Unveiling Chaos in Semiconductor Lasers: A Simulation-Based Study Using Vicente Equations

JOAN JANI
Department of Engineering Physics
Polytechnic University of Tirana
Rr. S. Delvina, 1001
ALBANIA

Abstract: - This paper presents a simulation study for a semiconductor laser with optical feedback based on the Lang-Kobayashi model. The analysis of the system's dynamical behavior, utilized using Vicente equations, has provided evidence of the presence of chaotic behavior. Performing detailed numerical simulations, we have explored the parameters, revealing the transitions of the system from stable operation to chaotic. Our findings corroborate previous theoretical predictions and offer new insights into the complex behavior of semiconductor lasers under feedback conditions. The results have significant implications for applications where controlled chaotic behavior is desirable, such as secure communications and random number generation.

Key-Words: - Optical feedback laser, Lang-Kobayashi model, Chaos theory, Vicente equations, Nonlinear dynamics, Chaotic systems

Received: March 21, 2023. Revised: September 11, 2024. Accepted: October 5, 2024. Published: November 7, 2024.

1 Introduction

The appearance of chaos in a deterministic system, in general, is characterized by its stochastic operation under complex conditions that affect its operation, [1]. With the evolution of our knowledge about dynamical systems, functions that we previously classified as noise are now presented in many physics, biology, and chemistry systems, [2].

Laser systems with optical feedback have attracted the scientific community's interest due to their rich characteristics, including oscillations, periodic operation, and chaotic regime, [3]. A widely studied model for understanding these functions is the Lang—Kobayashi model, developed by Lang and Kobayashi in 1980 to describe the effect of feedback's time lag on the system's output, [4]. Additionally, the Lang-Kobayashi model has become a central tool in experimental and theoretical studies to understand the fundamental mechanisms of chaos and bifurcation in nonlinear optical systems, [3]. Vicente and his team thoroughly investigated the possibility of generating chaotic signals through the Lang-Kobayashi model. The team also investigated the possible use of chaos in communications security. They proposed a system with synchronized lasers that could produce the same chaotic signal, [5].

In secure communications, encryption is crucial because it protects transmitted messages from interception and unauthorized access. Encryption transforms information into a coded format that authorized parties can only interpret with the correct decryption key, [6]. This ensures that sensitive personal, financial, or strategic information remains confidential, maintaining privacy and trust between communicating parties., [7]. With the development of more powerful computers, the traditional software cryptography algorithms look vulnerable, prompting the need to substitute the current encryption schemes, [8]. The application of chaotic signals in data encryption looks promising because chaotic signals look unpredictable, [9]. Various schemes have been proposed, with the chaotic masking being one of the most promising to be adapted, [10]. This technique is performed at the physical layer by adding the chaotic carrier. The generated signal is from a nonlinear optical element with the message's signal. The solid-state laser is a non-linear optical system that exhibits chaotic behavior, [11]. Since the laser is a nonlinear system, it typically exhibits three key characteristics. variables, field, polarization, and inversion, and it is an ideal element of a chaotic system exhibiting chaotic dynamics, [12].

The masking technique involves mixing the message to be transmitted with the signal from a chaotic carrier generated by a nonlinear optical component. The Lyapunov exponent calculation verifies a signal's chaotic behavior, [13]. Apart from chaotic behavior in optical systems, chaos has been experimentally verified in electronic circuits [14]. In addition, some circuits operate in chaotic mode with an external triggering signal or autonomous like the Chuas circuit, which can be combined with a laser,

[15]. In a chaotic scheme, the source of the chaotic signal could be an autonomous electronic circuit. The decoding of a message is done by the receiver, where a signal generator generates the same chaotic signal as the transmitter. The critical factor of message recovery is the synchronization of transmitter and receiver, [16].

In this paper, we simulate a model of a semiconductor laser proposed by Vicente using numerical techniques to solve the deference delay differential equations. The paper is organized as follows: section 1 is a literature review, followed by section 2, where the problem is formulated. Here, we present the details of the model. In section 3, we present the results of our simulations in semiconductor lasers, describe the findings, and finally, section 4 provides some concluding remarks and future work. Our results contribute to a deeper understanding of chaotic regimes in semiconductor lasers, providing insights that could inform future developments in optical communication, sensing, and other laser-based technologies.

