Preliminary Structure of Quasi Optimal Algorithm for Optimization of Analog Circuits

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Abstract— An analog circuit design methodology based on applications of control theory is the basis for constructing an optimal or quasi-optimal design algorithm. The main criterion for identifying the required structure of the algorithm is the behavior of the Lyapunov function, which was decisive for the circuit optimization process. The characteristics of the Lyapunov function and its derivative are the basis for finding the optimal structure of the control vector that determines the structure of the algorithm. A block diagram of a quasi-optimal algorithm that implements the main ideas of the methodology is constructed, and the main characteristics of this algorithm are presented in comparison with the traditional approach.

Keywords—Time-optimal design algorithm, control theory, Lyapunov function, quasi optimal algorithm.

I. Introduction

THE computer time reduction when designing a large system is one source of overall improvement in design quality. This problem is of great importance, since it has many applications, for example, in the design of VLSI electronic circuits. Any traditional system design strategy includes two main parts: a mathematical model of the physical system that can be defined by the algebraic equations or differential-integral equations and an optimization procedure that achieves the optimum point of objective function of designing. Within the framework of this concept, it is possible to change the optimization strategy and use different models and different analysis methods, but at each stage of the circuit optimization process there is a fixed number of equations of the mathematical model and a fixed number of independent parameters of the optimization procedure.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]–[2]. Other approach to reduce the amount of computational required for both linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem.

The extension of the direct solution methods can be obtained by hierarchical decomposition and macro model representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. The optimization technique that is used to optimize and design circuits also has a very strong impact on the total CPU time.

Different techniques for analog circuit optimization can be classified in two main groups: deterministic optimization algorithms and stochastic search algorithms. Some drawbacks of classic deterministic optimization algorithms consist in requirement of a good initial point in space of parameters, unsatisfactory local minimum that can be reached, and very often in requirement of continuity and differentiability of the objective function. To overcome these problems some special methods were applied. For example, the method of determining the starting point of the process by centering [7], the use of geometric programming methods [8], which guarantee convergence to the global minimum, but, on the other hand, this requires a special formulation of the design equation for which additional difficulties accompany. Other approach based on the idea of space mapping technique [9]. The aim of space-mapping is to achieve a satisfactory solution with a minimal number of computationally expensive fine model evaluations by means of optimization of coarse model. This technology successfully used for optimization of microwave systems but there are no experience for solution other problems.

Stochastic search algorithms, especially evolutionary computation algorithms like genetic algorithms, differential evaluation, genetic programming, particle swarm optimization, etc. have been developed in recent years [10]–[15]. Genetic algorithms have been employed as optimization routines for analog circuits due to the ability to find a satisfactory solution. A special algorithm defined as a particle swarm optimization technique is one of the evolutionary algorithms and competes with genetic algorithms. This method is successfully used for electromagnetic problems and for optimization of microwave systems [16]–[17].

The practical aspects of deterministic methods were developed for the electronic circuits design with the different optimization criterions [18]. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers [19]–[20].

The above described ideas of system design as deterministic and stochastic can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The idea of refusing of laws of Kirchhoff at designing of electronic circuits was outspoken in [21] and realized practically in two systems of designing [22]–[23]. The most general approach was realized at development of the generalized methodology of process of optimization of electronic circuit, defined as the controllable dynamic system [24]. This system is determined by differential or numerical equations for state variables and a system of constraints which is defined by the mathematical model of electronic circuit. The main conception of this theory is the introduction of the special control vector, which generalizes the network optimization process and gives the possibility to control the design process to achieve an optimum of the cost function of the designing for the minimal computer time. This possibility appears due to an infinite number of the different strategies of designing that exist within this theory. By this approach the traditional strategy of designing is only a one representative of a large set of different designing strategies. As shown in [24] the potential computer time gain that can be obtained by this approach is increased when the size and complexity of the system increase.

II. Problem Formulation

We will consider that the process of designing of electronic circuit is formulated as a task of minimization of not negative special objective function $C(X)$. It is assumed that all aims of designing are realized it in the point of a minimum of objective function $C(X)$.

