear Arrays.

The paper is organized as follows: Section 2 provides the foundational concepts necessary for understanding the analysis. Section 3 presents the derivation of the wavelength requirements for supporting all-to-all broadcast connections. Section 4 discusses the results obtained and their implications. Lastly, Section 5 concludes the study and highlights potential paths for future research.

2. Preliminaries

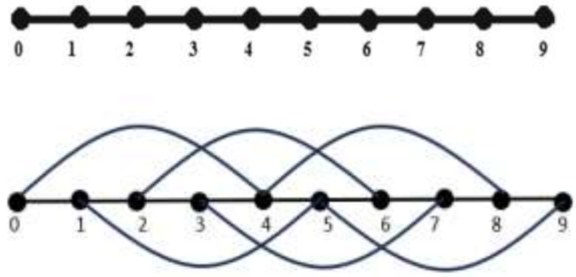

Fig. 1 A 10-node linear array and Augmented Linear Array

Fig.1 shows a 10-node linear array and its augmented version. In an augmented linear array terminal nodes are interconnected in a specific way to enhance communication efficiency. Along with the standard linear connections, each terminal node is linked to another terminal node whose index differs by $\left\lfloor \frac{T-1}{2} \right\rfloor$, where T is the total number of terminal nodes. This means data can directly hop not only to adjacent terminal nodes but also to terminal nodes separated by $\left\lfloor \frac{T-1}{2} \right\rfloor$, provided they exist. These additional connections create alternative pathways between terminal nodes, reducing the number of hops needed for data transfer and minimizing network wavelength demands.

The following definitions are required to understand the key results obtained in this paper.

Definition 1: An lightpath or optical connection is a dedicated path established between a source terminal node and a destination terminal node over a specific wavelength. This path follows a predetermined routing method to ensure packet transmission.

Definition 2: A longer link directly inter connects two different terminal nodes having index difference of $\left\lfloor \frac{T-1}{2} \right\rfloor$.

Definition 3: A shorter link straightly connects two neighboring terminal nodes having index difference of 1.

Definition 4: A connection is considered rightward if the destination terminal node's index is greater than the source terminal node's index. Conversely, it's considered leftward if the destination terminal node's index is less than the source terminal node's index.

Definition 5: Let a and b be two terminal nodes where a is the source and b is the destination. The length of the connection between these terminal nodes is the number of edges that need be traversed to reach one terminal node from the other.

Definition 6: A linear array is said to be augmented linear array, if each terminal node **x** of the network is directly connected to the terminal node $x + \left\lfloor \frac{T-1}{2} \right\rfloor$ away from it and if $(x + \left\lfloor \frac{T-1}{2} \right\rfloor) <= T-1$.

Example 1: Wavelength allotment for a 13- terminal node augmented linear array under shortest path routing