2 Problem formulation

Chaos is present in systems with nonlinear behavior. The nonlinearity in a system means that the measured values in the system's output are not proportional to the input values. The presence of nonlinearity in a system does not mean that the system will behave chaotically but requires a form of nonlinearity to achieve chaotic behavior, [17].

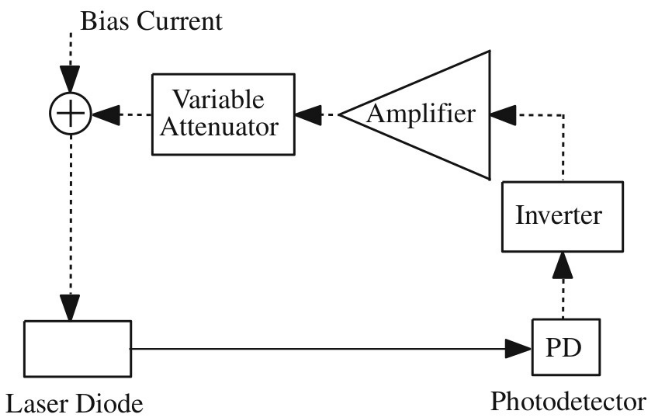

Fig. 1: Schematic diagram of the optoelectronic feedback system, [18]

A lot of optical elements exhibit nonlinear responses. The light amplification by stimulated emission of radiation (laser) is one of them since it is characterized by three parameters: the field, polarization of matter, and population inversion. Indeed, lasers were proven to be nonlinear systems similar to the Lorenz model and show chaotic dynamics in their output powers, [12].

However, lasers based on semiconductor gain media, whose equations portray the field and the carrier density (population inversion), can be disturbed by applying external perturbations such as external optical injection, optical feedback, or modulation for accessible laser parameters. In a semiconductor laser, the laser oscillation is affected considerably when the light is reflected from an external reflector coupled with the original field in the laser cavity, [4].

In the semiconductor laser, optoelectronic feedback is one of the perturbations of the injection current that induces instability. Phase sensitivity is essential for lasers with optical feedback. Fig. 1 shows optoelectronic feedback in a semiconductor laser. The photodetector detects the light emitted from the semiconductor laser, and then this current is fed back to the laser by a bias circuit. The current could be either positive or negative according to the polarity of the amplifier.

A prototypical model to describe single-mode semiconductor lasers subject to coherent optical feedback is the one described by the Lang-Kobayashi equations, [4] for the complex slowly varying amplitude of the electric field $E(t)$ and carrier number inside the cavity N(t):

$$\dot{E}(t) = \frac{1+ia}{2}\left(G + \frac{1}{\tau_{ph}}\right)E(t) + \kappa E(t-\tau)e^{-iC_p} \quad (1)$$

$$\dot{n}(t) = \frac{I}{e} - \frac{n(t)}{\tau_n} - G|E(t)|^2 \quad (2)$$

Where a denotes the line-width enhancement factor, κ is the feedback coefficient, τ is the external cavity roundtrip, τ_{ph} is the photon lifetime, τ_n is the carrier lifetime, and where C_p denotes the phase of the laser without feedback.

The pump current is fixed to I, where the laser operates in a chaotic regime. And finally, G is the optical gain, which is calculated by the following relation:

$$G = g\frac{n(t) - n_o}{1 + s|E(t)|^2} \quad (3)$$

g is the differential gain parameter, s is the saturation coefficient, and n_o is the carrier number of transparency.