In case of differential form for optimization procedure the system of equations for state variables can be written in procedure of optimization [24] by this form:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i = 1,2,...,N, \tag{1}$$

where N is an incurrence of variables in the task of optimization of electronic circuit, $U = (u_1, u_2, ..., u_M)$ is a vector of control functions, $u_j \in \Omega$, $\Omega = \{0;1\}$. According to the developed methodology, the system of constraints of the procedure of optimization, being at sense the mathematical model of electronic circuit, can be defined by means of the next equations:

$$(1-u_j)g_j(X) = 0, \quad j = 1,2,...,M, \tag{2}$$

where M is a number of dependency variables, coincide with the number of nods of circuit.

Functions $f_i(X,U)$ can be determined by one or another method of optimization and, in particular, for the gradient method of optimization, the functions $f_i(X,U)$ are given as follows [24]:

$$f_i(X,U) = -\frac{\delta}{\delta x_i} F(X,U), \quad i = 1,2,...,K, \tag{3}$$

$$f_i(X,U) = -u_{i-K}\frac{\delta}{\delta x_i} F(X,U) + \frac{(1-u_{i-K})}{dt}\left[-x_i^{'} + \eta_i(X)\right], \tag{3'}$$

$$i = K+1, K+2, ..., N,$$

where K is a number of independent variables in the traditional definition of task (N=K+M), function $\eta_i(X)$, written in implicit form and it defines a current value of variable x_i from the system (2), $x_i^{'}$ is a previous variable value x_i. The function $F(X,U)$ is a generalized objective function of the design process and can be defined by the following additive expression [24]:

$$F(X,U) = C(X) + \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j g_j^2(X). \tag{4}$$

It is necessary to find the optimal behavior of the control functions u_j during the design process in order to minimize the overall design time. Functions $f_i(X,U)$ are piecewise continuous as functions of time, and the structure of these functions can be found by approximate methods of the control theory [25]–[26].

In such definition the task of optimization of circuit is formulated as a controllable dynamic system, which needs to bring to a point of equilibrium. Thus the time of transient for the system is associated with the time of designing of electronic circuit. In this case a basic instrument is a control vector of U which changes the internal structure of task of optimization of circuit.

III. Lyapunov Function

Dynamic properties of designing process were analyzed [27] on the basis of the entered function of Lyapunov of process of optimization. The presence of the correlation was marked between processor time of optimization of circuit and properties of function of Lyapunov of process of optimization. It was showed that the function of Lyapunov can be defined on the basis of the generalized objective function of process of optimization $F(X,U)$ by means of the next formula:

$$V(X,U) = [F(X,U)]^r, \tag{5}$$

where degree of $r > 0$.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (final point of the optimization procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [28]–[29] to minimize the time of the transition process by means of the special choice of the right hand part of the principal system of equations; in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector U selection to obtain a maximum speed of the Lyapunov function decreasing (maximum absolute value of the Lyapunov function time derivative $\dot{V} = dV/dt$).

The problem of stability of designing trajectory is related to the analysis of conduct of derivative at times from the function of Lyapunov $\dot{V}$. However, more informative is the normalized time derivative, which is determined by the following formula:

$$W = \dot{V}/V. \tag{6}$$

In this case we can compare the different design strategies by means of the function *W(t)* behavior and we can search the optimal position for the control vector switch points.

It was showed [30] that on the basis of analysis of conduct of derivative function of Lyapunov it is possible to find optimum switch points of control vector U that is a basis of quasi optimal strategy of designing and allowing to minimizing processor time of designing.

IV. Optimal Strategy Prediction

The optimal structure of the control vector U is the principal aim of the analysis of process of designing that is based on generalized methodology. All examples were analyzed for the continuous form of the optimization procedure (1). Functions *V(t)* and *W(t)* were the main objects of the analysis and its behavior has been analyzed during the design process. As shown in [30] the behavior of the functions *V(t)* and *W(t)* can define the total computer time for each design strategy. It is very interesting to analyze the behavior of the function *V(t)* for determine the optimal position of the switch points of the control vector. This function serves as a sensitive criterion to detect the optimal switching of the control vector U. The Lyapunov function *V(t)* for all examples was calculated by formula (5) for $r= 0.5$.