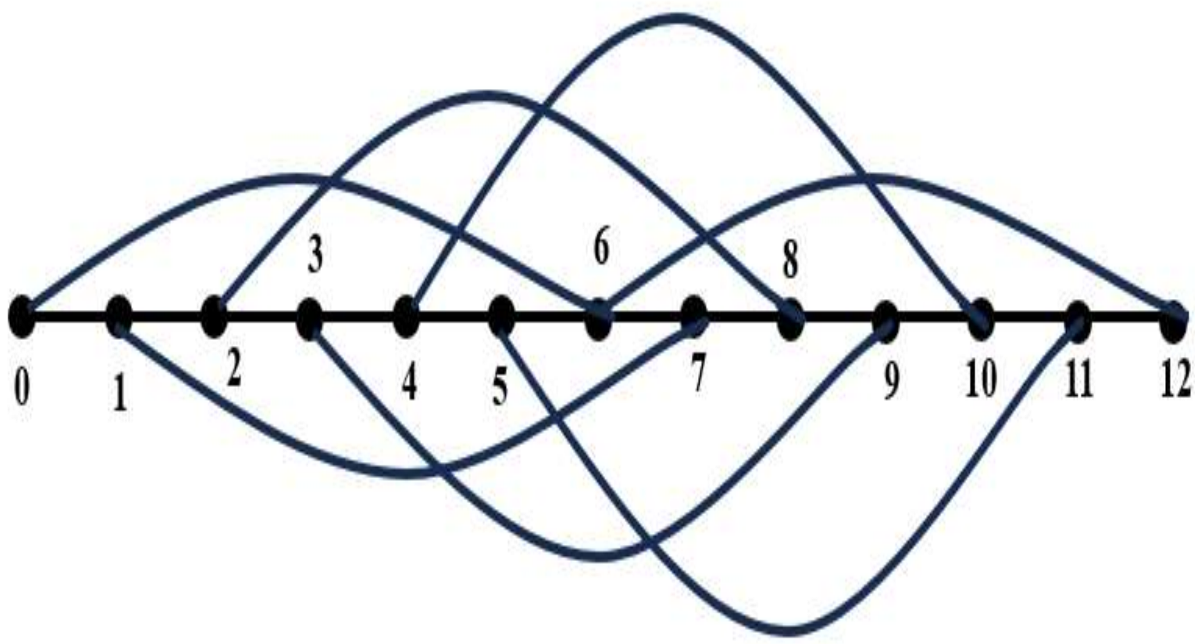

Fig. 2: A 13-Terminal node Augmented Linear array

Fig. 2.shows the 13- terminal node augmented linear array network. As rightward connections and leftward connections operate on separate fiber sets, same set of wavelengths can be allotted to them. Therefore, all rightward direction connections (without loss of generality) of all-to-all broadcast only listed below:

$(0,1), (1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,8), (8,9), (9,10), (10,11), (11,12)$
$(0,2), (1,3), (2,4), (3,5), (4,6), (5,7), (6,8), (7,9), (8,10), (9,11), (10,12)$
$(0,3), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11), (9,12)$
$(0,4), (1,5), (2,6), (3,7), (4,8), (5,9), (6,10), (7,11), (8,12)$
$(0,5), (1,6), (2,7), (3,8), (4,9), (5,10), (6,11), (7,12)$
$(0,6), (1,7), (2,8), (3,9), (4,10), (5,11), (6,12)$
$(0,7), (1,8), (2,9), (3,10), (4,11), (5,12)$
$(0,8), (1,9), (2,10), (3,11), (4,12)$
$(0,9), (1,10), (2,11), (3,12)$
$(0,10), (1,11), (2,12)$
$(0,11), (1,12)$
$(0,12)$

The listed connections is separated into several clusters. All connections inside each cluster are non-overlapping individually with other connections and are allotted a unique wavelength and is given below:

$\left\{\begin{matrix}(0,1), (1,2), (2,3), (3,4), (4,5), (5,6),\\ (6,7), (7,8), (8,9), (9,10), (10,11),\\ (11,12)(0,4), (3,7), (6,10)\end{matrix}\right\}$ − W_1
$\left\{\begin{matrix}(0,2), (2,4), (4,6), (6,8), (8,10),\\ (10,12)(1,5), (4,8), (7,11)\end{matrix}\right\}$ − W_2
$\left\{\begin{matrix}(1,3), (3,5), (5,7), (7,9), (9,11)\\ (2,6), (5,9), (8,12)\end{matrix}\right\}$ − W_3
$\left\{\begin{matrix}(0,3), (3,6), (6,9), (9,12)(0,5),\\ (2,7), (4,9), (6,11)\end{matrix}\right\}$ − W_4
$\left\{\begin{matrix}(1,4), (4,7), (7,10)(1,6), (3,8),\\ (5,10), (7,12)\end{matrix}\right\}$ − W_5
$\left\{\begin{matrix}(2,5), (5,8), (8,11), (0,6), (1,7),\\ (2,8), (3,9), (4,10), (5,11), (6,12)\end{matrix}\right\}$ − W_6
$\left\{\begin{matrix}(0,7), (1,8), (2,9), (3,10), (4,11),\\ (5,12)\end{matrix}\right\}$ − W_7
$\{(0,8), (2,10), (4,12)\}$ − W_8
$\{(1,9), (3,11)\}$ − W_9
$\{(0,9), (3,12),\}$ −W_{10}
$\{(1,10)\}$ −W_{11}
$\{(2,11)\}$ − W_{12}
$\{(0,10)\}$ − W_{13}
$\{(1,11)\}$ −W_{14}
$\{(2,12)\}$ − W_{15}
$\{(0,11)\}$ −W_{16}
$\{(1,12)\}$ − W_{17}
$\{(0,12)\}$ − W_{18}

Therefore, 18 unique wavelengths are sufficient for a 13-terminal node augmented linear array to facilitate all-to-all broadcast communication.

3. MAIN RESULTS

Let T denote the number of terminal nodes in a linear array network. Let W_T represent the number of wavelengths needed to facilitate all-to-all broadcast in an augmented linear array.

Lemma 1a: Let T be odd and $\frac{T-1}{2}$ be even number. Then for each g, such that $1 \le g \le \frac{T-1}{4}$,the total wavelengths sufficient to accommodate all g length connections, in an augmented linear array network with T terminal nodes are g under shortest path routing.

Proof: First, all g length connections in a network with T terminal nodes are listed below:

$(0, g)\, , (1, g+1)\, , (2, g+2)\, , \; . \; . \; . \; . \, , (m, g+m)$

where m is such that $g + m = T - 1$.

Then, the total g length connections are $m + 1 = T - g$.