3 Chaos in semiconductor lasers

The presence of chaos is explored in this section. The differential equations (1) and (2) are solved using numerical methods to unveil the chaotic behavior of

Symbol	Quantity	Typical orders of magnitude
a	Line-width enhancement factor	5
κ	Feedback coefficient	4 nsec^{-1}
τ	External cavity roundtrip time	$\tau = \frac{2L}{v_g}$
τ_{ph}	Photon lifetime	2 psec
τ_n	Carrier lifetime	2 nsec
C_p	Phase of laser without feedback	$\frac{3\pi}{2}$
$\dot{I}_{th}$	Pump current threshold value	$\frac{e}{\tau_n}\left(\frac{1}{\tau_{phi}\dot{g}} + n_o\right)$
I	Pump current	$1.5\dot{I}_{th}$
g	Differential gain parameter	1.5×10^4 sec
s	Saturation coefficient	5×10^7 sec
e	Electron charge	1.6×10^{-19} C
n_o	Carrier number of transparency	1.5×10^8

Table 1: Typical values of the Lang-Kobayashi simulation model

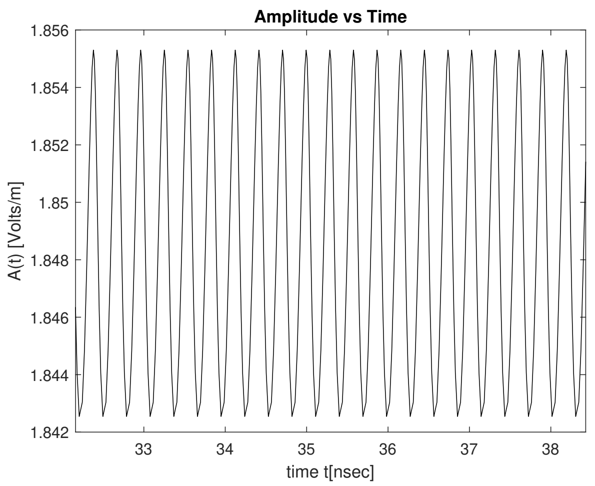

Fig. 2: Amplitude $A(t)$ vs time and single-sided spectrum for $A(t)$ when $\kappa = 3.7nsec^{-1}$ and ($C_p = 3\pi/2$ and $I = 1.5 \times I_{th}$)

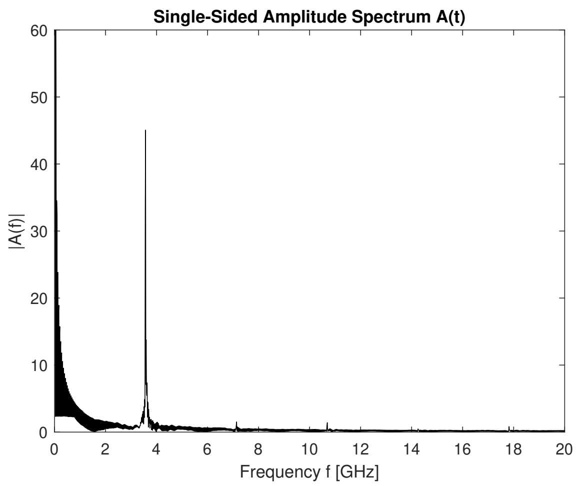

Fig. 3: Single-sided spectrum for $A(t)$ when $\kappa = 3.7nsec^{-1}$ and ($C_p = 3\pi/2$ and $I = 1.5 \times I_{th}$)

them. We express the electric field in the form of $E(t) = A(t)e^{i\phi t}$, and substituting this term in (1), we are taking a form of this equation that is easier to be simulated expressed in equations (4), (5), and (6):

$$\frac{dA(t)}{dt} = \frac{1}{2}\left(G - \frac{1}{\tau_{ph}}\right)A(t)\dots + \kappa A(t-\tau)\cos[C_p] + \phi(t) - \phi(t-\tau) \tag{4}$$

$$\frac{d\phi(t)}{dt} = \frac{1}{2}a\left(G - \frac{1}{\tau_{ph}}\right)\dots - \kappa\frac{A(t-\tau)}{A(t)}\sin[C_p + \phi(t) - \phi(t-\tau)] \tag{5}$$

$$\frac{dn(t)}{dt} = \frac{I}{e} - \frac{n(t)}{\tau_n} - G|A(t)|^2 \tag{6}$$

where A denotes the amplitude of electric field E_t and ϕ its phase, t is the time, C_p is the phase of laser without feedback, I is the pumping current, g is the parameter of deferential gain, and τ_{phi}, e, τ_n, and n_o are the lifetime of photons, the charge of electrons, the lifetime of carriers, and density in transparency, respectively.