A. Example 1

The analysis of the process of designing for two-node passive nonlinear network in Fig. 1 is presented below.

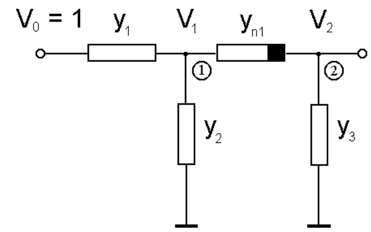

Fig. 1 Two-node nonlinear passive network

The nonlinear element is defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this network (2) includes two equations (*M*=2) and the optimization procedure (1) includes five equations. This network is characterized by two dependent parameters and the control vector includes two control functions: $U=(u_1, u_2)$. Structural basis includes four different strategies with corresponding control vector: (00), (01), (10), and (11). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector.

Taking into account the preliminary reasons about the optimal algorithm structure [24] we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (11) that corresponds to MTDS and the second part is defined by the control vector (00) that corresponds to TDS. So, the switching is realized between two strategies, (11) and (00).

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig. 2.

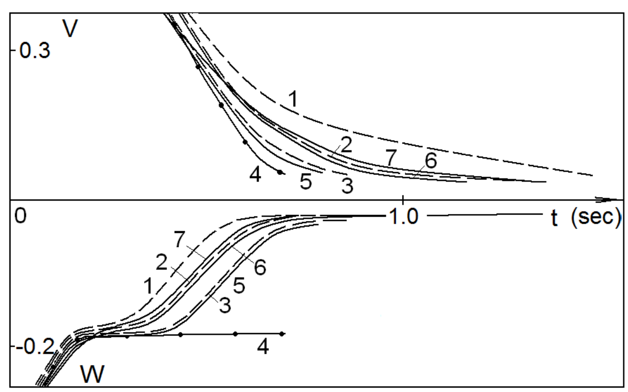

Fig. 2 Behavior of the functions *V(t)* and *W(t)* in the design process for seven different switch points (from 147 to 267)

The corresponding total iteration number and computer time are presented in Table 1.

The integration of the system (1) was realized by the constant integration step. The step for switch point increment is equal 20 to improve the identification of the difference between all curves.

Table 1. Iterations number and computer time for strategies with different switch points

The analysis shows that the optimal switch point corresponds to the step 207 (graph 4 with dots in Fig. 2). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

N	Switch point	Iterations number	Total design time (sec)
1	147	8319	0.221
2	167	6501	0.172
3	187	3697	0.096
4	207	2860	0.073
5	227	3383	0.087
6	247	5429	0.142
7	267	6682	0.175

The initial parts of *W(t)* dependencies of Fig. 2 are shown in Fig. 3 in large scale.

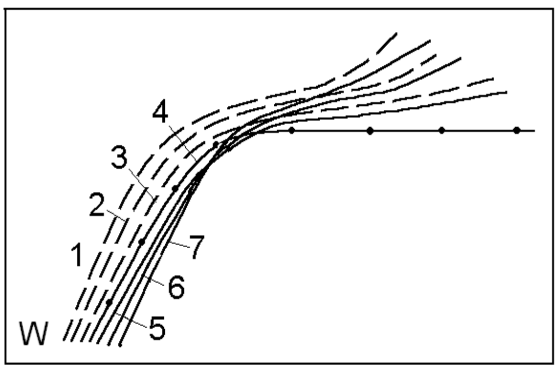

Fig. 3 Behavior of the functions *V(t)* and *W(t)* during the initial part of design process

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph *W(t)* for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below all of other graphs. This property serves as a principal criterion for the optimal switch point selection.

The function *W(t)* that corresponds to the optimal switch point has a maximum absolute value leading off the 340th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

B. Example 2

The next example corresponds to the two-stage transistor amplifier in Fig. 4.