The above set of connections is subdivided in such a way that in each sub set there is no overlapping of connections, as follows:

$\{(0, g), (g, 2g), (2g, 3g), \ldots, ((m_0 - 1)g, m_0 g)\}$,
$\{(1, g+1), (g+1, 2g+1), (2g+1, 3g+1), \ldots, ((m_1 - 1)g + 1, m_1 g + 1)\}$,
$\{(2, g+2), (g+2, 2g+2), (2g+2, 3g+2), \ldots, ((m_2 - 1)g + 2, m_2 g + 2)\}$,

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$\{(g-1, 2g-1), (2g-1, 3g-1), (3g-1, 4g-1), \ldots, ((m_{g-1} - 1)g + g - 1, m_{g-1}\, g + g - 1)\}$,

where mj's are suitable positive numbers and $m_j g + j \in \{T-1, T-2, \ldots, T-G\}$, $0 \le j \le g-1$, and $(m_a g + a) \ne (m_b g + b)$ if $a \ne b$where $a, b \in \{0,1,2, \ldots, j-1\}$. It can also be easily shown that the total connections of all sub-sets is $m_0 + m_1 + \ldots + m_{g-1} = T - g$. For each sub-set, a unique wavelength can be associated. Therefore, at least g wavelengths are needed to accommodate all g length connections.

Lemma 1b: Let T be odd and $\frac{T-1}{2}$ be even numbers. Then for each g, such that $\frac{T+3}{4} \leq g \leq \frac{T-1}{2}$, the total wavelengths sufficient to accommodate all g length connections, in augmented linear array network network with T terminal nodes is $\frac{T+1}{2} - g$ under shortest path routing.

Proof: First, all g length connections in a network with T terminal nodes are listed below:

$(0, g)\,, (1, g+1)\,, (2, g+2)\,, \;.\;.\;.\;.\,, (m, g+m)$ where m is such that $g + m = T - 1$.

Then, the total connections of length g is $m + 1 = T - g$.

The above set of connections is subdivided in such a way that in each sub -set there is no overlapping of connections, as follows:

$\left\{(0,g), \left(\frac{T+1}{2} - g, \frac{T+1}{2}\right), \left(2\left(\frac{T+1}{2} - g\right), T+1-g\right), \ldots, \left((m_0-1)g, m_0 g\right)\right\}$,

$\left\{(1,g+1), \left(\frac{T+1}{2} - g + 1, \frac{T+1}{2} + 1\right), \left(2\left(\frac{T+1}{2} - g\right) + 1, T+1-g+1\right), \ldots, \left((m_1-1)g + 1, m_1 g + 1\right)\right\}$,

$\left\{(2,g+2), \left(\frac{T+1}{2} - g + 2, \frac{T+1}{2} + 2\right), \left(2\left(\frac{T+1}{2} - g\right) + 2, T+1-g+2\right), \ldots, \left((m_2-1)g + 2, m_2 g + 2\right)\right\}$,

. . .

. . .

. . .

$\left\{\left(\frac{T+1}{2} - g - 1, \frac{T+1}{2} - 1\right), (T+1-2g-1, T+1-g-1), \left(3\left(\frac{T+1}{2}\right) - 3g - 1, 3\left(\frac{T+1}{2}\right) - 2g - 1\right), \ldots, \left(\left(m_{\frac{T+1}{2}-g-1} - 1\right)g + \frac{T+1}{2} - g - 1, \left(m_{\frac{T+1}{2}-g-1}\right)g + \frac{T+1}{2} - g - 1\right)\right\}$,

where mj's are suitable positive numbers and $m_j g + j \in \{T-1, T-2, \ldots, T-g\}$, $0 \leq j \leq g-1$, and $(m_a g + a) \neq (m_b g + b)$ if $a \neq b$ where $a, b \in \{0,1,2, \ldots, j-1\}$. It can also be easily shown that the total connections of all sub-sets is $m_0 + m_1 + \ldots + m_{g-1} = T - i$. For each sub-set, a unique wavelength is assigned. Therefore, at least g wavelengths are needed to accommodate all g length connections.

Lemma 1c: Let T be odd and $\frac{T-1}{2}$ be even numbers. Then for each g, such that $1 \leq g \leq \frac{T-1}{4}$, the total wavelengths sufficient to accommodate all $\left(\frac{T-1}{2} + g\right)$ length connections, in augmented linear array network with T terminal nodes are g under shortest path routing.

Proof:

First, all g length connections in a network with T terminal nodes are listed below:

$(0, \frac{T-1}{2} + g)\,, (1, \frac{T-1}{2} + g + 1)\,, (2, \frac{T-1}{2} + g + 2)\,, \;.\;.\;.\,, (m, \frac{T-1}{2} + g + m)$ where m is such that $\frac{T-1}{2} + g + m = T - 1$.