The equations 4, 5,and 6 are solved; for this purpose, we use the values of the parameters presented in Table 1. The amplitude of the signal $A(t)$ is calculated as a function of time and the number of carriers $n(t)$ for the corresponding values. After the end of the process, the construction of the diagrams, presented in Fig. 2 and 3, is done after the transient piece has been previously removed.

Then, a thorough study of the system's behavior for different feedback coefficient values is conducted

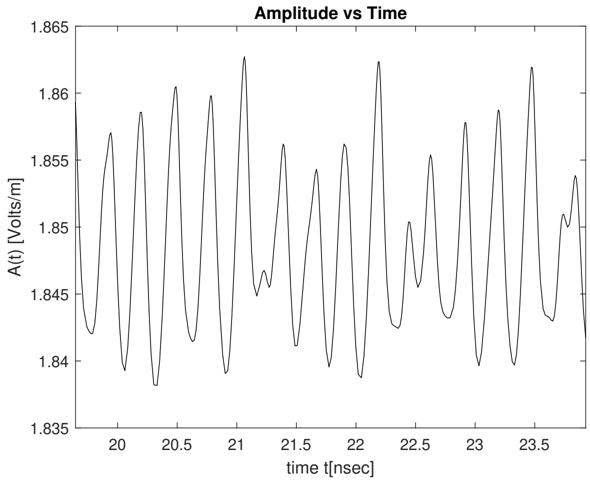

Fig. 4: Amplitude $A(t)$ as a function of time when $\kappa = 6.77nsec^{-1}$ and ($C_p = 3\pi/2$ and $I = 1.5 \times I_{th}$)

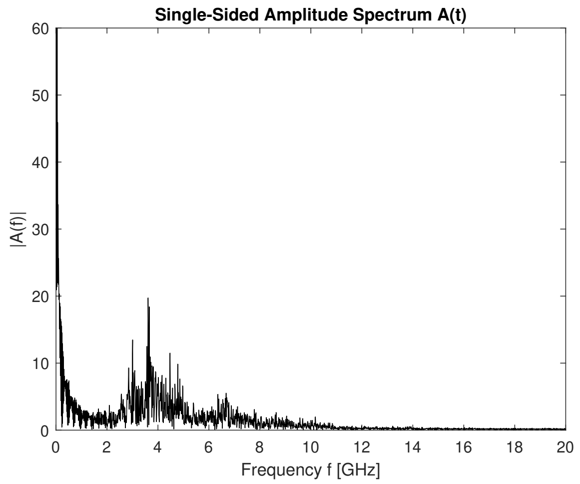

Fig. 5: Single-sided spectrum for $A(t)$ when $\kappa = 6.77nsec^{-1}$ and ($C_p = 3\pi/2$ and $I = 1.5 \times I_{th}$)

to analyze the behavior. From the shape of the output signal and the frequency spectrum diagram, it is categorized as whether it is chaotic or periodic. The transition to chaos is characterized by a doubling of the signal period. This is observed for values of $\kappa = 3.7$, and it is observed that the system has only one frequency, as it can be seen in Fig. 3 at the value of $\kappa = 5.14$, where the system exhibits the phenomenon of period doubling. The next period-doubling occurs for $\kappa = 5.39$ and then $\kappa = 5.67$. For values more significant than this, the system exhibits intensely chaotic behavior, as shown in Fig. 4, for $\kappa = 6.67$, where the signal looks stochastic without any periodicity. Moreover, from the single-sided frequency spectrum as shown in Fig. 5, this signal is characterized by the superposition of a wide range of frequencies.

The chaotic state of the system is very similar to a random noise signal. Driving the system from periodic to chaotic mode is done by changing the value of the feedback coefficient κ. In fact, as the value of κ increases, it is observed that the chaos is even more intense, and the spectrum of the system resembles the spectrum of Gaussian white noise.

4 Conclusion

From the data analysis from the simulation that solves the flow equations of a coherent optical feedback system with a semi-cavity DFB laser, the feedback coefficient κ increases the output waveform from periodic to chaotic. Further increase presents more intense chaos, consistent with greater apparent randomness, making its use in cryptography more effective. When the current I increases, the system becomes chaotic. However, the system stabilizes in periodic operation if its feedback coefficient is too small.