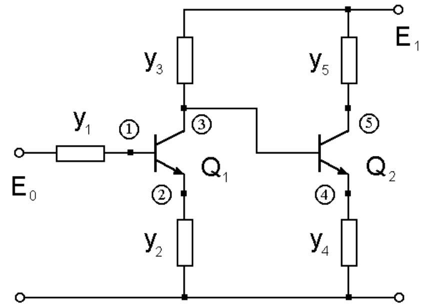

Fig. 4 Two-stage transistor amplifier

The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$. The model of this network (2) includes five equations (M=5) and the optimization procedure (5) includes ten equations. The total structural basis contains 32 different design strategies. The control vector includes five control functions: $U=(u_1,u_2,u_3,u_4,u_5)$. The Ebers-Moll static model of the transistor has been used [31]. Fig. 5 shows the behavior of the functions *V(t)* and *W(t)* for some design strategies with different switch points including the optimal one.

The data, which correspond to these graphs, are presented in Table 2. The integration of the system (1) was realized by the optimal variable integration step.

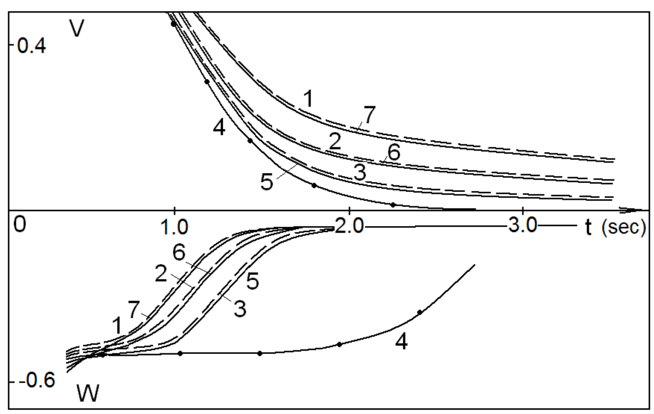

Fig. 5 Behavior of the functions *V(t)* and *W(t)* during the design process for seven switch points (from 7 to 13)

Table 2. Iterations number and computer time for strategies with different switch points

As for previous example, the design of two-transistor cell amplifier has been proposed as a combination of MTDS and TDS. In this case the quasi-optimal control vector includes two switch points. We changed the control vector from (11111) to (00000) and from (00000) to (11111). The consecutive change of the switch point was realized for the integration step's

N	Switch point 1	Switch point 2	Iterations number	Total design time (sec)
1	7	8	4900	9.912
2	8	9	4486	9.113
3	9	10	3785	7.691
4	10	11	1354	2.742
5	11	12	3618	7.341
6	12	13	4424	8.981
7	13	14	4882	9.893

number from 2 to 20. The behavior of the functions *V(t)* and *W(t)* for optimal switching steps and some steps close to optimal, confidently determines the optimal position of the switching points.

We observe a specific behavior of the function *W(t)* near the optimal switch point's position. Before the optimal switch point the function *W(t)* graphs are "parallel". Function *W(t)* has the maximum negative value for the optimal switch points. The graphs of the function *W(t)* that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. After the optimal points the graphs of the function *W(t)* intersect the graphs that correspond to the optimal switch point and before the optimal one. It means that we can detect the optimal position of the switch points during the initial design interval.

So, the structure of the optimal control vector i.e. the structure of the time optimal design strategy can be defined by means of the analysis of the relative time derivative of the Lyapunov function during the initial time interval of the design process.

We observe a behavior of the function *W(t)* near the optimal switch point's position similar the previous example. Function *W(t)* has the maximum negative value for the optimal switch points. The graphs of the function *W(t)* that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. After the optimal points the graphs of the function *W(t)* intersect the graphs that correspond to the optimal switch point. It means that we can detect the optimal position of the switch points during the initial design interval.

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V. Quasi Optimal Algorithm

We can mark two special strategies of optimization. The first strategy is determined by a control vector $U = (0,0,...,0)$. This strategy corresponds to the traditional approach for a circuit optimization, and in this case the system (2), being the system of constraints, must be solved on every step of procedure of optimization. We will name this strategy Traditional Strategy of Optimization (TSO). The second strategy is determined by a control vector $U = (1,1,...,1)$. In this case the system (2) disappears fully, but information about a circuit appears in the generalized objective function (4). We will name this strategy the Modified Traditional Strategy of Optimization (MTSO). Other trajectory corresponds to MTSO in a space of parameters. Flow-charts both TSO and MTSO is represented on Fig. 6 and Fig. 7.