Then, the total $\frac{T-1}{2} + g$ length connections are $m + 1 = \frac{T+1}{2} - g$.

The above set of connections is subdivided in such a way that in each subset there is no overlapping of connections, as follows:

$\left\{\left(0, \frac{T-1}{2} + g\right), \left(g, \frac{T-1}{2} + 2g\right), \ldots, \left((m_0 - 1)g, \frac{T-1}{2} + m_0 g\right)\right\}$,

$\left\{\left(1, \frac{T-1}{2} + g + 1\right), \left(g+1, \frac{T-1}{2} + 2g + 1\right), \ldots, \left((m_1 - 1)g + 1, \frac{T-1}{2} + m_1 g + 1\right)\right\}$,

$\left\{\left(2, \frac{T-1}{2} + g + 2\right), \left(g+2, \frac{T-1}{2} + 2g + 2\right), \ldots, \left((m_2 - 1)g + 2, \frac{T-1}{2} + m_2 g + 2\right)\right\}$,

. . .

. . .

. . .

$\left\{\left(g-1, \frac{T-1}{2} + 2g - 1\right), \left(2g-1, \frac{T-1}{2} + 3g - 1\right), \ldots, \left((m_{g-1} - 1)g + g - 1, \frac{T-1}{2} + m_{g-1} g + g - 1\right)\right\}$,

where mj's are suitable positive numbers and $m_j g + \frac{T-1}{2} + j \in \{T-1, T-2, \ldots, T-g\}$, $0 \leq j \leq g-1$, and $(m_a g + a) \neq (m_b g + b)$ if $a \neq b$ where $a, b \in \{0,1,2, \ldots, g-1\}$. It can also be easily shown that the total connections of all sub-sets is $m_0 + m_1 + \ldots + m_{g-1} = T - g$. For each sub-set, a unique wavelength is assigned. Therefore, at

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least g wavelengths are needed to accommodate all g length connections.

Lemma 1d: Let T be odd and $\frac{T-1}{2}$ be even numbers. Then for each p such that $1 \le p \le \frac{T-1}{4}$, the total number of $\frac{3(T-1)}{4} + p$ length connections in an augmented linear array network with T terminal nodes is $\frac{(T+3)}{4} - p$.

Proof: First all $\frac{3(T-1)}{4} + p$ length connections in T terminal nodes network is listed below:

$\left(0, \frac{3}{4}(T-1) + p\right)$

$\left(1, \frac{3}{4}(T-1) + p + 1\right)$

$\left(2, \frac{3}{4}(T-1) + p + 2\right)$

. .

. .

. .

$\left(m, \frac{3}{4}(T-1) + p + m\right)$

where m is a positive number such that $\frac{3}{4}(T-1) + p + m = T - 1$

Then the total connections of length $\frac{3}{4}(T-1) + p$ is $m + 1 = \left(\frac{T+3}{4}\right) - p$

Theorem 1: Let T be odd and $\frac{T-1}{2}$ be even, then $W_T = \frac{3(T-1)(T+3)}{32}$

Proof:

By lemma 1a, the total wavelengths sufficient to accommodate all connections of length shorter than or equal to $\frac{T-1}{4}$ is

$$\sum_{g=1}^{\frac{T-1}{4}} g = \frac{\left(\frac{T-1}{4}\right)\left(\frac{T-1}{4}+1\right)}{2} = \frac{(T-1)(T+3)}{32} \quad (1)$$

By lemma 1b, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+3}{4}$ and shorter than or equal to $\frac{T-1}{2}$ is

$$\sum_{g=\frac{T+3}{4}}^{\frac{T-1}{2}} \frac{T+1}{2} - g = \sum_{g=\frac{T+3}{4}}^{\frac{T-1}{2}} \frac{T+1}{2} - \sum_{g=\frac{T+3}{4}}^{\frac{T-1}{2}} g = \frac{(T-1)(T+3)}{32} \quad (2)$$

It is to be noted that all connections of length shorter than or equal to $\frac{T-1}{4}$ use shorter links only in the forward direction. Nevertheless, all connections of length greater than or equal to $\frac{T+3}{4}$ and shorter than or equal to $\frac{T-1}{2}$ use both shorter links in backward direction and longer links. Hence, the same set of wavelengths that was used under lemma 1a can be reused under lemma 1b.