Exploring semiconductor lasers in cryptography broadens the scope of secure communication technologies and opens new frontiers in utilizing chaotic systems for practical, high-stakes applications.

Since the system we modeled shows chaotic operation, the next goal is to couple two synchronized laser systems that produce the same chaotic waveform. After that, the next step is to use chaotic masking to explore the possibility of the system being proposed as a chaotic encryption scheme.

References:

[1] E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of atmospheric sciences*, vol. 20, no. 2, pp. 130–141, 1963.

[2] S. H. Strogatz, *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. CRC press, 2018.

[3] M. Sciamanna and K. A. Shore, "Physics and applications of laser diode chaos," *Nature photonics*, vol. 9, no. 3, pp. 151–162, 2015.

[4] R. Lang and K. Kobayashi, "External optical feedback effects on semiconductor injection laser properties," *IEEE journal of Quantum Electronics*, vol. 16, no. 3, pp. 347–355, 1980.

[5] R. Vicente, J. Daudén, P. Colet, and R. Toral, "Analysis and characterization of the hyperchaos generated by a semiconductor laser subject to a delayed feedback loop," *IEEE Journal of Quantum Electronics*, vol. 41, no. 4, pp. 541–548, 2005.

[6] M. I. Mihailescu and S. L. Nita, "Getting started in cryptography and cryptanalysis," in *Pro Cryptography and Cryptanalysis with C++ 23: Creating and Programming Advanced Algorithms*, pp. 3–18, Springer, 2023.

[7] C.-C. Lee, H.-C. Tseng, C.-C. Liu, and H.-J. Chou, "Using aes encryption algorithm to optimize high-tech intelligent platform," *WSEAS Transactions on Business and Economics*, vol. 18, pp. 1572–1579, 2021.

[8] L. M. Caligiuri and A. Manzalini, "Quantum hypercomputing and communications: Overview and future applications," *WSEAS Transactions on Computers*, vol. 20, pp. 247–257, 2021.

[9] B. Zhang and L. Liu, "Chaos-based image encryption: Review, application, and challenges," *Mathematics*, vol. 11, no. 11, p. 2585, 2023.

[10] P. Colet and R. Roy, "Digital communication with synchronized chaotic lasers," *Optics letters*, vol. 19, no. 24, pp. 2056–2058, 1994.

[11] L. Lugiato, F. Prati, and M. Brambilla, *Nonlinear optical systems*. Cambridge University Press, 2015.

[12] H. Haken, "Analogy between higher instabilities in fluids and lasers," *Physics Letters A*, vol. 53, no. 1, pp. 77–78, 1975.

[13] J. Jani and P. Malkaj, "Numerical calculation of lyapunov exponents in various nonlinear chaotic systems," *International Journal of Scientific & Technology Research*, vol. 3, no. 7, pp. 87–90, 2014.

[14] M. Hanias, I. Giannis, and G. Tombras, "Chaotic operation by a single transistor circuit in the reverse active region," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 20, no. 1, p. 013105, 2010.

[15] R. K. Jamal, F. H. Ali, and F. A. Mutlak, "Studying the chaotic dynamics using rossler-chua systems combined with a semiconductor laser," *Iraqi Journal of Science*, pp. 2213–2221, 2021.

[16] C. R. Mirasso, P. Colet, and P. García-Fernández, "Synchronization of chaotic semiconductor lasers: Application to encoded communications," *IEEE Photonics Technology Letters*, vol. 8, no. 2, pp. 299–301, 1996.

[17] H. K. Khalil, "Adaptive output feedback control of nonlinear systems represented by input-output models," *IEEE transactions on Automatic Control*, vol. 41, no. 2, pp. 177–188, 1996.

[18] J. Ohtsubo, *Semiconductor lasers: stability, instability and chaos*, vol. 111. Springer, 2012.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research at all stages, from the problem formulation to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself
No funding was received for conducting this study.

Conflicts of Interest
The authors have no conflicts of interest to declare that are relevant to the content of this article.