Dynamic properties of designing process were analyzed in [30] on the basis of the entered function of Lyapunov of process of optimization. The presence of the correlation was marked between processor time of optimization of circuit and properties of function of Lyapunov of process of optimization. It was showed that the function of Lyapunov can be defined on the basis of the generalized objective function of process of optimization $F(X,U)$ by the formula (5).

It was showed [32] that on the basis of analysis of conduct of derivative function of Lyapunov it is possible to find optimum switch points of control vector *U* that is a basis of quasi optimal strategy of designing and allowing to minimizing processor time.

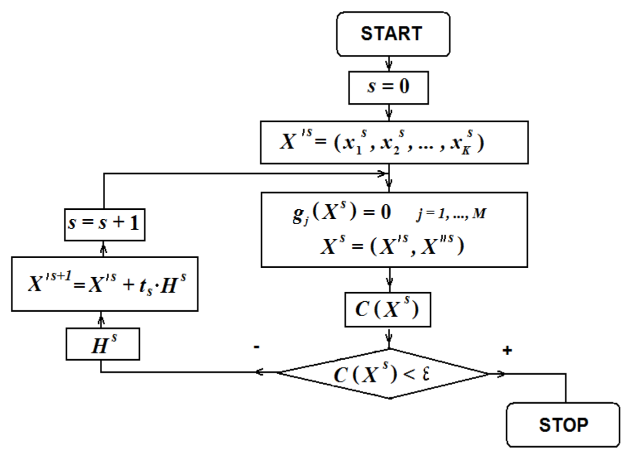

Fig. 6 Algorithm of TSO

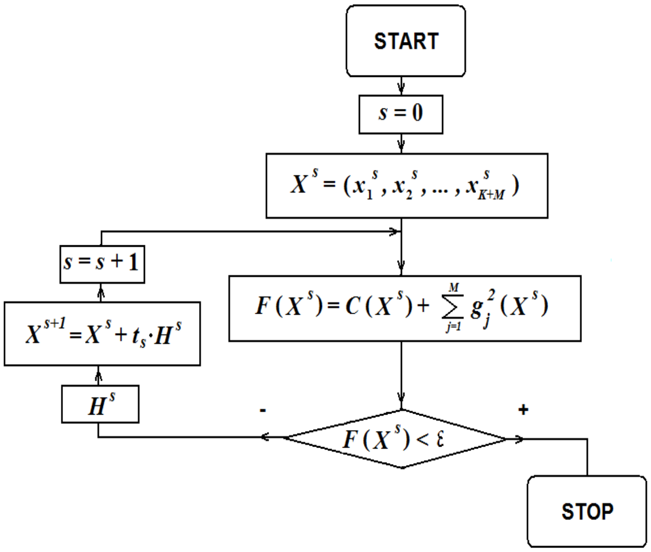

Fig. 7 Algorithm of MTSO

In Section IV, it was shown that, based on the analysis of the behavior of the derivative of the Lyapunov function, one can find the optimal switching points of the control vector *U*, which is the basis of the quasi-optimal design strategy and allows minimize the processor time. The flow-chart of algorithm is presented in Fig. 8. It is assumed that this variant of algorithm is based only on two switching of control vector.

We will define elementary structures which serve for the flow-chart construction of quasi optimal algorithm. There are two such structures.

We will define the first structure as a traditional strategy of optimization for one step of optimization procedure (TSO_1). This structure includes:

a) One step of procedure of unconstrained optimization is in space R^K of independent variables

$$X'^{s+1} = X'^{s} + t_s \cdot H^s, \tag{7}$$

where $X' \in R^K$, *H* is direction of decreasing of objective function $C(X)$, determined one or another method of descent.

b) Solution of the system of nonlinear equations that is the mathematical model of electronic circuit.

$$g_j(X) = 0, \quad j = 1,2,\ldots,M, \tag{8}$$

As a result of reproducing of both these steps we get the new values of all co-ordinates of vector *X*.