By lemma 1c, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+1}{2}$ and shorter than or equal to $3\left(\frac{T-1}{4}\right)$ is

$$\sum_{g=1}^{\frac{T-1}{4}} g = \frac{\left(\frac{T-1}{4}\right)\left(\frac{T-1}{4}+1\right)}{2} = \frac{(T-1)(T+3)}{32} \quad (3)$$

By lemma 1d, the total connections of length greater than $3\left(\frac{T-1}{4}\right)$ is

$$\sum_{p=1}^{\frac{T-1}{4}} \frac{T+3}{4} - p = \sum_{p=1}^{\frac{T-1}{4}} \frac{T+3}{4} - \sum_{p=1}^{\frac{N-1}{4}} p$$

$$= \frac{\left(\frac{T+3}{4}\right)\left(\frac{T-1}{4}\right) - \left(\frac{T-1}{4}\right)\left(\frac{T-1}{4}+1\right)}{2}$$

$$= \frac{(T+3)(T-1)}{32}$$

For each connection, we associate a unique wavelength. Hence, wavelength sufficient is

$$\frac{(T+3)(T-1)}{32} \quad (4)$$

Adding (1), (3) and (4), the total wavelength needed is $\frac{3(T-1)(T+3)}{32}$

Lemma 2a: Let both T and $\frac{(T-1)}{2}$ be odd numbers. Then for each g, such that $1 \le g \le \frac{T+1}{4}$, the total wavelengths sufficient to accommodate all g length connections in an augmented linear array network with T terminal nodes is g under shortest path routing.

Proof: Similar to lemma 1a.

Lemma 2b: Let both T and $\frac{(T-1)}{2}$ be odd numbers. Then for each g, such that $\frac{T+5}{4} \le g \le \frac{T-1}{2}$, the total wavelengths sufficient to accommodate all g length connections, in an augmented linear array network with T terminal nodes is $\frac{(T+1)}{2} - g$ under shortest path routing.

Proof: Similar to lemma 1b.

Lemma 2c: Let both T and $\frac{(T-1)}{2}$ be odd numbers. Then for each g, such that $1 \le g \le \frac{T+1}{4}$, the total wavelengths sufficient to accommodate all $\frac{(T-1)}{2} + g$ length connections in an augmented linear array network with T terminal nodes is g under shortest path routing.

Proof: Similar to lemma 1c.

Lemma 2d: Let both T and $\frac{(T-1)}{2}$ be odd numbers. Then for each p such that $0 \le p \le \frac{T-7}{4}$, the total $\frac{3(T+1)}{4} + p$ length connections in an augmented

linear array network with T terminal nodes are $\frac{(T-3)}{4} - p$.
Proof: Similar to lemma 1d.

Theorem 2: Let both T and $\frac{T-1}{2}$ be odd numbers. then $W_T = \frac{(T+1)(3T+7)}{32}$
Proof:
By lemma 2a, the total wavelengths needed to accommodate to all connections of length shorter than or equal to $\frac{T+1}{4}$ is

$$\sum_{g=1}^{\frac{T+1}{4}} g = \frac{\left(\frac{T+1}{4}\right)\left(\frac{T+1}{4}+1\right)}{2}$$
$$= \frac{\left(\frac{T+1}{4}\right)\left(\frac{T+5}{4}\right)}{2}$$
$$= \frac{(T+1)(T+5)}{32} \qquad (5)$$

By lemma 2b, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+5}{4}$ and shorter than or equal to $\frac{T-1}{2}$ is

$$\sum_{g=\frac{T+5}{4}}^{\frac{T-1}{2}} \frac{T+1}{2} - g = \sum_{g=\frac{T+5}{4}}^{\frac{T-1}{2}} \frac{T+1}{2} - \sum_{g=\frac{T+5}{4}}^{\frac{T-1}{2}} g$$
$$= \frac{(T+1)(T-3)}{32} \qquad (6)$$

It is to be noted that all connections of length shorter than or equal to $\frac{T-1}{4}$ use short links only in forward direction. But all connections of length greater than or equal to $\frac{T+3}{4}$ and shorter than or equal to $\frac{T-1}{2}$ use longer links and shorter links in reverse direction. Hence, the same set of wavelength that was used under lemma 2a can be reused under lemma 2b.

By lemma 2c, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+1}{2}$ and shorter than or equal to $\left(\frac{3T-1}{4}\right)$ is

$$\sum_{g=1}^{\frac{T+1}{4}} g = \frac{\left(\frac{T+1}{4}\right)\left(\frac{T+1}{4}+1\right)}{2}$$
$$= \frac{\left(\frac{T+1}{4}\right)\left(\frac{T+5}{4}\right)}{2}$$
$$= \frac{(T+1)(T+5)}{32} \qquad (7)$$