The second elementary structure is the modified traditional strategy of optimization for one step of optimization procedure ($MTSO_1$) and it includes one step of procedure of unconstrained optimization in space R^N of independent variables

$$X^{s+1} = X^{s} + t_s \cdot H^s, \tag{9}$$

where *H* is direction of decreasing of the generalized objective function $F(X)$.

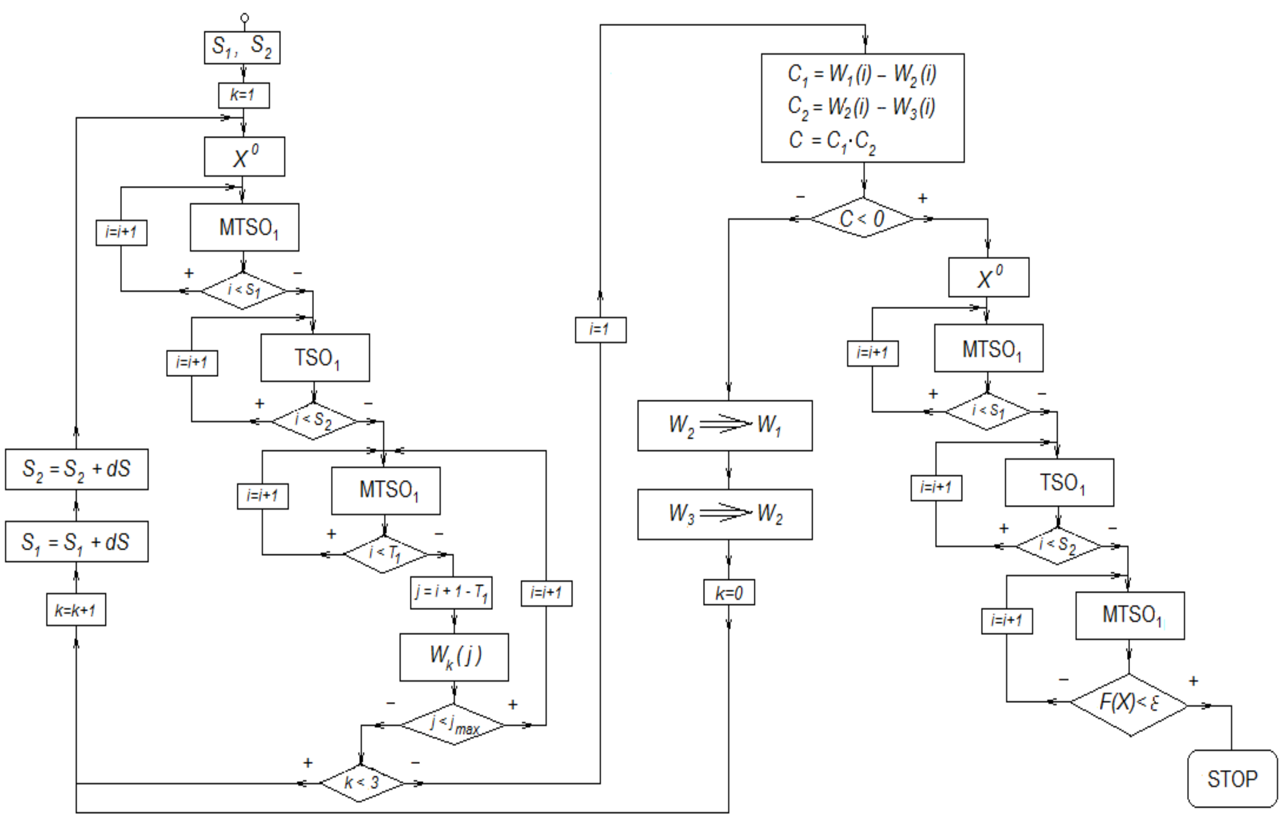

Fig.8 Quasi optimal algorithm

A vector of H is the function of objective function $F(X)$. As a result of reproducing of this step we get the new values of all co-ordinates of vector X.