By lemma 2d, the total connections of length greater than $\left(\frac{3T+3}{4}\right)$ is

$$\sum_{p=0}^{\frac{T-1}{4}} \frac{T-3}{4} - p = \sum_{p=0}^{\frac{T-3}{4}} \frac{T-3}{4} - \sum_{p=0}^{\frac{T-7}{4}} p$$
$$= \frac{\left(\frac{T-3}{4}\right)\left(\frac{T-7}{4}+1\right) - \left(\frac{T-7}{4}\right)\left(\frac{T-3}{4}\right)}{2}$$
$$= \frac{\left(\frac{T-3}{4}\right)\left(\frac{T-3}{4}\right) - \left(\frac{T-7}{4}\right)\left(\frac{T-3}{4}\right)}{2}$$
$$= \frac{(T-3)(T+1)}{32}$$

For each connection, we assign a unique wavelength. Hence, wavelength needed is $\frac{(T-3)(T+1)}{32}$ (8)

Adding (5), (7) and (8) the total wavelength needed is

$$= \frac{(T+1)(T+5)}{32} + \frac{(T+1)(T+5)}{32} + \frac{(T-3)(T+1)}{32}$$
$$= \frac{1}{32}[(T+1)(T+5) + (T+1)(T+5) + (T-3)(T+1)]$$
$$= \frac{(T+1)}{32}[(T+5) + (T+5) + (T-3)]$$
$$= \frac{(T+1)(3T+7)}{32}$$

Lemma 3a: Let both T and $\frac{T}{2}$ be even numbers. Then for each g, such that $1 \le g \le \frac{T}{4}$, the total wavelengths sufficient to accommodate all g length connections in an augmented linear array network with T terminal nodes are g under shortest path routing.

Proof: Similar to Lemma 1a.

Lemma 3b: Let both T and $\frac{T}{2}$ be even numbers. Then for each g, such that $\frac{T+4}{4} \le g \le \frac{T}{2}$, the total wavelengths sufficient to accommodate all g length connections in an augmented linear array network with T terminal nodes are $\frac{T}{2} - g + 1$ under shortest path routing.

Proof: Similar to Lemma 1b.

Lemma 3c: Let both T and $\frac{T}{2}$ be even numbers. Then for each g, such that $1 \le g \le \frac{T-4}{4}$, the total wavelengths sufficient to accommodate all $\frac{T}{2} + g$ length connections in an augmented linear array network with T terminal nodes are g under shortest path routing.

Proof: Similar to Lemma 1c.

Lemma 3d: Let both T and $\frac{T}{2}$ be even numbers. Then for each p, such that $0 \le p \le \frac{T-4}{4}$, the total connections of length $\frac{3T}{4} + p$ in an augmented linear array network with T terminal nodes is $\frac{T}{4} - p$.

Proof: Similar to Lemma 1d.

Theorem 3: Let both T and $\frac{T}{2}$ be even numbers, then $W_T = \frac{T(3T+4)}{32}$
Proof:

By lemma 3a, the total wavelengths needed to accommodate to all connections of length shorter than or equal to $\frac{T}{4}$ is

$$\sum_{g=1}^{\frac{T}{4}} g = \frac{\left(\frac{T}{4}\right)\left(\frac{T}{4}+1\right)}{2}$$
$$= \frac{\left(\frac{T}{4}\right)\left(\frac{T+4}{4}\right)}{2}$$
$$= \frac{(T)(T+4)}{32} \qquad (9)$$

By lemma 3b, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+4}{4}$ and shorter than or equal to $\frac{T}{2}$ is

$$\sum_{g=\frac{T+4}{4}}^{\frac{T}{2}} \frac{T}{2} - g + 1 = \sum_{g=\frac{T+4}{4}}^{\frac{T}{2}} \frac{T}{2} + 1 - \sum_{g=\frac{T+3}{4}}^{\frac{T-1}{2}} g$$
$$= \frac{(T)(T+4)}{32} \qquad (10)$$

It is to be noted that all connections of length shorter than or equal to $\frac{T-1}{4}$ use short links only in forward direction. But all connections of length greater than or equal to $\frac{T+4}{4}$ and shorter than or equal to $\frac{T}{2}$ use longer links and shorter links in reverse direction. Hence, the same set of wavelength that was used under lemma 3a can be reused under lemma 3b.

By lemma 3c, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T}{2}+1$ and shorter than or equal to $\left(\frac{3T}{4}-1\right)$ is

$$\sum_{g=1}^{\frac{T-4}{4}} g = \frac{\left(\frac{T-4}{4}\right)\left(\frac{T-4}{4}+1\right)}{2}$$
$$= \frac{\left(\frac{T-4}{4}\right)\left(\frac{T}{4}\right)}{2}$$
$$= \frac{(T-1)(T+3)}{32} \qquad (11)$$

By lemma 3d, the total connections of length greater than or equal $\left(\frac{3T}{4}\right)$ is

$$\sum_{p=0}^{\frac{T-4}{4}} \frac{T}{4} - p = \sum_{p=0}^{\frac{T-1}{4}} \frac{T}{4} - \sum_{p=0}^{\frac{T-4}{4}} p$$
$$= \frac{(T)(T+4)}{32}$$