On the basis of the ideas, presented in [32] and the determinations defined above, one of the possible variants of the quasi-optimal algorithm has been developed. It is clear that TSO_1 and $MTSO_1$ will implement their strategy on one step of the optimization method. In what follows, we will take into account that these strategies are defined as separate blocks and can be used in the main algorithm.

Initial switching points of control vector of S_1 and S_2 are set, where for example S_1=1 and S_2=S_1+n. The parameter of n can take on values 1, 2, ... An algorithm begins with MTSO. Then it is switching on TSO in the point of S_1. TSO is executed till switching on MTSO in the point of S_2. Then MTSO is further executed to the moment of T_1+ j_{max}, where T_1 is set a number in a range 10 – 40, and by a j_{max} number in a range 20 – 80. The values of derivative function of Lyapunov $W_1(t)$ are memorized from the step T_1 till the step T_1+ j_{max}. All calculation repeats oneself at the change of switch points on the step of dS. The values of relative derivative function of Lyapunov are again memorized, but it already $W_2(t)$. A calculation repeats oneself at a next change on the step of dS and $W_3(t)$ is determined. Then the obtained results are analyzed and the value of the main criterion C is produced.

VI. Basic Criterion of Principal Algorithm

We will consider the results of analysis of designing process of nonlinear circuit represented on Fig. 9. Ebers-Moll static model of transistor [31] is used for the analysis.

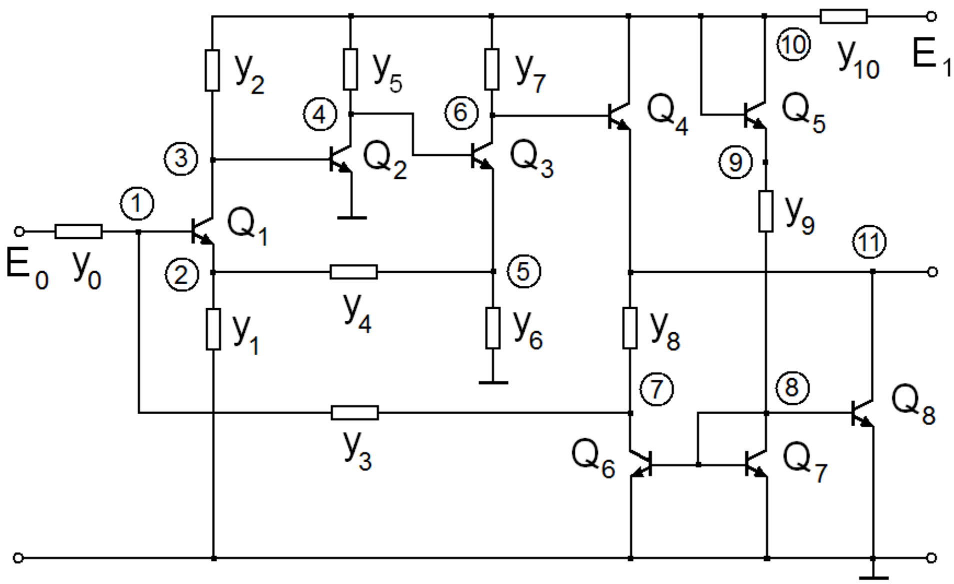

Fig.9 Operational amplifier

At designing of this amplifier, the quasi optimal strategy of designing has a time gain more than 1600 times comparing with traditional approach [33].

We will consider dependences of functions of $W(t)$ on the initial interval of designing process. These dependences are resulted in Fig. 10.

The curve 4 corresponds to the optimal point of switching. Three first curves correspond to the points of switching before the optimal point. Three last curves correspond to the points of switching after the optimal point. Distance between the dotted curves is increasing as approaching to the optimum switch point when the corresponding switch point lies before the optimum point. Opposite, a distance between continuous curves, which correspond of switching after an optimum point, is diminished as moving off from an optimum point. It is a good criterion to define an optimum switch point.

We will enter the function of P, determined as a difference of values of function of $W(t)$ for two nearby curves of Fig. 10 in certain moment of time of t.