For each connection, we assign a unique wavelength. Hence, wavelength needed is $\frac{(T)(T+4)}{32}$ (12)

Adding (9), (11) and (12) the total wavelength needed is

$$= \frac{T(T+4)}{32} + \frac{T(T-4)}{32} + \frac{T(T+4)}{32}$$
$$= \frac{T}{32}\left[(T+4) + (T-4) + (T+4)\right]$$
$$= \frac{T(3T+4)}{32}$$

Lemma 4a: Let T be even and $\frac{T}{2}$ be odd numbers. Then for each g, such that $1 \le g \le \frac{T-2}{4}$, the total wavelengths sufficient to accommodate all g length connections in an augmented linear array network with T terminal nodes are g under shortest path routing.

Proof: Similar to Lemma 1a.

Lemma 4b: Let T be even and $\frac{T}{2}$ be odd numbers. Then for each g, such that $\frac{T+2}{4} \le g \le \frac{T-2}{2}$, the total wavelengths sufficient to accommodate all g length connections in an augmented linear array network with T terminal nodes are $\frac{T}{2} - g$ under shortest path routing.

Proof: Similar to Lemma 1b.

Lemma 4c: Let T be even and $\frac{T}{2}$ be odd numbers. Then for each g, such that $0 \le g \le \frac{T-6}{4}$, the total wavelengths sufficient to accommodate all $\frac{T}{2} + g$ length connections in an augmented linear array network with T terminal nodes are $g+1$ under shortest path routing.

Proof: Similar to Lemma 1c.

Lemma 4d: Let T be even and $\frac{T}{2}$ be odd numbers. Then for each p, such that $1 \le p \le \frac{T+2}{4}$, the total connections of length $\frac{3(T-2)}{4} + p$ in an augmented linear array network with T terminal nodes is $\frac{T+6}{4} - p$.

Proof: Similar to Lemma 1d.

Theorem 4: Let T be even and $\frac{T}{2}$ be odd numbers, then $W_T = \frac{(T+2)(3T+2)}{32}$

Proof:

By lemma 4a, the total wavelengths needed to accommodate to all connections of length shorter than or equal to $\frac{T-2}{4}$ is

$$\sum_{g=1}^{\frac{T-2}{4}} g = \frac{\left(\frac{T-2}{4}\right)\left(\frac{T-2}{4}+1\right)}{2}$$
$$= \frac{\left(\frac{T-2}{4}\right)\left(\frac{T+2}{4}\right)}{2}$$
$$= \frac{(T-2)(T+2)}{32} \qquad (13)$$

By lemma 4b, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T+2}{4}$ and shorter than or equal to $\frac{T-2}{2}$ is

$$\sum_{g=\frac{T+2}{4}}^{\frac{T-2}{2}} \frac{T}{2} - g = \sum_{g=\frac{T+2}{4}}^{\frac{T-2}{2}} \frac{T}{2} - \sum_{g=\frac{T+2}{4}}^{\frac{T-2}{2}} g$$
$$= \frac{(T-2)(T+2)}{32} \qquad (14)$$

It is to be noted that all connections of length shorter than or equal to $\frac{T-1}{4}$ use short links only in forward direction. But all connections of length greater than or equal to $\frac{T+3}{4}$ and shorter than or equal to $\frac{T-1}{2}$ use longer links and shorter links in reverse direction. Hence, the same set of wavelength that was used under lemma 4a can be reused under lemma 4b.

By lemma 4c, the total wavelengths needed to accommodate to all connections of length greater than or equal to $\frac{T}{2}$ and shorter than or equal to $\left(\frac{3T-6}{4}\right)$ is

$$= \sum_{g=0}^{\frac{T-6}{4}} g + 1 = \sum_{p=0}^{\frac{T-6}{4}} g + \sum_{p=0}^{\frac{T-6}{4}} 1$$
$$= \frac{\left(\frac{T-2}{4}\right)\left(\frac{T+2}{4}\right)}{2}$$
$$= \frac{(T-2)(T+2)}{32} \qquad (15)$$

By lemma 4d, the total connections of length greater than $3\left(\frac{T-2}{4}\right)$ is equal to

$$\sum_{p=1}^{\frac{T+2}{4}} \frac{T+6}{4} - p = \sum_{p=1}^{\frac{T+2}{4}} \frac{T+6}{4} - \sum_{p=1}^{\frac{T+2}{4}} p$$
$$= \frac{(T+6)(T+2)}{32}$$

For each connection, we assign a unique wavelength. Hence, wavelength needed is $\frac{(T+6)(T+2)}{32}$ (16)

Adding (13) , (15) and (16) the total wavelength needed is

$$= \frac{(T-2)(T+2)}{32} + \frac{(T-2)(T+2)}{32} + \frac{(T+6)(T+2)}{32}$$
$$= \frac{(T+2)}{32}[(T-2) + (T-2) + (T+6)]$$
$$= \frac{(T+2)(3T+2)}{32}$$

4. Results and Discussion

The augmented linear array is analyzed to assess its effectiveness in reducing the wavelength needed for all-to-all broadcast communication. This is achieved by grouping non-overlapping connections onto the same wavelength, optimizing wavelength usage for efficient network operation.