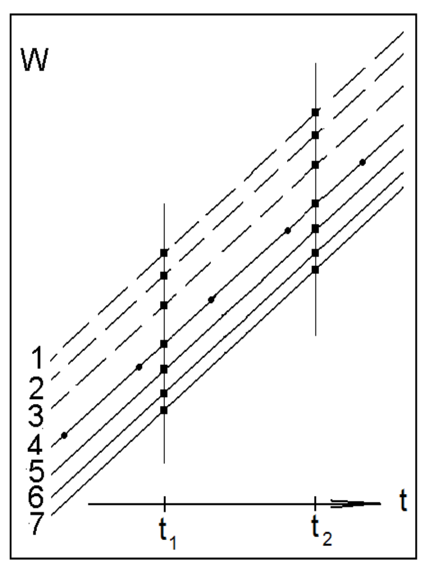

Fig.10 *W(t)* for some switching points

The function *P* has an argument *m*, which can be defined as the number of the corresponding curve (1, 2, …), i.e. *P* is the function of discrete argument. This function was built for two different values of time of t_1 and t_2, certain on a Fig. 10. Time of t_1 corresponds to 20th step of optimization procedure after a switching point and time of t_2 corresponds to 40th step. The corresponding curves (continuous) are represented in Fig. 11.

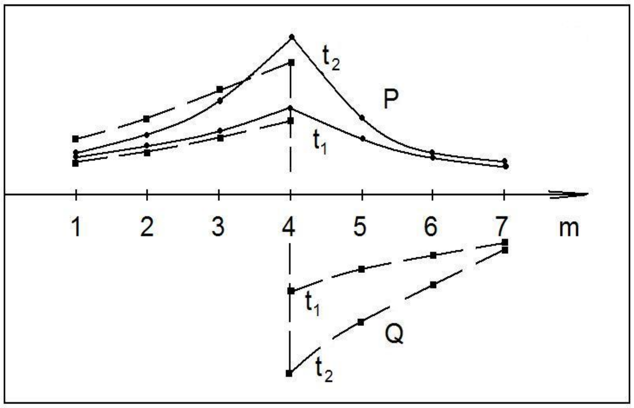

Fig.11 *P* and *Q* functions of curve number *m*

This function increases at approaching to the optimum point of switching of control vector. Attaining a maximum the function of *P* is diminished in further. We will define another function of *Q*, that is a discrete derivative of the function of *P* ($Q=P(m+1) -P(m)$) and which is built on the same figure. The conduct of function of *Q* is such, that it can afford basic for making of basic criterion of quasi optimal algorithm.

If we choose as a criterion of *C* the product of values of function of *Q* in two nearby points of the argument ($C=Q(m)xQ(m+1)$), a positive value *C* testifies that an optimum switch point is not yet attained, and the negative value corresponds to an optimum switch point. Thus, calculating the value of functions of *P* and *Q* and checking the value of criterion of *C* into an algorithm, we can exactly define an optimum switch point of control vector. The quasi optimal strategy will be realized after authentication of optimum points of switching. All these details are reflected in an algorithm that presented on a Fig. 8.

The developed quasi optimal algorithm give a time gain approximately on 25% less than quasi optimal strategy of designing, that allows to get the real time gain approximately in 1200 times as compared to TSO.

VII. CONCLUSION

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process is formulated as the controllable dynamic system. The Lyapunov function of the design process and its time derivative include the sufficient information to select more perspective design strategies from infinite set of the different design strategies that exist into the general design methodology. The special function *W(t)* was proposed to predict the structure of the time optimal design strategy. This function can be used as a main tool to construct the optimal sequence of the control vector switch points. This is the basis for the optimal design algorithm construction for the system design.

Additional expense of computer time associated with the search for optimal switching points of the control vector reduce the gain in time, which corresponds to the quasi-optimal strategy.

However, taking into account the fact that the quasi-optimal strategy makes it possible to obtain a total gain in processor time by several hundred times, the quasi-optimal algorithm, i.e. practical implementation of the quasi-optimal strategy gives a gain in time by the same order of magnitude.

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