Table 1 Wavelength requirement for all-to-all broadcast in augmented linear array

Number of nodes T	Number of wavelengths used
T - odd, $\frac{T-1}{2}$ even	$W_T = \frac{3(T-1)(T+3)}{32}$
T - odd, $\frac{T-1}{2}$ odd	$W_T = \frac{(T+1)(3T+7)}{32}$
T - even, $\frac{T}{2}$ even	$W_T = \frac{T(3T+4)}{32}$
T - even, $\frac{T}{2}$ odd	$W_T = \frac{(T+2)(3T+2)}{32}$

Table 1 provides the simplified expression to calculate the wavelength requirement for all-to-all broadcast in an augmented linear array across various cases of T values. Table 2 presents the wavelength requirements and maximum hop count involved for typical values of node size T. Compared to a linear array extended by two lengths, the augmented configuration demonstrates a reduction in wavelength requirements of approximately 10% to 24%, underscoring its ability to manage high communication demands while conserving spectral resources. In addition to wavelength efficiency, the augmented linear array achieves a 50% reduction in hop count relative to the two-length extension, significantly lowering communication latency and enabling faster data transmission.

Table 2 Comparison of Number of wavelengths needed in a linear array with Jump 2, 3 and $\left\lfloor \frac{T-1}{2} \right\rfloor$

Number of Terminal nodes T	Number of wavelengths needed in an augmented linear array with			Maximum hop involved in the communication		
	Jump=2 [29]	Jump=3 [30]	Jump=$\left\lfloor \frac{T-1}{2} \right\rfloor$	Jump=2 [29]	Jump=3 [30]	Jump=$\left\lfloor \frac{T-1}{2} \right\rfloor$
12	18	13	15	6	5	3
13	21	16	18	6	5	4
14	24	18	22	7	5	4
15	28	19	26	7	5	4
16	32	23	26	8	7	4
17	36	26	30	8	6	5
18	40	28	35	9	6	5
19	45	33	40	9	7	5
20	50	36	40	10	8	5
21	55	38	45	10	7	6
22	60	44	51	11	8	6
23	66	48	57	11	8	6
24	72	50	57	12	9	6
27	91	62	77	13	9	7
30	112	77	92	15	10	8
33	136	93	108	16	11	9
36	162	111	126	18	13	9
39	190	129	155	19	13	10
42	220	150	176	21	14	11
45	253	172	198	22	15	12
48	288	196	222	24	17	12
51	325	220	260	25	17	13
60	450	305	345	30	21	15
70	612	420	477	35	24	18
80	800	546	610	40	28	20
90	1012	682	782	45	30	23
100	1250	849	950	50	35	25

In contrast to the linear array with a three-length extension, the augmented linear array shows an increase of about 10% in wavelength usage. Despite this increase, it offers a notable advantage by reducing the hop count by over 20%. This reduction in hop count is critical, as it shortens the communication path, decreases transmission delays, and reduces the number of intermediate links involved, thereby enhancing the reliability of data delivery. Thus, the augmented linear array effectively balances the trade-off between wavelength utilization and hop count reduction, making it a practical choice for networks where these parameters are vital for performance optimization.

5. Conclusion and Future Work

This study introduces an enhancement to the linear array by connecting terminal nodes separated by an index difference of $\left\lfloor \frac{T-1}{2} \right\rfloor$ using additional fibers, forming what is termed the Augmented Linear Array. This modification aims to reduce the wavelength requirements for all-to-all broadcast communication. The wavelengths needed atmost for this network configuration is derived, supported by detailed lemmas and theorems outlining the wavelength allocation process. The results reveal that the Augmented Linear Array achieves a reduction of approximately 10% to 24% in wavelength requirements compared to a standard linear array with a two-length extension achieving 50% reduction in Hop count. However, it exhibits a 10% increase in wavelength usage when compared to a linear array with a three-length extension, it offers a hop count reduction more than 20%.

Future research directions include extending this analysis to other network topologies with similar modifications. Further research is necessary to assess the impact of physical layer impairments on these networks and to investigate wavelength requirements in scenarios involving link or terminal node failures.

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